Questions from Homework

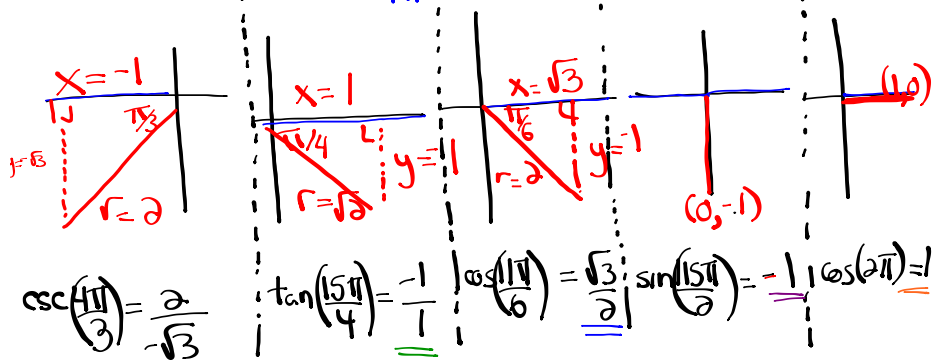
$-\frac{13\pi}{6}+\frac{24\pi}{6}=\frac{11\pi}{6}$ $\quad -14\pi+16\pi=2\pi$

⑤ $\csc^2\left(\frac{4\pi}{3}\right)\tan\left(\frac{15\pi}{4}\right)+\cos\left(-\frac{13\pi}{6}\right)-\sin\left(\frac{115\pi}{2}\right)+\cos(-14\pi)$

$\csc^2\left(\frac{4\pi}{3}\right)\tan\left(\frac{15\pi}{4}\right)+\cos\left(\frac{11\pi}{6}\right)-\sin\left(\frac{115\pi}{2}\right)+\cos(2\pi)$

$\frac{3\pi}{3}, \frac{4\pi}{3}, \frac{5\pi}{3}, \frac{14\pi}{4}, \frac{15\pi}{4}, \frac{16\pi}{4}, \frac{10\pi}{6}, \frac{11\pi}{6}, \frac{12\pi}{6}, \frac{114\pi}{2}, \frac{115\pi}{2}, \frac{116\pi}{2}$ $\quad 2\pi$

$1\pi \quad 4\pi \quad 2\pi \quad 57\pi \quad 58\pi$



$\csc\left(\frac{4\pi}{3}\right)=\frac{2}{-\sqrt{3}}$ $\quad \tan\left(\frac{15\pi}{4}\right)=\frac{-1}{1}$ $\quad \cos\left(\frac{11\pi}{6}\right)=\frac{\sqrt{3}}{2}$ $\quad \sin\left(\frac{115\pi}{2}\right)=-1$ $\quad \cos(2\pi)=1$

$\csc^2\left(\frac{4\pi}{3}\right)\tan\left(\frac{15\pi}{4}\right)+\cos\left(\frac{11\pi}{6}\right)-\sin\left(\frac{115\pi}{2}\right)+\cos(2\pi)$

$\left(-\frac{2}{\sqrt{3}}\right)^2(-1)+\left(\frac{\sqrt{3}}{2}\right)-(-1)+(1)$

$\left(\frac{4}{3}\right)(-1)+\frac{\sqrt{3}}{2}+1+1$

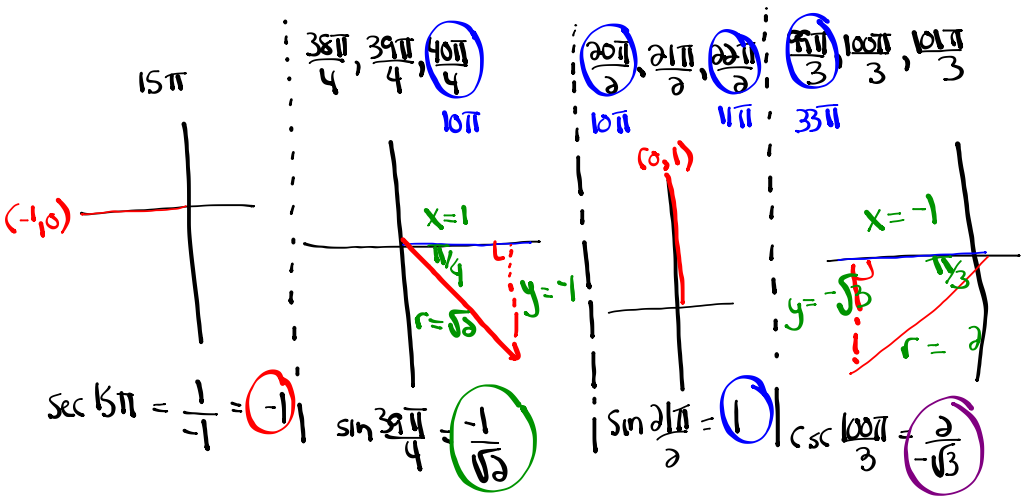
$-\frac{4}{3}+\frac{\sqrt{3}}{2}+\frac{2}{1}$

$-\frac{8}{6}+\frac{3\sqrt{3}}{6}+\frac{12}{6}$

$\boxed{\frac{4+3\sqrt{3}}{6}}$ or $\frac{3\sqrt{3}+4}{6}$

Questions from Homework

⑥ $\sec 15\pi + \sqrt{2}\sin\frac{39\pi}{4}\sin\frac{21\pi}{2} - \csc^2\frac{100\pi}{3}$



$\sec 15\pi = \frac{1}{-1} = -1$ $\quad \sin\frac{39\pi}{4} = \frac{-1}{\sqrt{2}}$ $\quad \sin\frac{21\pi}{2} = 1$ $\quad \csc\frac{100\pi}{3} = \frac{2}{-\sqrt{3}}$

$$\sec 15\pi + \sqrt{2}\sin\frac{39\pi}{4}\sin\frac{21\pi}{2} - \csc^2\frac{100\pi}{3}$$

$$(-1) + \sqrt{2}\left(\frac{-1}{\sqrt{2}}\right)(1) - \left(\frac{2}{-\sqrt{3}}\right)^2$$

$$-1 + \sqrt{2}\left(\frac{-1}{\sqrt{2}}\right)(1) - \frac{4}{3}$$

$$-1 - \frac{\sqrt{2}}{\sqrt{2}} - \frac{4}{3}$$

$$-1 - 1 - \frac{4}{3}$$

$$-\frac{3}{3} - \frac{3}{3} - \frac{4}{3}$$

$$\boxed{-\frac{10}{3}}$$

Questions from Homework

⑧ $\dfrac{\tan\left(\frac{\pi}{4}\right)+\sec\left(\frac{301\pi}{3}\right)}{\csc\left(\frac{25\pi}{2}\right)+\cot\left(\frac{5\pi}{6}\right)}$

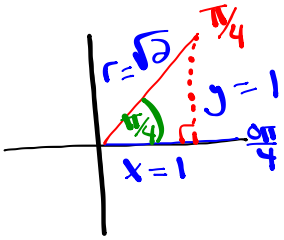
* $-\frac{15\pi}{4}+\frac{4\pi}{1}$ | $-\frac{31\pi}{6}+\frac{6\pi}{1}$

$-\frac{15\pi}{4}+\frac{16\pi}{4}$ | $-\frac{31\pi}{6}+\frac{36\pi}{6}$

$\frac{\pi}{4}$ | $\frac{5\pi}{6}$

① $\frac{9\pi}{4}, \frac{\pi}{4}, \frac{2\pi}{4}$

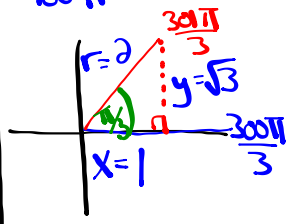
0



$\tan\frac{\pi}{4}=\frac{1}{1}=1$

② $\frac{3001\pi}{3}, \frac{301\pi}{3}, \frac{302\pi}{3}$

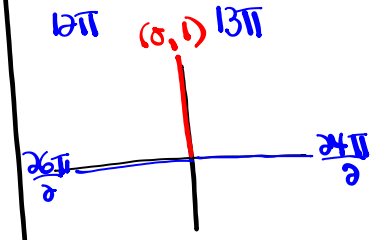
100π



$\sec\frac{301\pi}{3}=\frac{2}{1}=2$

③ $\frac{24\pi}{2}, \frac{25\pi}{2}, \frac{26\pi}{2}$

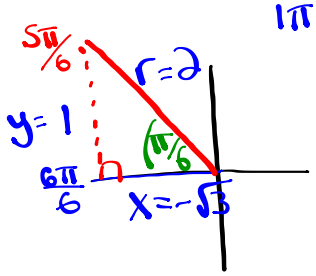
12π



$\csc\frac{25\pi}{2}=\frac{1}{1}=1$

④ $\frac{4\pi}{6}, \frac{5\pi}{6}, \frac{6\pi}{6}$

1π



$\cot\frac{5\pi}{6}=\frac{-\sqrt{3}}{1}=-\sqrt{3}$

$\dfrac{\tan\left(\frac{\pi}{4}\right)+\sec\left(\frac{301\pi}{3}\right)}{\csc\left(\frac{25\pi}{2}\right)+\cot\left(\frac{5\pi}{6}\right)}$

$\dfrac{1+2}{1-\sqrt{3}}$

$\dfrac{3(1+\sqrt{3})}{(1-\sqrt{3})(1+\sqrt{3})}$

$\dfrac{3+3\sqrt{3}}{1+\sqrt{3}-\sqrt{3}-3}$

$\dfrac{3+3\sqrt{3}}{-2}$ or $\boxed{\dfrac{-3-3\sqrt{3}}{2}}$

Introduction to Trigonometric Equations

trigonometric equation

- an equation involving trigonometric ratios

Focus on...

- algebraically solving first-degree and second-degree trigonometric equations in radians and in degrees
- verifying that a specific value is a solution to a trigonometric equation
- identifying exact and approximate solutions of a trigonometric equation in a restricted domain
- determining the general solution of a trigonometric equation

Did You Know?

In equations, mathematicians often use the notation $\cos^2 \theta$. This means the same as $(\cos \theta)^2$.

Let's start with basic LINEAR trigonometric equations...

...Pre-Calculus 110

Solve: (Approx) $\sin\theta = 0.9659\ ,\ -360° < \theta < 720°$

Domain in degrees

- Reference angle?
- Which quadrants?
- Any co-terminal angles acceptable?

- If the domain is in degrees, give solutions in degrees.
- If the domain is in radians, give solutions in radians.

$\sin\theta = 0.9659$

① Find $\bar{\theta}$

② where is $\sin\theta > 0$

S	A
T	C

$\bar{\theta} = \sin^{-1}(0.9659)$

③ Find θ

$\bar{\theta} = 75°$

Q1	Q2
$\theta = \bar{\theta}$	$\theta = 180° - \bar{\theta}$
$\theta = 75°$	$\theta = 180° - 75° = 105°$
$\theta = 75° - 360° = -285°$	$\theta = 105° - 360° = -255°$
$\theta = 75° + 360° = 435°$	$\theta = 105° + 360° = 465°$

Solve: $\sec\theta = -1.3054\ ,\ -2\pi \le \theta \le 2\pi$ (Approx.) (Radians) $-6.28 \le \theta \le 6.28$

$\cos\theta = \frac{1}{-1.3054}$

$\cos\theta = -0.7660$

① Find $\bar{\theta}$

② where is $\cos\theta < 0$

S	A
T	C

$\bar{\theta} = \cos^{-1}(0.7660)$

$\bar{\theta} = 0.7$ rads

ignore negative

③ Find θ

Q2	Q3
$\theta = \pi - \bar{\theta}$	$\theta = \pi + \bar{\theta}$
$\theta = 3.14 - 0.7 = 2.44$	$\theta = 3.14 + 0.7 = 3.84$
$\theta = 2.44 - 6.28 = -3.84$	$\theta = 3.84 - 6.28 = -2.44$

Warm-up

Ex: $\tan\theta = -0.8524$ (Approx.), $-360^\circ \le \theta \le 360^\circ$ (Degrees)

① Find $\bar{\theta}$

$\bar{\theta} = \tan^{-1}(0.8524)$ ← ignore negative

$\bar{\theta} = 40.4^\circ$

② Where is $\tan\theta < 0$

S ✓	A
T	C ✓

③ Find θ

Q2	Q4
$\theta = 180^\circ - \bar{\theta}$	$\theta = 360^\circ - \bar{\theta}$
$\theta = 180^\circ - 40.4^\circ = 139.6^\circ$	$\theta = 360^\circ - 40.4^\circ = 319.6^\circ$
$\theta = 139.6^\circ - 360^\circ = -219.4^\circ$	$\theta = 319.6^\circ - 360^\circ = -40.4^\circ$

(Exact) $\frac{-1}{\sqrt{2}} = -0.7071$

Ex. $\sqrt{2}\cos\theta + 1 = 0, \quad -360° \le \theta \le 720°$ (Degrees)

$\frac{\sqrt{2}\cos\theta}{\sqrt{2}} = \frac{-1}{\sqrt{2}}$

$\cos\theta = \frac{-1}{\sqrt{2}}$

① Find $\bar{\theta}$

$\bar{\theta} = \cos^{-1}\left(\frac{1}{\sqrt{2}}\right)$ (Triangle #1) $\cos\theta = \frac{a}{h} = \frac{x}{r}$

$\bar{\theta} = 45°$

② Where is $\cos\theta < 0$

S	A
T	C

③ Find θ

Q2	Q3
$\theta = 180° - \bar{\theta}$	$\theta = 180° + \bar{\theta}$
$\theta = 180° - 45° = 135°$	$\theta = 180° + 45° = 225°$
$\theta = 135° - 360° = -225°$	$\theta = 225° - 360° = -135°$
$\theta = 135° + 360° = 495°$	$\theta = 225° + 360° = 585°$

(Exact)

Ex. $\sin x + 1 = 0, \; -2\pi \le x \le 4\pi$ (Radians) $-\frac{4\pi}{2} \le x \le \frac{8\pi}{2}$

$\sin x = -1$

① Find x (Unit Circle)

$x = \frac{3\pi}{2}$

$x = \frac{3\pi}{2} - \frac{2\pi}{1} = \frac{3\pi}{2} - \frac{4\pi}{2} = -\frac{\pi}{2}$

$x = \frac{3\pi}{2} + \frac{2\pi}{1} = \frac{3\pi}{2} + \frac{4\pi}{2} = \frac{7\pi}{2}$

Your Turn

Solve each trigonometric equation in the specified domain.

a) $3\cos\theta - 1 = \cos\theta + 1,\ -2\pi \le \theta \le 2\pi$

b) $4\sec x + 8 = 0,\ 0° \le x < 360°$

a) $3\cos\theta - 1 = \cos\theta + 1$ (Exact), $-2\pi \le \theta \le 2\pi$ (Radians)

$3\cos\theta - \cos\theta = 1 + 1$

$\frac{2\cos\theta}{2} = \frac{2}{2}$

$\cos\theta = 1$

① Find θ (unit circle)

$\theta = 0, 2\pi$

$\theta = 0 - 2\pi = -2\pi$

---

b) $4\sec x + 8 = 0$, $0° \le x < 360°$

$4\sec x = -8$

$\sec x = -2$

$\cos x = \frac{-1}{2}$

① Find $\bar{x}$

$\bar{x} = \cos^{-1}\left(\frac{1}{2}\right)$ (Triangle #2)

$\bar{x} = 60°$

② where is $\cos x < 0$

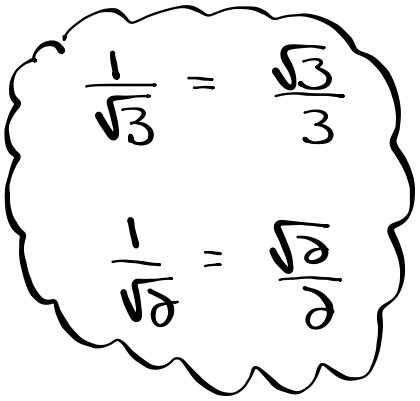
S	A
T	C

③ Find x

Q2	Q3
$x = 180° - \bar{x}$	$x = 180° + \bar{x}$
$x = 180° - 60° = 120°$	$x = 180° + 60° = 240°$

Homework

Finish worksheet and Page 211 #1-5



Worksheet Solutions

① $\sin\theta = -\frac{\sqrt{3}}{2}$ where is $\sin\theta < 0$ (S \| A; T \| C)

use positive

$\bar{\theta} = \sin^{-1}\left(\frac{\sqrt{3}}{2}\right)$

$\bar{\theta} = 60°$

Q3	Q4
$\theta = 180° + 60° = 240°$	$\theta = 360° - 60° = 300°$
$240° \pm 360°n, n \in \mathbb{N}$	$300° \pm 360°n, n \in \mathbb{N}$

② $\cos\theta = -\frac{1}{2}$ where is $\cos\theta < 0$ (S \| A; T \| C)

$\bar{\theta} = \cos^{-1}\left(\frac{1}{2}\right)$

$\bar{\theta} = 60°$

Q2	Q3
$\theta = 180° - 60° = 120°$	$\theta = 180° + 60° = 240°$
$120° \pm 360°n, n \in \mathbb{N}$	$240° \pm 360°n, n \in \mathbb{N}$

③ $\tan\theta = -\frac{\sqrt{3}}{3}$ where is $\tan\theta < 0$ (S \| A; T \| C)

$\tan\theta = -\frac{1}{\sqrt{3}}$

$\bar{\theta} = \tan^{-1}\left(\frac{1}{\sqrt{3}}\right)$

$\bar{\theta} = 30°$

Q2	Q4
$\theta = 180° - 30° = 150°$	$\theta = 360° - 30° = 330°$
$150° \pm 360°n, n \in \mathbb{N}$	$330° \pm 360°n, n \in \mathbb{N}$

④ $\tan\theta = \frac{\sqrt{3}}{1}$ Where is $\tan\theta > 0$ (S \| A; T \| C)

$\bar{\theta} = \tan^{-1}\left(\frac{\sqrt{3}}{1}\right)$

$\bar{\theta} = 60°$

Q1	Q4
$\theta = 60°$	$\theta = 360° - 60° = 300°$
$60° \pm 360°n, n \in \mathbb{N}$	$300° \pm 360°n, n \in \mathbb{N}$

⑥ $\sin\theta = -1$

* Unit Circle

$\theta = 270° \rightarrow 270° \pm 360°n, n \in \mathbb{N}$

⑬ $5\sin\theta - 4 = 0$ where is $\sin\theta > 0$ (S \| A; T \| C)

$\sin\theta = \frac{4}{5}$

$\sin\theta = 0.8$ (approx. value)

$\bar{\theta} = \sin^{-1}(0.8)$

$\bar{\theta} = 53.1°$

Q1	Q2
$\theta = 53.1°$	$\theta = 180° - 53.1 = 126.9°$
$53.1° \pm 360°n, n \in \mathbb{N}$	$126.9° \pm 360°n, n \in \mathbb{N}$

Worksheet Solutions

⑧ $2\sin\theta + 1 = 0$

$\frac{2\sin\theta}{2} = \frac{-1}{2}$

$\sin\theta = -\frac{1}{2}$

① Find $\bar{\theta}$

$\bar{\theta} = \sin^{-1}\left(\frac{1}{2}\right)$ (Triangle #2)

$\bar{\theta} = 30°$

② where is $\sin\theta < 0$ S | A / T | C

③

Q3	Q4
$\theta = 180° + \bar{\theta}$	$\theta = 360° - \bar{\theta}$
$\theta = 180° + 30° = 210°$	$\theta = 360° - 30° = 330°$
$210° \pm 360°n, n \in \mathbb{N}$	$330° \pm 360°n, n \in \mathbb{N}$

⑨ $\sqrt{3} - 2\sin\theta = 0$

$\frac{-2\sin\theta}{-2} = \frac{-\sqrt{3}}{-2}$

$\sin\theta = \frac{\sqrt{3}}{2}$

① Find $\bar{\theta}$

$\bar{\theta} = \sin^{-1}\left(\frac{\sqrt{3}}{2}\right)$ (Triangle #2)

$\bar{\theta} = 60°$

② where is $\sin\theta > 0$ S | A / T | C

③

Q1	Q2
$\theta = \bar{\theta}$	$\theta = 180° - \bar{\theta}$
$\theta = 60°$	$\theta = 180° - 60° = 120°$
$60° \pm 360°n, n \in \mathbb{N}$	$120° \pm 360°n, n \in \mathbb{N}$

⑩ $\sqrt{\tan^2\theta} = \sqrt{3}$

$\tan\theta = \pm\sqrt{3}$

① Find $\bar{\theta}$

$\bar{\theta} = \tan(\sqrt{3})$ (Triangle #2)

$\bar{\theta} = 60°$

② where is $\tan\theta$ positive/negative S | A / T | C

③

Q1	Q2	Q3	Q4
$\theta = \bar{\theta}$	$\theta = 180° - \bar{\theta}$	$\theta = 180° + \bar{\theta}$	$\theta = 360° - \bar{\theta}$
$\theta = 60°$	$\theta = 120°$	$\theta = 240°$	$\theta = 300°$
$60° \pm 360°n$	$120° \pm 360°n$	$240° \pm 360°n$	$300° \pm 360°n$

⑪ $5\sin\theta - 4 = 0$

$\frac{5\sin\theta}{5} = \frac{4}{5}$

$\sin\theta = \frac{4}{5} = 0.8$ (Approx)

① Find $\bar{\theta}$

$\bar{\theta} = \sin^{-1}(0.8)$

$\bar{\theta} = 53.1°$

② where is $\sin\theta > 0$ S | A / T | C

③

Q1	Q2
$\theta = \bar{\theta}$	$\theta = 180° - \bar{\theta}$
$\theta = 53.1°$	$\theta = 126.9°$
$53.1° \pm 360°n, n \in \mathbb{N}$	$126.9 \pm 360°n, n \in \mathbb{N}$

Worksheet Solutions

Backside of sheet $0 \le \theta \le 2\pi$

③ $\tan\theta = \frac{\sqrt{3}}{3} = \frac{1}{\sqrt{3}}$

① Find $\bar{\theta}$

$\bar{\theta} = \tan^{-1}\left(\frac{1}{\sqrt{3}}\right)$ (Triangle #4)

$\bar{\theta} = \frac{\pi}{6}$

② where is $\tan\theta > 0$

S	A ✓
T ✓	C

③ Find θ

Q1	Q3
$\theta = \bar{\theta}$	$\theta = \pi + \bar{\theta}$
$\theta = \frac{\pi}{6}$	$\theta = \pi + \frac{\pi}{6} = \frac{6\pi}{6} + \frac{\pi}{6} = \frac{7\pi}{6}$

Check-Up:

Solve:

① $\cot\theta = 0.7834$, $-\frac{\pi}{2} < \theta < -\pi$

② $3\cos x + 5 = 6$, $-360° \leq x \leq 720°$

③ $2\sin x(1-\sin x) = 0$, $-4\pi < x < 4\pi$

Factoring trinomials:

① Hard Trinomial

$2x^2+7x+6$ $\quad 2\times 6=12$, $2+6=7$

$(x+2)(x+6)$

$(2x+3)(x+2)$

② Simple trinomial

x^2+7x+6 $\quad 6\times 1=6$, $6+1=7$

$(x+1)(x+6)$

③ Difference of Squares

x^2-16

$(x-4)(x+4)$

④ Common Factor

$3x^2-9x^3+21x$

$3x(x-3x^2+7)$

Let's move onto QUADRATIC trigonometric equations...

...Pre-Calculus 110

- What strategies can we use to solve quadratic equations? (Factor)
- Quadratic trigonometric equations will ultimately become TWO linear trigonometric equations.

Solve: $2\sin^2 x + \sin x = 1 \quad 0 \le x \le 4\pi$ (Radians)

$2\sin^2 x + \sin x - 1 = 0$ Hard Trinomial

$(\sin x - 1)(\sin x + 2) = 0$ $-1 + 2 = 1$, $-1 \times 2 = -2$ (2, -1)

$(2\sin x - 1)(\sin x + 1) = 0$

$2\sin x - 1 = 0$

$\frac{2\sin x}{2} = \frac{1}{2}$

$\sin x = \frac{1}{2}$

① Find $\bar{x}$:

$\bar{x} = \sin^{-1}\left(\frac{1}{2}\right)$ (Triangle #4)

$\bar{x} = \frac{\pi}{6}$

② where is $\sin x > 0$

S	A
T	C

③ Find x:

Q1	Q2
$x = \bar{x}$	$x = \pi - \bar{x}$
$x = \frac{\pi}{6}$	$x = \pi - \frac{\pi}{6}$
	$x = \frac{6\pi}{6} - \frac{\pi}{6} = \frac{5\pi}{6}$
$x = \frac{\pi}{6} + 2\pi$	$x = \frac{5\pi}{6} + 2\pi$
$x = \frac{\pi}{6} + \frac{12\pi}{6} = \frac{13\pi}{6}$	$x = \frac{5\pi}{6} + \frac{12\pi}{6} = \frac{17\pi}{6}$

$\sin x + 1 = 0$

$\sin x = -1$ (Unit Circle)

①Find x.

$x = \frac{3\pi}{2}$

$x = \frac{3\pi}{2} + 2\pi$

$x = \frac{3\pi}{2} + \frac{4\pi}{2} = \frac{7\pi}{2}$

Ex. $\cos^2\theta - \frac{1}{2}\cos\theta = 0, \ -2\pi \le \theta \le 4\pi$ (Common Factor)

(Radians)

$\cos\theta\left(\cos\theta - \frac{1}{2}\right) = 0$

$\cos\theta = 0$ (Unit Circle)

① Find θ:

$\theta = \frac{\pi}{2}$ $\quad \theta = \frac{3\pi}{2}$

$\theta = \frac{\pi}{2} - 2\pi$ $\quad \theta = \frac{3\pi}{2} - 2\pi$

$\theta = \frac{\pi}{2} - \frac{4\pi}{2} = \boxed{\frac{-3\pi}{2}}$ $\quad \theta = \frac{3\pi}{2} - \frac{4\pi}{2} = \boxed{\frac{-\pi}{2}}$

$\theta = \frac{\pi}{2} + 2\pi$ $\quad \theta = \frac{3\pi}{2} + 2\pi$

$\theta = \frac{\pi}{2} + \frac{4\pi}{2} = \boxed{\frac{5\pi}{2}}$ $\quad \theta = \frac{3\pi}{2} + \frac{4\pi}{2} = \boxed{\frac{7\pi}{2}}$

$\cos\theta - \frac{1}{2} = 0$

$\cos\theta = \frac{1}{2}$

① Find $\bar{\theta}$

$\bar{\theta} = \cos^{-1}\left(\frac{1}{2}\right)$ (Triangle)

$\bar{\theta} = \frac{\pi}{3}$

② where is $\cos\theta > 0$

S	~~A~~
T	C

③ Find θ:

Q1

$\theta = \bar{\theta}$ $\quad \theta = 2\pi - \bar{\theta}$

$\theta = \boxed{\frac{\pi}{3}}$ $\quad \theta = \frac{6\pi}{3} - \frac{\pi}{3} = \boxed{\frac{5\pi}{3}}$

$\theta = \frac{\pi}{3} - 2\pi$ $\quad \theta = \frac{5\pi}{3} - 2\pi$

$\theta = \frac{\pi}{3} - \frac{6\pi}{3} = \boxed{\frac{-5\pi}{3}}$ $\quad \theta = \frac{5\pi}{3} - \frac{6\pi}{3} = \boxed{\frac{-\pi}{3}}$

$\theta = \frac{\pi}{3} + 2\pi$ $\quad \theta = \frac{5\pi}{3} + 2\pi$

$\theta = \frac{\pi}{3} + \frac{6\pi}{3} = \boxed{\frac{7\pi}{3}}$ $\quad \theta = \frac{5\pi}{3} + \frac{6\pi}{3} = \boxed{\frac{11\pi}{3}}$

$-\frac{12\pi}{6} \le \theta \le \frac{24\pi}{6}$

Ex. $6\sin^2 x - \sin x = 2 \quad -2\pi \le \theta \le 4\pi$ (Radians)

(Hard Trinomial)

$3 + -4 = -1$

$3 \times -4 = -12$

$6\sin^2\theta - \sin\theta - 2 = 0$

$(\sin\theta + \frac{3}{6})(\sin\theta - \frac{4}{6}) = 0$

$(\sin\theta + \frac{1}{2})(\sin\theta - \frac{2}{3}) = 0$

$(2\sin\theta + 1)(3\sin\theta - 2) = 0$

$2\sin\theta + 1 = 0$

$\frac{2\sin\theta}{2} = \frac{-1}{2}$

$\sin\theta = -\frac{1}{2}$

① Find $\bar{\theta}$:

$\bar{\theta} = \sin^{-1}(\frac{1}{2})$ (Triangle #4)

$\bar{\theta} = \frac{\pi}{6}$ (Exact)

② where is $\sin\theta < 0$ S | A / T | C

② Find θ:

Q3	Q4
$\theta = \pi + \bar{\theta}$	$\theta = 2\pi - \bar{\theta}$
$\theta = \pi + \frac{\pi}{6}$	$\theta = 2\pi - \frac{\pi}{6}$
$\theta = \frac{6\pi}{6} + \frac{\pi}{6} = \frac{7\pi}{6}$	$\theta = \frac{12\pi}{6} - \frac{\pi}{6} = \frac{11\pi}{6}$
$\theta = \frac{7\pi}{6} - 2\pi$	$\theta = \frac{11\pi}{6} - 2\pi$
$\theta = \frac{7\pi}{6} - \frac{12\pi}{6} = \frac{-5\pi}{6}$	$\theta = \frac{11\pi}{6} - \frac{12\pi}{6} = \frac{-\pi}{6}$
$\theta = \frac{7\pi}{6} + 2\pi$	$\theta = \frac{11\pi}{6} + 2\pi$
$\theta = \frac{7\pi}{6} + \frac{12\pi}{6} = \frac{19\pi}{6}$	$\theta = \frac{11\pi}{6} + \frac{12\pi}{6} = \frac{23\pi}{6}$

$3\sin\theta - 2 = 0$

$\frac{3\sin\theta}{3} = \frac{2}{3}$

$\sin\theta = \frac{2}{3} = 0.\overline{6}$

① Find $\bar{\theta}$

$\bar{\theta} = \sin^{-1}(0.\overline{6})$

$\bar{\theta} = 0.73$ (Approx)

② where is $\sin\theta > 0$ ~~S~~ | ~~A~~ / T | C

③ Find θ:

Q1	Q2
$\theta = \bar{\theta}$	$\theta = 3.14 - \bar{\theta}$
$\theta = 0.73$	$\theta = 3.14 - 0.73 = 2.41$
$\theta = 0.73 - 6.28$	$\theta = 2.41 - 6.28$
$\theta = -5.55$	$\theta = -3.87$
$\theta = 0.73 + 6.28$	$\theta = 2.41 + 6.28$
$\theta = 7.01$	$\theta = 8.69$

Your Turn

Solve for θ.

$\cos^2\theta - \cos\theta - 2 = 0,\ 0° \le \theta < 360°$ (Simple Trinomial)

Give solutions as exact values where possible. Otherwise, give approximate measures to the nearest thousandth of a degree. (Degrees)

$\cos^2\theta - \cos\theta - 2 = 0$

$(\cos\theta - 2)(\cos\theta + 1) = 0$

$-2 + 1 = -1$

$-2 \times 1 = -2$

$\cos\theta - 2 = 0$

$\cos\theta = 2$ (No diagram)

$\cos\theta + 1 = 0$

$\cos\theta = -1$ (Unit Circle)

① Find $\bar{\theta}$:

$\bar{\theta} = \cos^{-1}(2)$

$\bar{\theta}$ = Math Error

Not Possible

(Extraneous)

① Find θ:

$\theta = 180°$

General Solution of a Trigonometric Equation

↖ All angles

Solve: $3\cos^2\theta - \cos\theta = 2; \ \theta \in \mathbb{R}$ (Assume Degrees)

$3\cos^2\theta - 1\cos\theta - 2 = 0$ Hard Trinomial

$-3 + 2 = -1$

$-3 \times 2 = -6$

$(\cos\theta - 3)(\cos\theta + 2) = 0$

$(\cos\theta - 1)(3\cos\theta + 2) = 0$

$\cos\theta - 1 = 0$

$\cos\theta = 1$ (unit circle)

① Find θ:

$\theta = 0°$ or $\theta = 360°$

$0° \pm 360°n, n \in \mathbb{N}$

$360° \pm 360°n, n \in \mathbb{N}$ not necessary

$3\cos\theta + 2 = 0$

$\frac{3\cos\theta}{3} = \frac{-2}{3}$

$\cos\theta = \frac{-2}{3} = -0.\overline{6}$

① Find $\bar{\theta}$

$\bar{\theta} = \cos^{-1}(0.\overline{6})$ (Approx)

$\theta = 48.2°$

② where is $\cos\theta < 0$ S | A / T | C

③ Find θ:

Q2	Q3
$\theta = 180° - \bar{\theta}$	$\theta = 180° + \bar{\theta}$
$\theta = 180° - 48.2° = 131.8°$	$\theta = 180° + 48.2° = 228.2°$

$131.8° \pm 360°n, n \in \mathbb{N}$

$228.2° \pm 360°n, n \in \mathbb{N}$

Determine the general solution for $\sin^2 x - 1 = 0$ over the real numbers if x is measured in radians.

(Diff. of squares)

$\sin^2 x - 1 = 0$

$(\sin x + 1)(\sin x - 1) = 0$

$\sin x + 1 = 0$ $\qquad$ $\sin x - 1 = 0$

$\sin x = -1$ ←(Unit circle)→ $\sin x = 1$

① $x = 270°$ $\qquad$ ① $x = 90°$

$270° \pm 360°n, n \in \mathbb{N}$ $\qquad$ $90° \pm 360°n, n \in \mathbb{N}$

General Solution is all angles

Did You Know?

$2n$, where $n \in \mathrm{I}$, represents all even integers.

$2n + 1$, where $n \in \mathrm{I}$, is an expression for all odd integers.

$$x = \frac{\pi}{2} + 2\pi n, \text{ where } n \in \mathrm{I}$$

$$x = \frac{3\pi}{2} + 2\pi n, \text{ where } n \in \mathrm{I}$$

or

$$x = \frac{\pi}{2} + \pi n, \text{ where } n \in \mathrm{I}$$

or

$$(2n + 1)\left(\frac{\pi}{2}\right), n \in \mathrm{I}$$

Determine the general solution for $\cos^2 x - 1 = 0$, where the domain is real numbers measured in degrees.

Chapter 4 Test

Open Response:

① Trig Expression (# 2 on Review)

② Trig Equation (# 1 e) on Review)

③ Problem involving Trig Ratios (# 3 or 4 on Review)

Ch. 4 Review

① a) $\cos\theta = \frac{\sqrt{3}}{2}$, $0° < \theta < 360°$

① Find $\bar{\theta}$:

$\bar{\theta} = \cos^{-1}\left(\frac{\sqrt{3}}{2}\right)$ (Triangle #2)

$\bar{\theta} = 30°$

② where is $\cos\theta > 0$

S	A
T	C

③ Find θ:

Q1	Q4
$\theta = \bar{\theta}$	$\theta = 360° - \bar{\theta}$
$\theta = 30°$	$\theta = 360° - 30° = 330°$

b) $\sin\theta = -\frac{\sqrt{2}}{2} = -\frac{1}{\sqrt{2}}$ $0 < \theta < 2\pi$

① Find $\bar{\theta}$

$\bar{\theta} = \sin^{-1}\left(\frac{1}{\sqrt{2}}\right)$ (Triangle #3)

$\bar{\theta} = \frac{\pi}{4}$

② where is $\sin\theta < 0$

S	A
T	C

③ Find θ:

Q3	Q4
$\theta = \pi + \bar{\theta}$	$\theta = 2\pi - \bar{\theta}$
$\theta = \pi + \frac{\pi}{4}$	$\theta = 2\pi - \frac{\pi}{4}$
$\theta = \frac{4\pi}{4} + \frac{\pi}{4} = \frac{5\pi}{4}$	$\theta = \frac{8\pi}{4} - \frac{\pi}{4} = \frac{7\pi}{4}$

Ch. 4 Review

① c) $\cot\theta = \text{undefined}$, $0° \le \theta \le 270°$ (Unit Circle) $\cot\theta = \frac{x}{y}$

* anywhere $y=0$

(i) Find θ:

$\theta = 0°$

$\theta = 180°$

~~$\theta = 360°$~~

① d) $2\sin\theta - 1 = 0$ $-2\pi \le \theta \le 2\pi$

$\frac{2\sin\theta}{2} = \frac{1}{2}$

$\sin\theta = \frac{1}{2}$ (Triangle #4)

(i) Find $\bar{\theta}$:

$\bar{\theta} = \sin^{-1}\left(\frac{1}{2}\right)$

$\bar{\theta} = \frac{\pi}{6}$

(ii) where is $\sin\theta > 0$

S	A
T	C

(iii) Find θ:

Q1	Q2
$\theta = \bar{\theta}$	$\theta = \pi - \bar{\theta}$
$\theta = \frac{\pi}{6}$	$\theta = \pi - \frac{\pi}{6}$
$\theta = \frac{\pi}{6} - 2\pi$	$\theta = \frac{6\pi}{6} - \frac{\pi}{6} = \frac{5\pi}{6}$
$\theta = \frac{\pi}{6} - \frac{12\pi}{6} = \frac{-11\pi}{6}$	$\theta = \frac{5\pi}{6} - 2\pi$
	$\theta = \frac{5\pi}{6} - \frac{12\pi}{6} = \frac{-7\pi}{6}$

Ch.4 Review

① e) $\cos^2\theta + \frac{1}{2}\cos\theta = 0$, $0° \le \theta \le 360°$

$\cos\theta\left(\cos\theta + \frac{1}{2}\right) = 0$

$\cos\theta = 0$ (Unit Circle) $\theta = 90°, 270°$

$\cos\theta + \frac{1}{2} = 0$

$\cos\theta = -\frac{1}{2}$ where is $\cos\theta < 0$

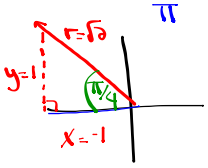
(Triangle #2) $\bar{\theta} = 60°$

Q2	Q3
$\theta = 180° - 60°$	$\theta = 180° + 60°$
$\theta = 120°$	$\theta = 240°$

$\frac{2\pi}{4}, \frac{3\pi}{4}, \frac{4\pi}{4}$

π

② a) $\frac{3}{1 - 2\sin\frac{3\pi}{4}}$



$\sin\frac{3\pi}{4} = \frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2}$

$\frac{3}{1 - 2\left(\frac{\sqrt{2}}{2}\right)}$

$\frac{3}{(1-\sqrt{2})(1+\sqrt{2})}$ Rationalize

$\frac{3+3\sqrt{2}}{1+\sqrt{2}-\sqrt{2}-2}$

$\frac{3+3\sqrt{2}}{-1}$ or $-3-3\sqrt{2}$

② d) $\frac{2\cos 3\pi + \sin 11\pi/4}{\cos^2 \pi/6}$

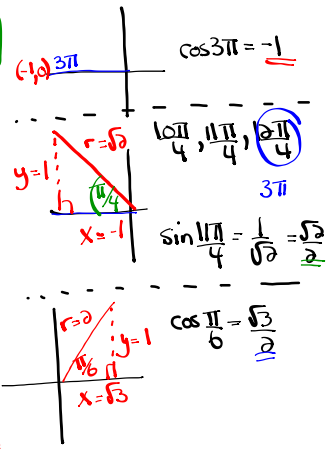
$\frac{2(-1) + \frac{\sqrt{2}}{2}}{\left(\frac{\sqrt{3}}{2}\right)^2}$

$\frac{-2 + \frac{\sqrt{2}}{2}}{\frac{3}{4}}$

$\left(-\frac{4}{2} + \frac{\sqrt{2}}{2}\right) \cdot \frac{4}{3}$ ← multiply by reciprocal

$\left(\frac{-4+\sqrt{2}}{2}\right) \cdot \frac{4}{3}$

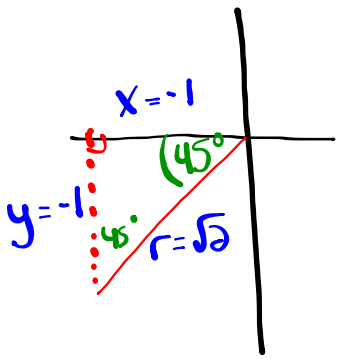
$\frac{-16+4\sqrt{2}}{6} \rightarrow$ Reduce $\frac{-8+2\sqrt{2}}{3}$



Ch. 4 Review

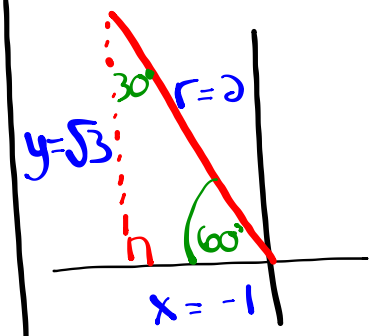
② b) $\dfrac{\sin^2 225^\circ}{8\sin 120^\circ}$

Sketch 225°



$\sin 225^\circ = \dfrac{-1}{\sqrt{2}}$

Sketch 120°



$\sin 120^\circ = \dfrac{\sqrt{3}}{2}$

$\dfrac{\left(\frac{-1}{\sqrt{2}}\right)^2}{8\left(\frac{\sqrt{3}}{2}\right)}$

$\dfrac{\frac{1}{2}}{\frac{8\sqrt{3}}{2}}$

$\dfrac{\frac{1}{2}}{4\sqrt{3}}$

$\frac{1}{2} \times \frac{1}{4\sqrt{3}}$

$\frac{1}{8\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}}$

$\dfrac{\sqrt{3}}{8(3)}$

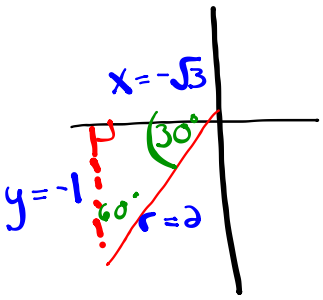
$\boxed{\dfrac{\sqrt{3}}{24}}$

Ch.4 Review:

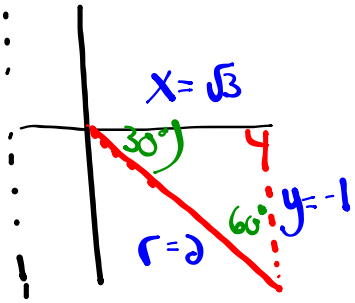
② c) $\cos^2 210° + \sin^2(-30°) - \sin 90° + \tan 480°$

($-30° = 330°$, $480° = 120°$)

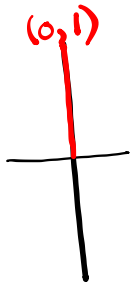
Sketch 210°



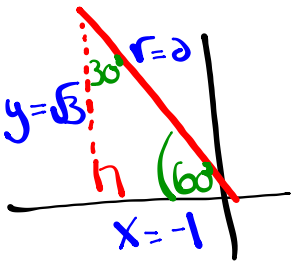
Sketch 330°



Sketch 90°



Sketch 120°



$\cos 210° = -\frac{\sqrt{3}}{2}$

$\sin 330° = \frac{-1}{2}$

$\sin 90° = 1$

$\tan 120° = \frac{\sqrt{3}}{-1} = -\sqrt{3}$

$\cos^2 210° + \sin^2(-30°) - \sin 90° + \tan 480°$

$\left(\frac{-\sqrt{3}}{2}\right)^2 + \left(\frac{-1}{2}\right)^2 - (1) + (-\sqrt{3})$

$\frac{3}{4} + \frac{1}{4} - 1 - \sqrt{3}$

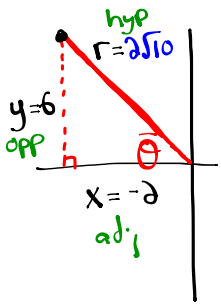
$\frac{3}{4} + \frac{1}{4} - \frac{4}{4} - \frac{4\sqrt{3}}{4}$

$-\frac{4\sqrt{3}}{4}$

$\boxed{-\sqrt{3}}$

Ch. 4 Review

③



① Find r:

$x^2+y^2=r^2$

$(-2)^2+(6)^2=r^2$

$4+36=r^2$

$40=r^2$

$\sqrt{40}=r$

$\sqrt{2\cdot2\cdot2\cdot5}=r$

$2\sqrt{10}=r$

(ii) $\sin\theta=\frac{6}{2\sqrt{10}}=\frac{3}{\sqrt{10}}=\frac{3\sqrt{10}}{10}$

$\cos\theta=\frac{-2}{2\sqrt{10}}=\frac{-1}{\sqrt{10}}=\frac{-\sqrt{10}}{10}$

$\tan\theta=\frac{6}{-2}=-3$

$\csc\theta=\frac{\sqrt{10}}{3}$

$\sec\theta=-\sqrt{10}$

$\cot\theta=-\frac{1}{3}$

} Reciprocal Ratios

④ $\sec\theta=-\frac{\sqrt{11}}{3}$ hyp / adj and $\tan\theta>0$

$\cos\theta<0$

Given:

$r=\text{hyp}=\sqrt{11}$

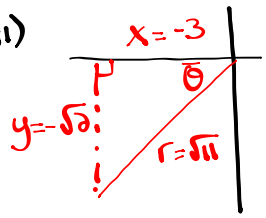
$x=\text{adj}=-3$

(i) • $\cos\theta<0$
• $\tan\theta>0$

~~S~~	~~A~~
~~T~~	C

θ is in Q3

(ii)



$x^2+y^2=r^2$

$(-3)^2+y^2=(\sqrt{11})^2$

$9+y^2=11$

$y^2=2$

$y=\pm\sqrt{2}$

$y=-\sqrt{2}$ Q3

(iii) $\sin\theta=\frac{-\sqrt{2}}{\sqrt{11}}=\frac{-\sqrt{22}}{11}$

$\cos\theta=\frac{-3}{\sqrt{11}}=\frac{-3\sqrt{11}}{11}$

$\tan\theta=\frac{-\sqrt{2}}{-3}=\frac{\sqrt{2}}{3}$

$\csc\theta=\frac{\sqrt{11}}{\sqrt{2}}=\frac{\sqrt{22}}{2}$

$\cot\theta=\frac{3}{\sqrt{2}}=\frac{3\sqrt{2}}{2}$

⑤ Find one positive and one negative co-terminal angle:

a) $\frac{\pi}{9}$

$\frac{\pi}{9} - \pi$

$\frac{\pi}{9} - \frac{18\pi}{9} = \boxed{\frac{-16\pi}{9}}$

$\frac{\pi}{9} + \pi$

$\frac{\pi}{9} + \frac{18\pi}{9} = \boxed{\frac{20\pi}{9}}$

b) -900°

$-900^\circ - 360^\circ = \boxed{-1260^\circ}$

$-900^\circ + 1080^\circ = \boxed{180^\circ}$

-900°
-180°

c) 300°

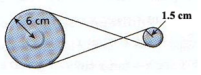
$300^\circ - 360^\circ = \boxed{-60^\circ}$

$300^\circ + 360^\circ = \boxed{660^\circ}$

Practice Problems:

Pages 212 - 214
#11 - 23

If the belt in the pulley system below travels 30 cm, what is the angle of rotation of the smaller pulley?



[A] $\frac{\pi}{9}$ radians [B] 20°

[C] 20 radians [D] 5°

Given:
$a = 30cm$
$r = 1.5cm$

$\theta = \frac{a}{r} = \frac{30cm}{1.5cm} = 20\ rads$

Nibbles the hamster is running at 0.02 m/s on an exercise wheel of radius 8 cm. What is the angular velocity of this wheel?

[A] 0.15 rad/minute [B] 240 rad/minute [C] 0.25 rad/minute [D] 15 radians/minute

Given:
$r = 8cm = 0.08m$
$a = 0.02m$ (after 1 sec)

① $\theta = \frac{a}{r} = \frac{0.02m}{0.08m} = 0.25\ rads$

② $v_a = \frac{\theta}{t} = \frac{0.25rads}{1\ sec} = \frac{15\ rads}{60sec}$

Solve: $2(1-\sin\theta)^2 + \sin\theta = 2(3-4\sin^2\theta),\ -360° \le \theta \le 720°$ (Degrees)

$2(1-\sin\theta)(1-\sin\theta) + \sin\theta = 6 - 8\sin^2\theta$

$2(1-\sin\theta-\sin\theta+\sin^2\theta) + \sin\theta = 6-8\sin^2\theta$

$2(1-2\sin\theta+\sin^2\theta) + \sin\theta = 6-8\sin^2\theta$

$2-4\sin\theta+2\sin^2\theta+\sin\theta = 6-8\sin^2\theta$

$10\sin^2\theta - 3\sin\theta - 4 = 0$ $\quad -8+5=-3$, $-8\times 5 = -40$

$(\sin\theta - \frac{8}{10})(\sin\theta + \frac{5}{10}) = 0$

$(\sin\theta - \frac{4}{5})(\sin\theta + \frac{1}{2}) = 0$

$(5\sin\theta - 4)(2\sin\theta + 1) = 0$

$5\sin\theta - 4 = 0$

$\sin\theta = \frac{4}{5}$

① Find $\bar{\theta}$:

$\bar{\theta} = \sin^{-1}(\frac{4}{5})$

$\bar{\theta} = 53°$

② where is $\sin\theta > 0$

S | A
T | C

③ Find θ:

Q1	Q2
$\theta = \bar{\theta}$	$\theta = 180° - \bar{\theta}$
$\theta = 53°$	$\theta = 180° - 53° = 127°$
$\theta = 53° - 360° = -307°$	$\theta = 127° - 360° = -233°$
$\theta = 53° + 360° = 413°$	$\theta = 127° + 360° = 487°$

$2\sin\theta + 1 = 0$

$\sin\theta = -\frac{1}{2}$

① Find $\bar{\theta}$:

$\bar{\theta} = \sin^{-1}(\frac{1}{2})$

$\bar{\theta} = 30°$

② where is $\sin\theta < 0$

S | A
T | C

③ Find θ:

Q3	Q4
$\theta = 180° + \bar{\theta}$	$\theta = 360° - \bar{\theta}$
$\theta = 180° + 30° = 210°$	$\theta = 330°$
$\theta = -150°$	$\theta = -30°$
$\theta = 570°$	$\theta = 690°$

Solve: $6\sin^2\theta - 3\sin\theta = 0$, $0 \le \theta \le 360°$ (Open Response)

[A] $0°, 30°, 180°, 330°, 360°$
[B] $0°, 30°, 180°, 150°, 360°$
[C] $30°, 90°, 120°, 270°$
[D] $0°, 180°, 210°, 330°, 360°$

$6\sin^2\theta - 3\sin\theta = 0$ (Common Factor)

$3\sin\theta(2\sin\theta - 1) = 0$

$\frac{3\sin\theta}{3} = \frac{0}{3}$

$\sin\theta = 0$ (Unit Circle)

(i) Find θ

$\theta = 0°$
$\theta = 180°$
$\theta = 360°$

$2\sin\theta - 1 = 0$

$\frac{2\sin\theta}{2} = \frac{1}{2}$

$\sin\theta = \frac{1}{2}$ (Triangle #2)

(i) Find $\bar{\theta}$:

$\bar{\theta} = \sin^{-1}\left(\frac{1}{2}\right)$

$\bar{\theta} = 30°$

(ii) where is $\sin\theta > 0$? ~~S~~ A / T C

(iii) Find θ:

Q1	Q2
$\theta = \bar{\theta}$	$\theta = 180° - \bar{\theta}$
$\theta = 30°$	$\theta = 180° - 30° = 150°$

If $\csc\theta < 0$ and $\tan\theta > 0$, then which of the following could be a possible measure of angle θ?

[A] $\frac{11\pi}{6}$ [B] $\frac{4\pi}{3}$ [C] $\frac{3\pi}{4}$ [D] $\frac{\pi}{2}$

~~$\frac{10\pi}{6}, \frac{11\pi}{6}, \frac{12\pi}{6}$~~ 2π ~~$\frac{3\pi}{3}, \frac{4\pi}{3}, \frac{5\pi}{3}$~~ 1π

Q4 Q3 Q2

Given:
$\csc\theta < 0$ (negative)
$\tan\theta > 0$ (positive)

S | ~~A~~
T | C

Angle θ is in Q3

What is the principal angle of $-\frac{25\pi}{4}$?

[A] $\frac{3\pi}{4}$ [B] $\frac{\pi}{4}$ [C] $-\frac{\pi}{4}$ [D] $\frac{7\pi}{4}$

$\frac{2\pi}{1} = \frac{8\pi}{4}$

① $-\frac{25\pi}{4} \div 2\pi = -\frac{25\pi}{4} \times \frac{1}{2\pi} = -\frac{25}{8} = -3\frac{1}{8}$

② $-3\frac{1}{8} + 3 = -\frac{1}{8}$

③ $-\frac{1}{8} \times 2\pi = -\frac{2\pi}{8} = -\frac{\pi}{4}$

④ $-\frac{\pi}{4} + 2\pi = -\frac{\pi}{4} + \frac{8\pi}{4} = \frac{7\pi}{4}$

Check-Up problem...

Solve: $\sin x \sec x + 2\sin x = 0 \ , \ x \in R$ (x is measured in radians)

Review...

C4 **a)** Determine all solutions for the equation $2\sin^2\theta = 1 - \sin\theta$ in the domain $0° \leq \theta < 360°$.

b) Are your solutions exact or approximate? Why?

c) Show how you can check one of your solutions to verify its correctness.

A grandfather clock shows a time of 7 o'clock. What is the exact radian measure of the angle between the hour hand and the minute hand?

Determine the angular velocity of the minute hand on a clock.

Little Johnny has a rock tied to the end of a piece of rope 1.5 m long and he is swinging it around his head in a circular pattern. Mrs. Centripetal, his physics teacher, is watching Johnny out the window of her physics lab and notes that the rock is making 12 revolutions every 48 seconds.

(a) Determine the angular velocity with which little Johnny is twirling the rope above his head. [2]

(b) The rock comes flying from the rope 3 minutes after Mrs. Centripetal started to time little Johnny. How far did the rock travel during the 3 minutes? [2]

$$\frac{1}{2} \div 4\sqrt{3}$$

$$\frac{1}{2} \cdot \frac{1}{4\sqrt{3}}$$

$$\frac{1}{8\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}}$$

$$\frac{\sqrt{3}}{8\sqrt{9}}$$

$$\frac{\sqrt{3}}{24}$$

Attachments

Worksheet - Sketching Angles in Radians.doc