

Correct Homework Sheet

$$\textcircled{1} \text{ a) } f(x) = 3x^{-3} + 3x^3 + 1$$

$$f'(x) = -6x^{-3} + 9x^2$$

$$\text{b) } y = \frac{4x+1}{6-2x^5}$$

$$y' = \frac{4(6-2x^5) - (-10x^4)(4x+1)}{(6-2x^5)^2}$$

$$\text{c) } f(x) = 3(2x^5 + x - 5)^{10}$$

$$f'(x) = 30(2x^5 + x - 5)^9 (10x^4 + 1)$$

$$\text{d) } h(x) = (x^2 - x)\sqrt{4 - 9x} = (x^2 - x)(4 - 9x)^{\frac{1}{2}}$$

$$h'(x) = (2x - 1)(4 - 9x)^{\frac{1}{2}} + (x^2 - x)\left(\frac{1}{2}\right)(4 - 9x)^{-\frac{1}{2}}(-9)$$

$$\textcircled{2} \text{ a) } y = \frac{2}{x} + \frac{3}{5x^3} - 6\sqrt{x} + \sqrt[3]{9x^3} - 8\pi$$

$$y = 2x^{-1} + \frac{3}{5}x^{-3} - 6x^{\frac{1}{2}} + (9x^3)^{\frac{1}{3}} - 8\pi$$

$$y' = -2x^{-2} - \frac{9}{5}x^{-4} - 3x^{-\frac{1}{2}} + \frac{1}{3}(9x^3)^{-\frac{2}{3}}(27x^2) - 0$$

$$\text{b) } y = \sqrt[3]{\frac{1-x^6}{2+(5x-1)^4}} = \left(\frac{1-x^6}{2+(5x-1)^4}\right)^{\frac{1}{3}}$$

$$y' = \frac{1}{3}\left(\frac{1-x^6}{2+(5x-1)^4}\right)^{-\frac{2}{3}} \left[\frac{-6x^5(2+(5x-1)^4) - (1-x^6)(4)(5x-1)^3(5)}{[2+(5x-1)^4]^2} \right]$$

$$\text{c) } g(x) = (x-5)^3(7x^5+2x)^9(4-2x^3)^5$$

$$g'(x) = 3(x-5)^2(1)(7x^5+2x)^9(4-2x^3)^5 + 9(7x^5+2x)^8(35x^4+2)(x-5)^3(4-2x^3)^5 + 5(4-2x^3)^4(4x^2)(x-5)^3(7x^5+2x)^9$$

$$\textcircled{3} \text{ d) } f(x) = \sqrt{25+4(2x-1)^4} = (25+4(2x-1)^4)^{\frac{1}{2}}$$

$$f'(x) = \frac{1}{2}(25+4(2x-1)^4)^{-\frac{1}{2}}(16(2x-1)^3(2))$$

Correct Homework Sheet

$$\textcircled{3} \text{ a) } y = \sqrt{x^2 - 5x\sqrt{2x^3 + 3}\sqrt{x}} = [x^2 - 5x(2x^3 + 3x^{\frac{1}{2}})^{\frac{1}{2}}]^{\frac{1}{2}}$$

$$y' = \frac{1}{2} [x^2 - 5x(2x^3 + 3x^{\frac{1}{2}})^{\frac{1}{2}}]^{-\frac{1}{2}} \left(2x - [5(2x^3 + 3x^{\frac{1}{2}})^{\frac{1}{2}} + 5x \left(\frac{1}{2} \right) (2x^3 + 3x^{\frac{1}{2}})^{-\frac{1}{2}} (6x^2 + \frac{3}{2}x^{-\frac{1}{2}})] \right)$$

Correct Homework Sheet

$$\textcircled{3} \text{ b) } f(x) = \frac{8x^3(12x^2 - 5x)^8}{2 - 3\sqrt[5]{(1 - 32x^{10})}} = \frac{8x^3(12x^2 - 5x)^8}{2 - 3(1 - 32x^{10})^{1/5}}$$

$$f'(x) = \frac{[24x^2(12x^2 - 5x)^8 + 8x^3(8)(12x^2 - 5x)^7(24x - 5)](2 - 3(1 - 32x^{10})^{1/5}) - [8x^3(12x^2 - 5x)^8] \left[-\frac{3}{5}(1 - 32x^{10})^{-4/5}(-320x^9) \right]}{[2 - 3(1 - 32x^{10})^{1/5}]^2}$$

Correct Homework Sheet

$$\textcircled{3} \text{ c) } f(x) = \frac{[x^5 - x\sqrt{4-x^2}]^6}{12x(5x^3-8)^7} = \frac{[x^5 - x(4-x^2)^{1/2}]^6}{12x^{1/2}(5x^3-8)^7} \quad \begin{matrix} f(x) \\ g(x) \end{matrix}$$

$$f'(x) = \frac{\overbrace{6[x^5 - x(4-x^2)^{1/2}]^5}^{f'(x)} \left[\overbrace{5x^4 - (1(4-x^2)^{1/2} + x(\frac{1}{2})(4-x^2)^{-1/2})(-2x))}^{g'(x)} \right] \left[\overbrace{12x^{1/2}(5x^3-8)^7}^{g(x)} \right] - \overbrace{[x^5 - x(4-x^2)^{1/2}]^6}^{f(x)} \left[\overbrace{6x^{-1/2}(5x^3-8)^7 + 12x^{1/2}(7)(5x^3-8)^6(15x^2)}^{g'(x)} \right]}{[12x^{1/2}(5x^3-8)^7]^2}$$

To be handed in today

$$f(x) = \sqrt[7]{\frac{9+16x^4}{[4x^5(3x^8+8x-2)]^5}} = \left[\frac{9+16x^4}{[4x^5(3x^8+8x-2)]^5} \right]^{1/7}$$

$$f'(x) = \frac{1}{7} \left[\frac{9+16x^4}{[4x^5(3x^8+8x-2)]^5} \right]^{-6/7} \left[\frac{64x^4(4x^5(3x^8+8x-2))^5 - (9+16x^4)(5)[4x^5(3x^8+8x-2)]^4[(20x^4)(3x^8+8x-2) + 4x^5(24x^7+8)]}{[4x^5(3x^8+8x-2)]^{10}} \right]$$

Review

$$\textcircled{1} a) f(x) = \sqrt{x-5} \quad f(x+h) = \sqrt{x+h-5}$$

$$f'(x) = \lim_{h \rightarrow 0} \frac{(\sqrt{x+h-5} - \sqrt{x-5})}{h} \cdot \frac{(\sqrt{x+h-5} + \sqrt{x-5})}{(\sqrt{x+h-5} + \sqrt{x-5})}$$

$$f'(x) = \lim_{h \rightarrow 0} \frac{\cancel{x+h-5} - \cancel{(x-5)}}{h(\sqrt{x+h-5} + \sqrt{x-5})}$$

$$f'(x) = \lim_{h \rightarrow 0} \frac{\cancel{h}}{\cancel{h}(\sqrt{x+h-5} + \sqrt{x-5})} = \boxed{\frac{1}{2\sqrt{x-5}}}$$

$$\textcircled{2} b) f(x) = \frac{3}{\sqrt{x}} = \frac{3}{x^{1/2}} = 3x^{-1/2}$$

$$f'(x) = -\frac{3}{2} x^{-3/2} = \left(\frac{-3}{2x^{3/2}} \right) = \frac{-3}{2\sqrt{x^3}}$$

Review

$$\textcircled{1} \text{ b) } f(x) = \frac{2x-2}{x+3} \quad f(x+h) = \frac{2(x+h)-2}{(x+h)+3} = \frac{2x+2h-2}{x+h+3}$$

$$f'(x) = \lim_{h \rightarrow 0} \frac{2x+2h-2}{x+h+3} - \frac{2x-2}{x+3} \quad \text{CO: } (x+3)(x+h+3)$$

$$f'(x) = \lim_{h \rightarrow 0} \frac{(x+3)(2x+2h-2) - (2x-2)(x+h+3)}{h(x+3)(x+h+3)}$$

$$f'(x) = \lim_{h \rightarrow 0} \frac{\cancel{2x^2} + \cancel{2xh} - \cancel{2x} + \cancel{6x} + \cancel{6h} - \cancel{6} - (\cancel{2x^2} + \cancel{2xh} + \cancel{6x} - \cancel{2x} - \cancel{2h} - \cancel{6})}{h(x+3)(x+h+3)}$$

$$f'(x) = \lim_{h \rightarrow 0} \frac{8h}{h(x+3)(x+h+3)} = \boxed{\frac{8}{(x+3)^2}}$$

Review:

$$\textcircled{a}) f(x) = 3x^2 + 5x - 2$$

$$f'(x) = 6x + 5$$

$$\textcircled{c}) f(x) = 2x^4 + \sqrt{x} = 2x^4 + x^{1/2}$$

$$f'(x) = 8x^3 + \frac{1}{2}x^{-1/2}$$

$$f'(x) = 8x^3 + \frac{1}{2x^{1/2}}$$

$$\textcircled{d}) f(x) = \sqrt[3]{x^2} = x^{2/3}$$

$$f'(x) = \frac{2}{3}x^{-1/3}$$

$$f'(x) = \frac{2}{3x^{1/3}}$$

Review:

$$\textcircled{3} \text{ a) } y = (3x^2 - 2)(4x + 5)$$

$$y' = (6x)(4x + 5) + 4(3x^2 - 2)$$

$$y' = 24x^2 + 30x + 12x^2 - 8$$

$$y' = 36x^2 + 30x - 8$$

$$\textcircled{3} \text{ b) } g(x) = (x^2 - 5x + 2)(4x + 1)$$

$$g'(x) = (2x - 5)(4x + 1) + 4(x^2 - 5x + 2)$$

$$g'(x) = \underline{8x^2} + \underline{2x} - \underline{20x} - 5 + \underline{4x^2} - \underline{20x} + 8$$

$$g'(x) = 12x^2 - 38x + 3$$

Review:

$$\textcircled{4} \text{ a) } f(x) = \frac{2x^2 + 3}{3x - 2}$$

$$f'(x) = \frac{4x(3x-2) - 3(2x^2+3)}{(3x-2)^2}$$

$$f'(x) = \frac{12x^2 - 8x - 6x^2 - 9}{(3x-2)^2}$$

$$f'(x) = \frac{6x^2 - 8x - 9}{(3x-2)^2}$$

$$\textcircled{4} \text{ b) } y = \frac{\sqrt{x}}{3+x^2} = \frac{x^{1/2}}{3+x^2} \quad \begin{matrix} f(x) \\ g(x) \end{matrix}$$

$$y' = \frac{\frac{1}{2}x^{-1/2}(3+x^2) - (2x)(x^{1/2})}{(3+x^2)^2}$$

$$y' = \frac{\cancel{2x^{1/2}}(3+x^2)^{2x^{1/2}} - \cancel{2x^{1/2}} \cdot 2x^{3/2}}{(3+x^2)^2 \cdot 2x^{1/2}} \quad \text{CD: } 2x^{1/2}$$

$$y' = \frac{3+x^2 - 4x^2}{2\sqrt{x}(3+x^2)^2} = \frac{3-3x^2}{2\sqrt{x}(3+x^2)^2}$$

Review

⑤ Find the equation of the tangent line to the curve $y = (x^2 - 3)^8$ at $x = 2 \leftarrow x_1$.

(i) Find y :

$$y = (2^2 - 3)^8$$

$$y = (4 - 3)^8$$

$$y = 1^8$$

$$y = 1 \leftarrow y_1$$

(ii) Find y'

$$y = (x^2 - 3)^8$$

$$y' = 8(x^2 - 3)^7 (2x)$$

$$y' = 16x(x^2 - 3)^7$$

(iii) Find m ($y'(a)$)

$$y' = 16(2)(2^2 - 3)^7$$

$$y' = 32(1)$$

$$y' = 32 \leftarrow m$$

(iv) Find equation:

$$y - y_1 = m(x - x_1)$$

$$y - 1 = 32(x - 2)$$

$$y - 1 = 32x - 64$$

$$\boxed{y = 32x - 63} \quad \text{or} \quad 32x - y - 63 = 0$$

Review:

$$\textcircled{6} \text{ a) } f(x) = 3(2x^2 - 4)^4$$

$$f'(x) = 12(2x^2 - 4)^3(4x)$$

$$f'(x) = 48x(2x^2 - 4)^3$$

$$\text{b) } y = \frac{16}{\sqrt{x-1}} = \frac{16}{(x-1)^{1/2}} = 16(x-1)^{-1/2}$$

$$y' = -8(x-1)^{-3/2}(1)$$

$$y' = -8(x-1)^{-3/2}$$

$$y' = \frac{-8}{(x-1)^{3/2}}$$

Review:

$$\textcircled{1} \text{ a) } f(x) = \left(\frac{2x+1}{x-1} \right)^5 = \frac{(2x+1)^5}{(x-1)^5}$$

$$f'(x) = 5 \left(\frac{2x+1}{x-1} \right)^4 \left[\frac{2(x-1) - 1(2x+1)}{(x-1)^2} \right]$$

$$f'(x) = 5 \frac{(2x+1)^4}{(x-1)^4} \left[\frac{\cancel{2x} - 2 - \cancel{2x} - 1}{(x-1)^2} \right]$$

$$f'(x) = 5 \cdot \frac{(2x+1)^4}{(x-1)^4} \cdot \frac{-3}{(x-1)^2} = \boxed{\frac{-15(2x+1)^4}{(x-1)^6}}$$

Review

$$\textcircled{7} \text{ b) } y = (x^2 - 1)^3 (3x - 2)^2$$

$$y' = 3(x^2 - 1)^2 (2x)(3x - 2)^2 + (x^2 - 1)^3 (2)(3x - 2)(3)$$

$$y' = 6x(x^2 - 1)^2 (3x - 2)^2 + 6(x^2 - 1)^3 (3x - 2)$$

$$y' = 6(x^2 - 1)^2 (3x - 2) \left[\overset{3x^2 - 2x}{x(3x - 2)} + \overset{x^2 - 1}{(x^2 - 1)} \right]$$

$$y' = 6(x^2 - 1)^2 (3x - 2) (4x^2 - 2x - 1)$$

Review

$$\textcircled{1} \Rightarrow y = \frac{(2x+1)^2}{(x^4-x+1)^2} f(x) g(x)$$

$$\frac{f'(x)g(x) - f(x)g'(x)}{[g(x)]^2}$$

$$y' = \frac{2(2x+1)'(2)(x^4-x+1)^0 - (2x+1)^2(2)(x^4-x+1)'(4x^3-1)}{(x^4-x+1)^4}$$

$$y' = \frac{4(2x+1)(x^4-x+1)^0 - 2(4x^3-1)(2x+1)^2(x^4-x+1)}{(x^4-x+1)^4}$$

$$y' = \frac{2(2x+1)(x^4-x+1) \left[\begin{array}{l} 2x^4 - 2x + 2 \\ - (8x^4 + 4x^3 - 2x - 1) \end{array} \right] - (4x^3-1)(2x+1)^2}{(x^4-x+1)^4}$$

$$y' = \frac{2(2x+1)(\cancel{x^4-x+1})(-6x^4-4x^3+3)}{(x^4-x+1)^{\cancel{4}3}}$$

$$y' = \frac{-2(2x+1)(6x^4+4x^3-3)}{(x^4-x+1)^3}$$

Example 1

Let $F(x) = f(g(x))$ $F'(x) = f'(g(x)) \cdot g'(x)$

If $f(2) = 3$, $f'(2) = \underline{5}$, $g(1) = \underline{2}$ and $g'(1) = \underline{4}$ find $F'(1)$.

$$\begin{aligned} F'(1) &= f'(g(1)) \cdot g'(1) \\ &= \underline{f'(2)} \cdot \underline{g'(1)} \\ &= 5 \cdot 4 \\ &= 20 \end{aligned}$$

Question #6 ex. 2.4

Given Find $(fg)'(2) = f'(2)g(2) + f(2)g'(2)$

$$\begin{aligned} f(2) &= 3 && = (5)(-1) + (3)(-4) \\ f'(2) &= 5 && = -5 - 12 \\ g(2) &= -1 && = -5 - 12 \\ g'(2) &= -4 && = -17 \end{aligned}$$

Find $\left(\frac{f}{g}\right)'(2) = \frac{f'(2)g(2) - f(2)g'(2)}{(g(2))^2}$

$$\begin{aligned} &= \frac{(5)(-1) - (3)(-4)}{(-1)^2} \\ &= \frac{-5 + 12}{1} \\ &= 7 \end{aligned}$$

Example 2

If $y = u^{10} + u^5 + 2$, where $u = 1 - 3x^2$, find $\left. \frac{dy}{dx} \right|_{x=1}$

$$\frac{dy}{du} = 10u^9 + 5u^4 \quad \frac{du}{dx} = -6x$$

when $x = 1$
 $u = 1 - 3(1)^2 = -2$

$$\frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx}$$

$$\begin{aligned} \left. \frac{dy}{dx} \right|_{x=1} &= (10u^9 + 5u^4)(-6x) \\ &= (10(-2)^9 + 5(-2)^4)(-6(1)) \\ &= (-5120 + 80)(-6) \\ &= (-5040)(-6) \\ &= 30240 \end{aligned}$$

If $y = u^{10} + u^5 + 2$, where $u = 1 - 3x^2$, find $\left. \frac{dy}{dx} \right|_{x=1}$

$$y = (1 - 3x^2)^{10} + (1 - 3x^2)^5 + 2$$

$$\frac{dy}{dx} = 10(1 - 3x^2)^9(-6x) + 5(1 - 3x^2)^4(-6x)$$

$$\left. \frac{dy}{dx} \right|_{x=1} = 10(-2)^9(-6) + 5(-2)^4(-6)$$

$$= 30720 - 480$$

$$= 30240$$

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3-10

③ Find $\left. \frac{dy}{dx} \right|_{x=4}$

if $y = u^2 - 2u^5$

$$\frac{dy}{du} = 2u - 10u^4$$

and $u = x - \sqrt{x}$

$$\frac{du}{dx} = 1 - \frac{1}{2}x^{-1/2}$$

$$\frac{du}{dx} = 1 - \frac{1}{2\sqrt{x}}$$

$$\left. \frac{dy}{dx} \right|_{x=4} = \left[\frac{dy}{du} \right] \left[\frac{du}{dx} \right]$$

$$= [2u - 10u^4] \left[1 - \frac{1}{2\sqrt{x}} \right]$$

$$= [2(2) - 10(2)^4] \left[1 - \frac{1}{2\sqrt{4}} \right]$$

$$= (-156) \left(\frac{3}{4} \right)$$

$$= \boxed{-117}$$

when $x=4$ $u=2$

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④ Find $\left. \frac{dy}{dt} \right|_{t=1} \rightarrow$ when $t=1$: $r = \frac{1+1}{2(1)+1} = \frac{2}{3}$

$$y = \sqrt{1+r^2} = (1+r^2)^{1/2} \quad \left| \quad r = \frac{t+1}{2t+1} \right.$$

$$\frac{dy}{dr} = \frac{1}{2} (1+r^2)^{-1/2} (2r) \quad \left| \quad \frac{dr}{dt} = \frac{1(\cancel{2t+1}) - 2(\cancel{t+1})}{(2t+1)^2} \right.$$

$$\frac{dy}{dr} = \frac{\cancel{2}r}{2(1+r^2)^{1/2}} = \frac{r}{\sqrt{1+r^2}} \quad \left| \quad \frac{dr}{dt} = \frac{2t+1-2t-2}{(2t+1)^2} = \frac{-1}{(2t+1)^2} \right.$$

$$\left. \frac{dy}{dt} \right|_{t=1} = \left[\frac{r}{\sqrt{1+r^2}} \right] \left[\frac{-1}{(2t+1)^2} \right]$$

$$= \left[\frac{2/3}{\sqrt{1+(2/3)^2}} \right] \left[\frac{-1}{(2(1)+1)^2} \right]$$

$$= \left[\frac{2/3}{\sqrt{1+4/9}} \right] \left[\frac{-1}{9} \right]$$

$$= \left[\frac{2/3}{\sqrt{13/9}} \right] \left[\frac{-1}{9} \right]$$

$$= \left[\frac{2/3}{\sqrt{13}/3} \right] \left[\frac{-1}{9} \right]$$

$$= \left[\frac{2}{\cancel{3}} \cdot \frac{\cancel{3}}{\sqrt{13}} \right] \left[\frac{-1}{9} \right]$$

$$= \frac{-2}{9\sqrt{13}}$$

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⑥ e) $F(x) = \frac{x}{\sqrt{2x+3}} = x(2x+3)^{-1/2}$

$$F'(x) = 1(2x+3)^{-1/2} + x \left(-\frac{1}{2}\right)(2x+3)^{-3/2} (2)$$

$$F'(x) = (2x+3)^{-1/2} - x(2x+3)^{-3/2}$$

$$F'(x) = (2x+3)^{-3/2} [(2x+3) - x]$$

$$F'(x) = \frac{x+3}{(2x+3)^{3/2}} = \frac{x+3}{\sqrt{(2x+3)^3}}$$

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⑨ $F(x) = f(g(x))$ ← Composite function

Given:

$$\underline{g(a) = 4}$$

$$\underline{g'(a) = 3}$$

$$\underline{f'(4) = 5}$$

$$f(a) = -1$$

$$f'(a) = 2$$

$$F'(x) = f'(g(x)) \cdot g'(x)$$

$$F'(a) = f'(g(a)) \cdot g'(a)$$

$$F'(a) = \underline{f'(4)} \cdot \underline{g'(a)}$$

$$F'(a) = 5 \cdot 3$$

$$F'(a) = 15$$

Find $F'(a)$