

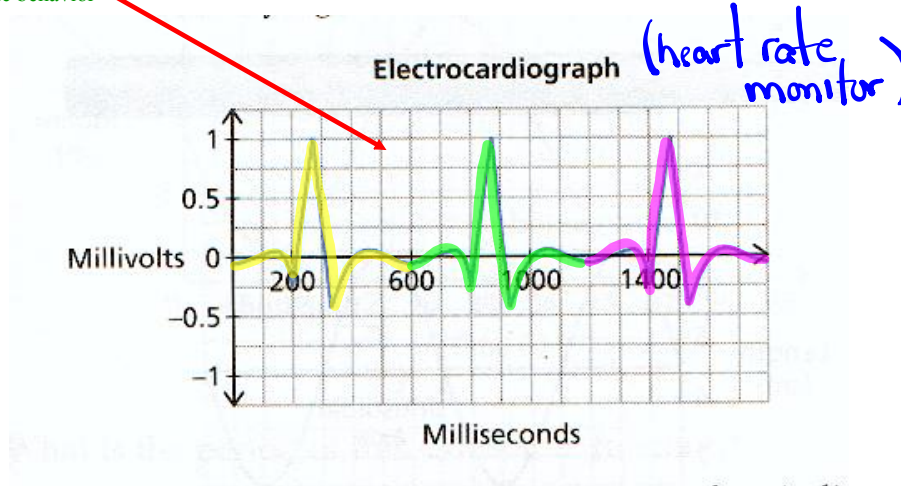
# Sinusoidal Relations (Trig Graphs)

$y = \sin x$   
 $y = \cos x$

**Periodic Function:** A function for which the dependent variable takes on the same set of values over and over again as the independent variable changes.

*(a function that repeats)*

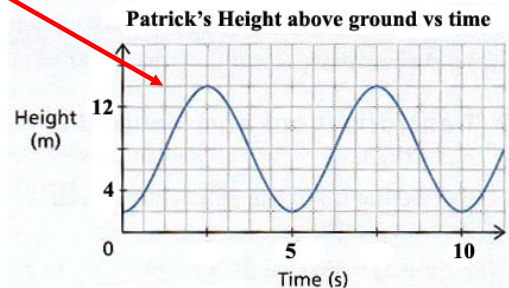
Example of periodic behavior



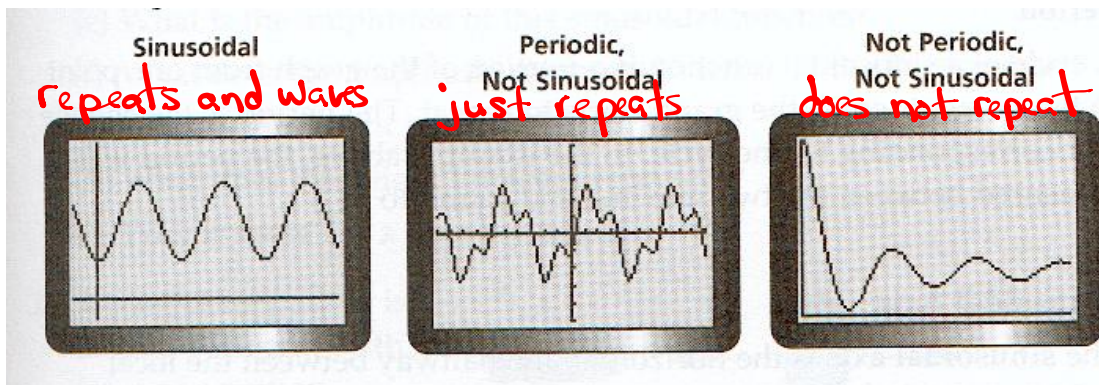
**Sinusoidal Function:** A periodic function that looks like waves, where any portion of the curve can be translated onto another portion of the curve.

*(Repeats and looks like a smooth wave).*

Example of sinusoidal behavior



These illustrations should summarize periodic and sinusoidal...

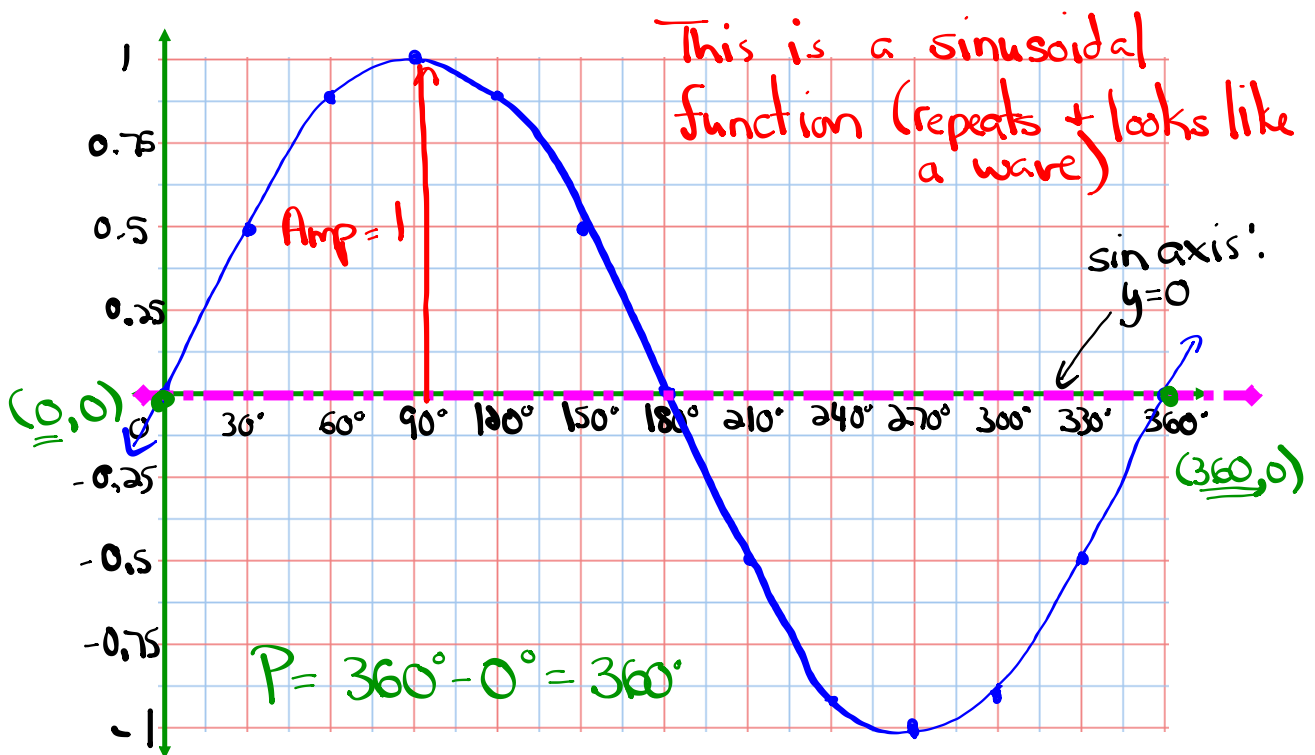


Let's examine the graph of  $y = \sin \theta$

$$y = \sin x$$

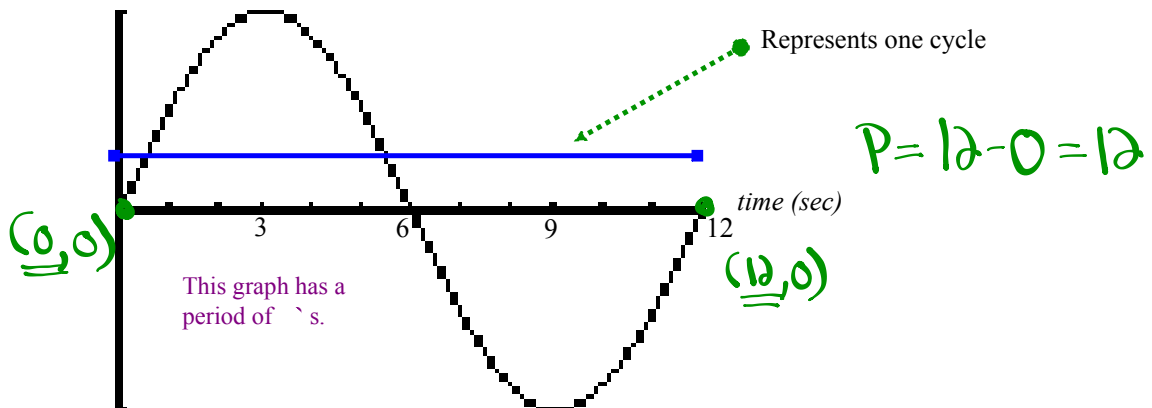
$\theta$	$0$	$30$	$60$	$90$	$120$	$150$	$180$	$210$	$240$	$270$	$300$	$330$	$360$
$y$	0	0.5	0.87	1	0.87	0.5	0	-0.5	-0.87	-1	-0.87	-0.5	0

Now plot the above points...

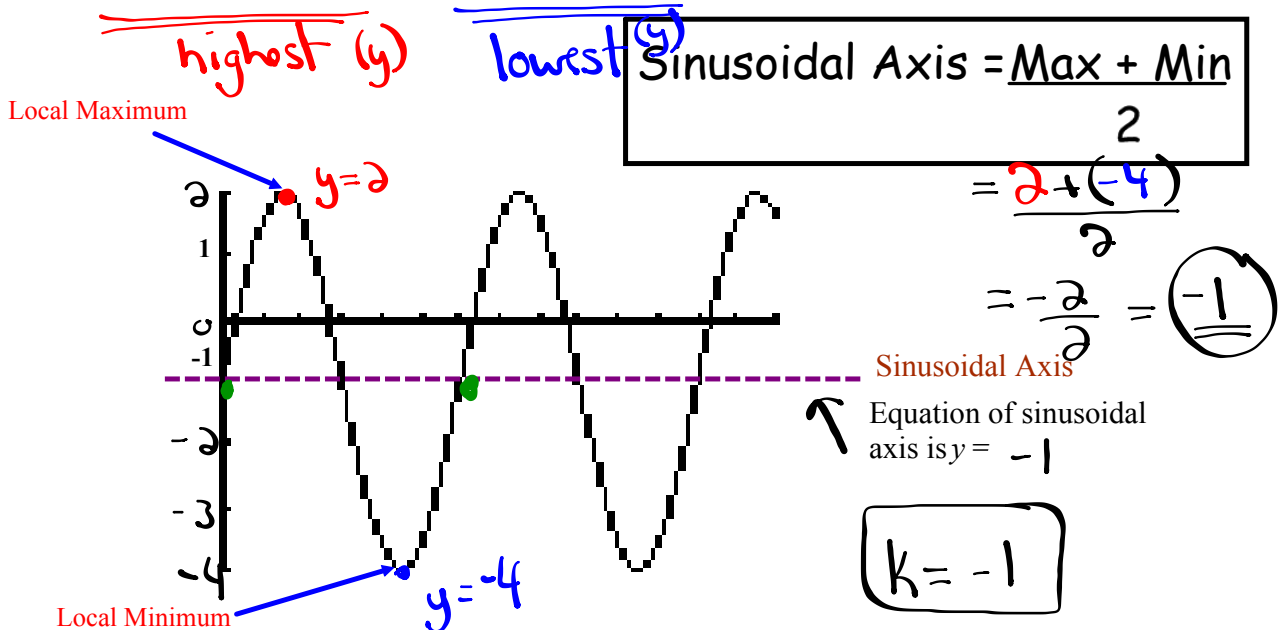


## Vocabulary of Sinusoidal Functions

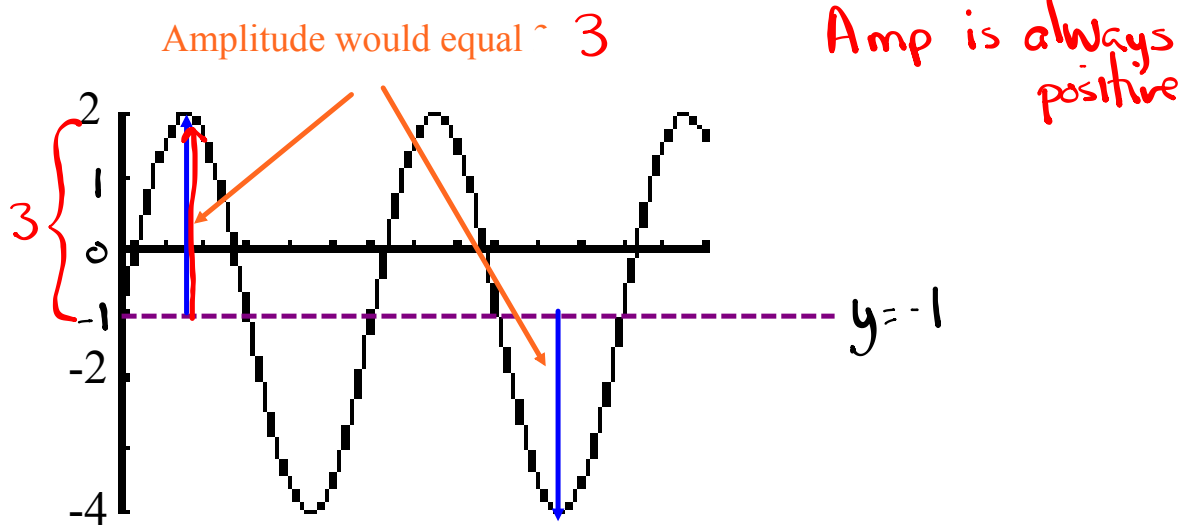
I. **Period:** The change in x corresponding to one cycle. *(one repetition)*



II. **Sinusoidal Axis:** The horizontal line halfway between the local maximum and local minimum.

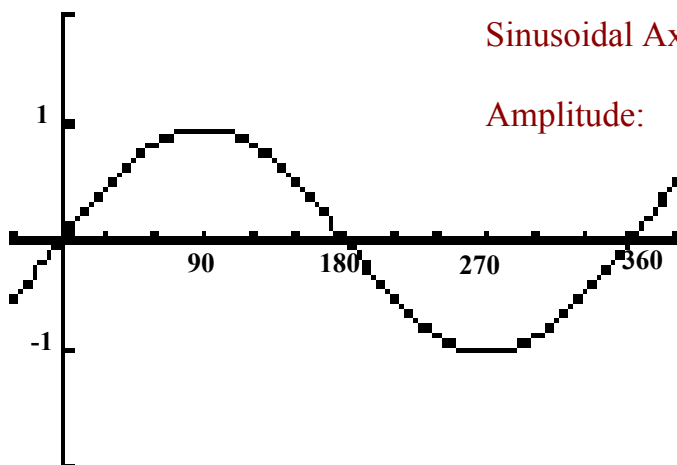


III. **Amplitude:** The vertical distance from the sinusoidal axis to a local maximum or local minimum. *Amplitude = |a|*



## Summarize...

Here is the graph of  $y = \sin \theta$



Period :

Sinusoidal Axis:

Amplitude:

What about  $y = \cos \theta$  ?

$y = \cos x$

Complete the table of values and sketch below

$\theta$	$\theta$	30	60	90	120	150	180	210	240	270	300	330	360
$y$		0.87	0.5	0	-0.5	-0.87	-1	-0.87	-0.5	0	0.5	0.87	1



Is this a sinusoidal function? *Yes (repeats + looks like waves)*  
 What about the period, sinusoidal axis, and amplitude?

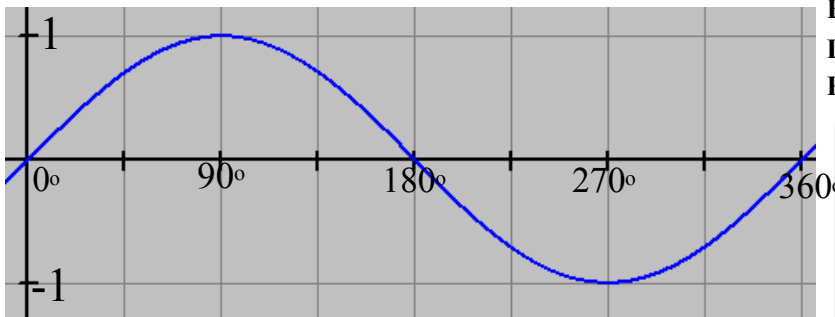
$Period = 360^\circ - 0^\circ = 360^\circ$

$sinusoidal\ axis = \frac{max + min}{2} = \frac{1 + (-1)}{2} = \frac{0}{2} = 0 \ (y=0)$

$Amplitude = 1$

## Basic Trig Graphs (Base Functions)

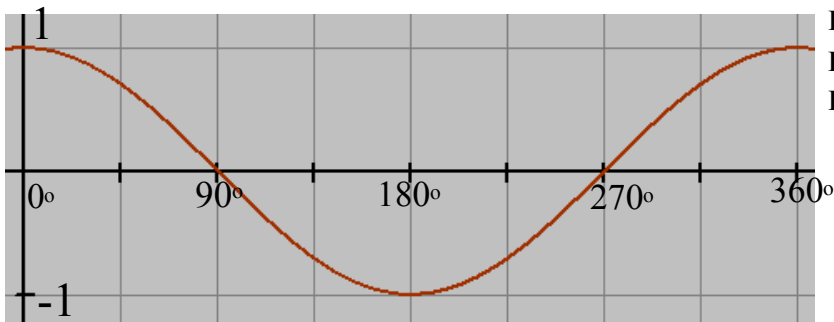
$$y = \sin \theta$$



Period =  $360^\circ$   
 Amplitude = 1  
 Eq'n of Sinusoidal Axis:  $y = 0$   
 Domain:  $\{\theta \in \mathbb{R}\}$   
 Range:  $\{-1 \leq y \leq 1\}$

$\theta$	$y$
$0^\circ$	0
$90^\circ$	1
$180^\circ$	0
$270^\circ$	-1
$360^\circ$	0

$$y = \cos \theta$$



Period =  $360^\circ$   
 Amplitude = 1  
 Eq'n of Sinusoidal Axis:  $y = 0$   
 Domain:  $\{\theta \in \mathbb{R}\}$   
 Range:  $\{-1 \leq y \leq 1\}$

$\theta$	$y$
$0^\circ$	1
$90^\circ$	0
$180^\circ$	-1
$270^\circ$	0
$360^\circ$	1

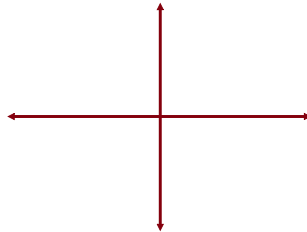
## Transformations of the Sinusoidal Function

Recall...

$$y = -2(x-3)^2 + 4$$

Vertex  $\Rightarrow$

Sketch  $\Rightarrow$



Now, let's look at a sinusoidal function...

$$y = -2 \sin[3(\theta - 60^\circ)] - 1$$

$a = -2 \rightarrow$  reflected in the x-axis and vertically stretched by a factor of 2 (Amp = 2)

$b = 3 \rightarrow$  horizontally stretched by a factor of  $\frac{1}{3}$ .

$$* P = \frac{360^\circ}{b} = \frac{360^\circ}{3} = 120^\circ$$

$h = 60^\circ \rightarrow$  translated  $60^\circ$  right (Phase Shift)

$k = -1 \rightarrow$  " 1 unit down

\* Sin axis:  $y = -1$

Mapping Rule:  $(x, y) \rightarrow \left[ \frac{1}{3}x + 60^\circ, -2y - 1 \right]$

$y = \sin x$		$\rightarrow$		
$x$	$y$		$x$	$y$
$0^\circ$	$0$		$60^\circ$	$-1$
$90^\circ$	$1$		$90^\circ$	$-3$
$180^\circ$	$0$		$120^\circ$	$-1$
$270^\circ$	$-1$		$150^\circ$	$1$
$360^\circ$	$0$		$180^\circ$	$-1$

## Equations in Standard Form

$$y = a \sin[b(x - c)] + d \quad \text{or} \quad y = a \cos[b(x - h)] + k$$

$a = \text{Amplitude}$  → influences how tall the sine curve is.  $\text{Amp} = |a|$   
or cosine

$b = \frac{360^\circ}{P}$  → influences how often the pattern repeats.  $b = \frac{2\pi}{P}$

$h = \text{Horizontal Translation}$  → Influences how far to the left or the right that the graph will shift.  $h = \text{Phase Shift}$

- If  $h$  is positive → Shift Left
  - If  $h$  is negative → Shift Right
- } Inside Brackets

$k = \text{Vertical Translation}$  → influences how far up and down the graph will shift.

- If  $k$  is positive → Shift Up
- If  $k$  is negative → Shift Down
- equal to the sinusoidal axis:  $y = k$

Example:

$$2y + 5 = -6 \sin\left(\frac{1}{3}x - 30^\circ\right) - 3 \quad (\text{Subtract 5 from both sides})$$

$$\frac{2y}{2} = \frac{-6 \sin\left(\frac{1}{3}x - 30^\circ\right) - 8}{2} \quad (\text{Divide by 2 a+k})$$

$$y = -3 \sin\left(\frac{1}{3}x - 30^\circ\right) - 4 \quad (\text{Factor out a } \frac{1}{3})$$

$$y = -3 \sin\left[\frac{1}{3}(x - 90^\circ)\right] - 4$$

$$y = \underline{-3} \sin\left[\underline{\frac{1}{3}}(x - \underline{90^\circ})\right] - \underline{4}$$

$$a = -3 \quad b = \frac{1}{3} \quad h = 90^\circ \quad k = -4$$

$$\text{Amp} = 3 \quad P = \frac{360^\circ}{\frac{1}{3}}$$

equation of sinusoidal axis:  $y = -4$

$$P = 360 \times 3$$

$$P = 1080^\circ$$



# Homework

Page 233 #1-9

ex:  $\frac{y}{2} - 5 = -4 \cos[3x - 90^\circ] - 7$

a.  $\frac{y}{2} = -4 \cos[3x - 90^\circ] - 2$

$$y = -8 \cos[3x - 90^\circ] - 4$$

$$y = -8 \cos[3(x - 30^\circ)] - 4$$

$$y = -8 \cos[3(x - 30^\circ)] - 4$$

$a = -8 \rightarrow$  vertically stretched by a factor of 8.  
vertically reflected in the x-axis.

$b = 3 \rightarrow$  horizontally stretched by a factor of  $\frac{1}{3}$ .

$h = 30^\circ \rightarrow$  horizontally translated  $30^\circ$  right.

$k = -4 \rightarrow$  vertically translated 4 units down.

# Questions from Homework

6. Match each function with its graph.

a)  $y = 3 \cos x$

$a = 3$

b)  $y = \cos 3x$

$b = 3$

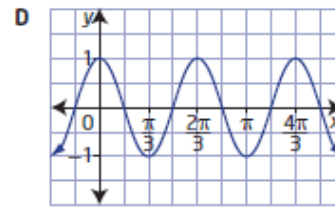
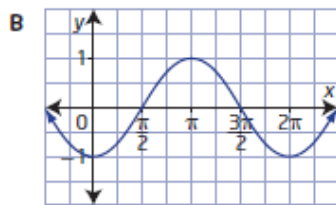
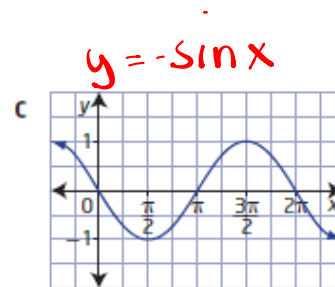
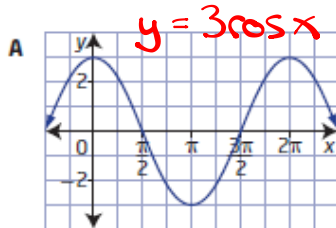
$\rightarrow b = \frac{2\pi}{3}$

c)  $y = -\sin x$

$a = -1$

d)  $y = -\cos x$

$a = -1$



$y = -\cos x$

$y = \cos 3x$

From Sheet:

$$h^{\circ} \frac{1}{a}(y+a) = 3 \cos(x-90^{\circ})$$

$$y+a = 6 \cos(x-90^{\circ})$$

$$y = \underline{6} \cos(x - \underline{90^{\circ}}) - a$$

$a = 6$      $h = 90^{\circ}$     equation of sin axis:  $y = -a$

$b = 1$      $k = -a$      $P = \frac{360^{\circ}}{b} = \frac{360^{\circ}}{1} = 360^{\circ}$

## Worksheet

$$\textcircled{1} \text{ d) } y - 5 = 6 \cos\left[\frac{1}{3}\left(x - \frac{\pi}{2}\right)\right] - 2$$

$$y = \underline{6} \cos\left[\frac{1}{3}\left(x - \frac{\pi}{2}\right)\right] + \underline{3}$$

$$a = 6 \quad h = \frac{\pi}{2} \quad \text{equation of sin. axis: } y = 3$$

$$b = \frac{1}{3} \quad k = 3 \quad P = \frac{2\pi}{b} = 2\pi \div \frac{1}{3} = 2\pi \cdot \frac{3}{1} = 6\pi$$

$$\text{g) } y + 5 = -2 \sin\left(4x + \frac{\pi}{3}\right)$$

$$y = -2 \sin\left(4x + \frac{\pi}{3}\right) - 5 \quad (\text{Factor out a 4})$$

$$y = \underline{-2} \sin\left[\underline{4}\left(x + \frac{\pi}{12}\right)\right] - \underline{5}$$

$$\frac{\pi}{3} \div 4$$

$$\frac{\pi}{3} \times \frac{1}{4} = \frac{\pi}{12}$$

$$a = -2 \quad h = -\frac{\pi}{12} \quad \text{equation of sin. axis: } y = -5$$

$$b = 4 \quad k = -5 \quad P = \frac{2\pi}{b} = \frac{2\pi}{4} = \frac{\pi}{2}$$

## Worksheet

$$h) \frac{1}{2}(y+d) = 3\cos(x-90^\circ)$$

$$(y+d) = 6\cos(x-90^\circ)$$

$$y+d = 6\cos(x-90^\circ)$$

$$y = \underline{6}\cos(x - \underline{90^\circ}) - d$$

$$a = 6$$

$$b = 1$$

$$h = 90^\circ$$

$$k = -d$$

$$\text{Amp} = 6$$

$$p = \frac{360^\circ}{1} = 360^\circ$$

$$\text{sin axis: } y = -d$$

## Sketching Sinusoidal Functions using Transformations

Development of a standard form for sinusoidal functions...

Standard Form  $\longrightarrow y = a \sin[b(x - h)] + k$

1. Reflection: If  $a < 0$  the graph will be reflected in the  $x$ -axis.  
 *$b < 0$  " " " " " " " "  $y$ -axis*
2. Amplitude: The amplitude of the graph will be equal to  $|a|$ .
3. Period: The period of the graph will be equal to  $\frac{360^\circ}{b}$  or  $\frac{2\pi}{b}$
4. Horizontal Phase Shift: The graph will shift  $h$  units to the right/*(Left)*  
*(Translation)*
5. Vertical Translation: The graph will shift  $k$  units up/*down*

Mapping Notation:  $(x, y) \rightarrow \left(\frac{1}{b}x + h, ay + k\right)$

## Transformations of Sinusoidal Functions

Example:  $f(\theta) = \underline{-2} \sin \underline{3}(\theta + 30^\circ) - 2$

$a = -2$        $b = 3$        $h = -30^\circ$        $k = -2$

Range:  $\max = k + \text{Amp} = -2 + 2 = 0$

$\min = k - \text{Amp} = -2 - 2 = -4$

Domain	$\{ \theta \mid \theta \in \mathbb{R} \}$ or $(-\infty, \infty)$
Range	$\{ y \mid -4 \leq y \leq 0, y \in \mathbb{R} \}$ or $[-4, 0]$
Reflection	<i>in the <math>x</math>-axis (<math>a &lt; 0</math>)</i>
Amplitude	$\text{Amp} =  a  =  -2  = 2$
Horizontal Phase Shift	$30^\circ$ left ( $h = -30^\circ$ )
Vertical Translation	$2$ down ( $k = -2$ )
Period	$P = \frac{360^\circ}{b} = \frac{360^\circ}{3} = 120^\circ$

sinusoidal axis

$y = -2$

### EXAMPLE #1

Now let's sketch a graph of  $y = \underline{3} \cos[2(\theta - \underline{135^\circ})] + \underline{2}$

$a = 3$

$b = 2$

$h = 135^\circ$

$k = 2$

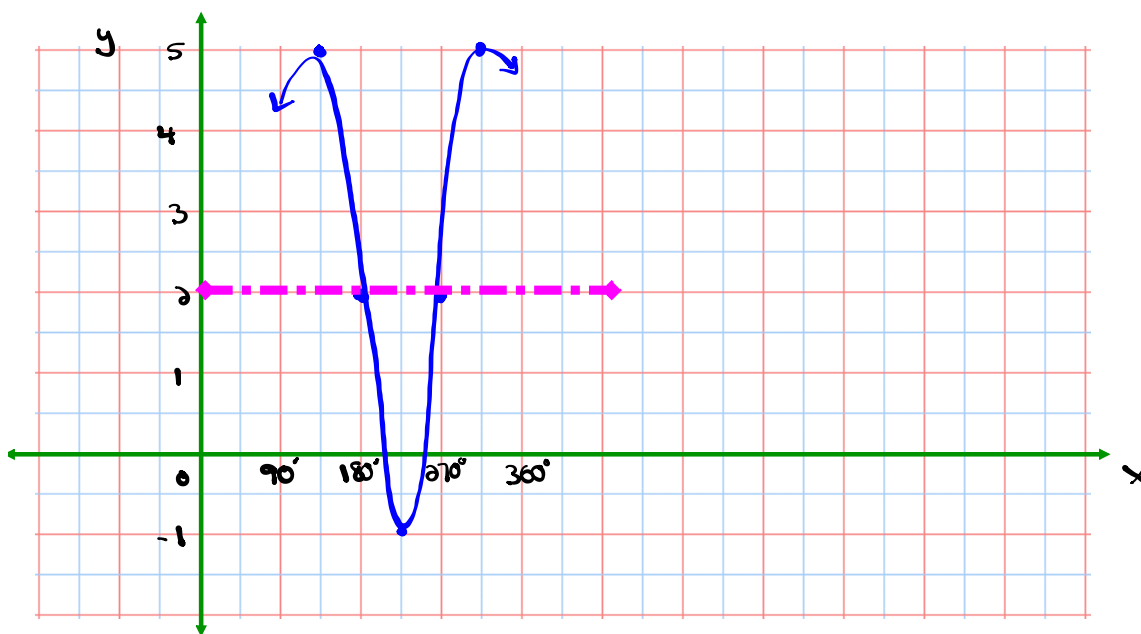
$y = \cos \theta$

$(\theta, y) \rightarrow \left[ \frac{1}{2}\theta + 135^\circ, 3y + 2 \right]$

$\theta$	$y$
$0^\circ$	1
$90^\circ$	0
$180^\circ$	-1
$270^\circ$	0
$360^\circ$	1

New points after mapping

$\theta$	$y$
$135^\circ$	5
$180^\circ$	2
$225^\circ$	-1
$270^\circ$	2
$315^\circ$	5



DOMAIN	$\{\theta   \theta \in \mathbb{R}\}$
RANGE	$\{y   -1 \leq y \leq 5, y \in \mathbb{R}\}$
AMPLITUDE	$Amp =  a  =  3  = 3$
PERIOD	$P = \frac{360^\circ}{b} = \frac{360^\circ}{2} = 180^\circ$
PHASE SHIFT	$135^\circ$ right ( $h = 135^\circ$ )
VERTICAL TRANSLATION	2 up ( $k = 2$ )
EQUATION OF SINUSOIDAL AXIS	$y = 2$

## Use Mapping to Graph

$$\frac{3y}{3} = \frac{-6 \cos(3x - \pi) - 9}{3}$$

$$a = -2$$

$$b = 3$$

$$y = -2 \cos(3x - \pi) - 3$$

$$h = \frac{\pi}{3}$$

$$y = -2 \cos\left[3\left(x - \frac{\pi}{3}\right)\right] - 3$$

$$k = -3$$

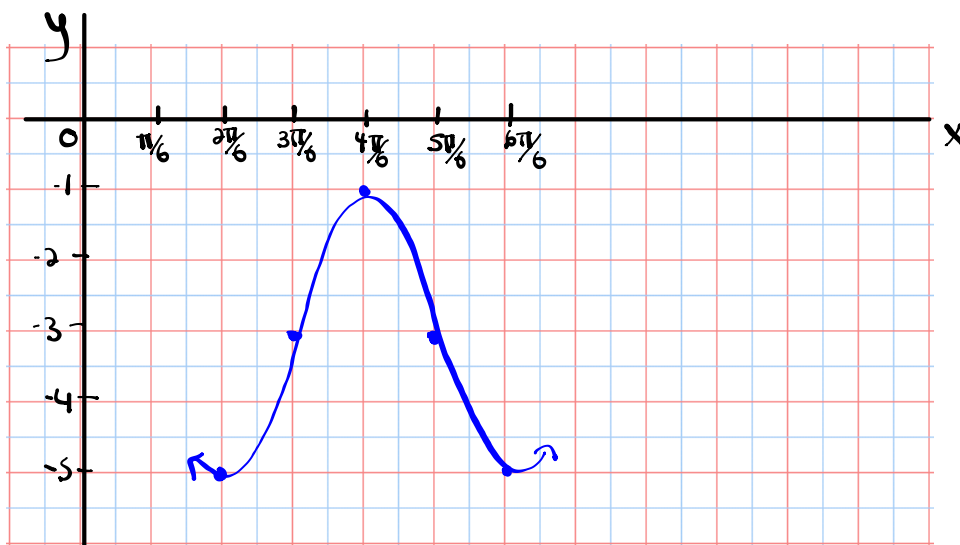
$$y = -2 \cos\left[3\left(x - \frac{\pi}{3}\right)\right] - 3$$

$$y = \cos x \quad (x, y) \rightarrow \left[\frac{1}{3}x + \frac{\pi}{3}, -2y - 3\right]$$

x	y
0	1
$\frac{\pi}{2}$	0
$\pi$	-1
$\frac{3\pi}{2}$	0
$2\pi$	1

New points after mapping

x	y
$\frac{2\pi}{6} = \frac{\pi}{3}$	-5
$\frac{3\pi}{6} = \frac{\pi}{2}$	-3
$\frac{4\pi}{6} = \frac{2\pi}{3}$	-1
$\frac{5\pi}{6}$	-3
$\frac{6\pi}{6} = \pi$	-5



### Homework

DOMAIN	
RANGE	
AMPLITUDE	
PERIOD	
PHASE SHIFT	
VERTICAL TRANSLATION	
EQUATION OF SINUSOIDAL AXIS	

$$\frac{1}{3}x + \frac{\pi}{3}$$

$(i) \frac{1}{3}(0) + \frac{\pi}{3}$ $0 + \frac{\pi}{3}$ $\left(\frac{\pi}{3}\right)$ $\frac{2\pi}{6}$	$(ii) \frac{1}{3}\left(\frac{\pi}{2}\right) + \frac{\pi}{3}$ $\frac{\pi}{6} + \frac{\pi}{3}$ $\frac{\pi}{6} + \frac{2\pi}{6}$ $\frac{3\pi}{6}$ $\left(\frac{\pi}{2}\right)$ $\frac{3\pi}{6}$	$(iii) \frac{1}{3}(\pi) + \frac{\pi}{3}$ $\frac{\pi}{3} + \frac{\pi}{3}$ $\left(\frac{2\pi}{3}\right)$ $\frac{4\pi}{6}$	$(iv) \frac{1}{3}\left(\frac{3\pi}{2}\right) + \frac{\pi}{3}$ $\frac{3\pi}{6} + \frac{\pi}{3}$ $\frac{3\pi}{6} + \frac{2\pi}{6}$ $\left(\frac{5\pi}{6}\right)$ $\frac{5\pi}{6}$	$(v) \frac{1}{3}(2\pi) + \frac{\pi}{3}$ $\frac{2\pi}{3} + \frac{\pi}{3}$ $\frac{3\pi}{3}$ $\left(\pi\right)$ $\frac{6\pi}{6}$
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### Homework

State ***a, b, h, k, and P*** from the following sinusoidal equations:

$$2y + 6 = 4\sin\left(4x + \frac{\pi}{2}\right) - 2$$

Given the function  $y = -2 \sin(2\theta - 30^\circ) + 3$



Remember...Put in standard form first!!

Complete the chart below and sketch the graph of this function.

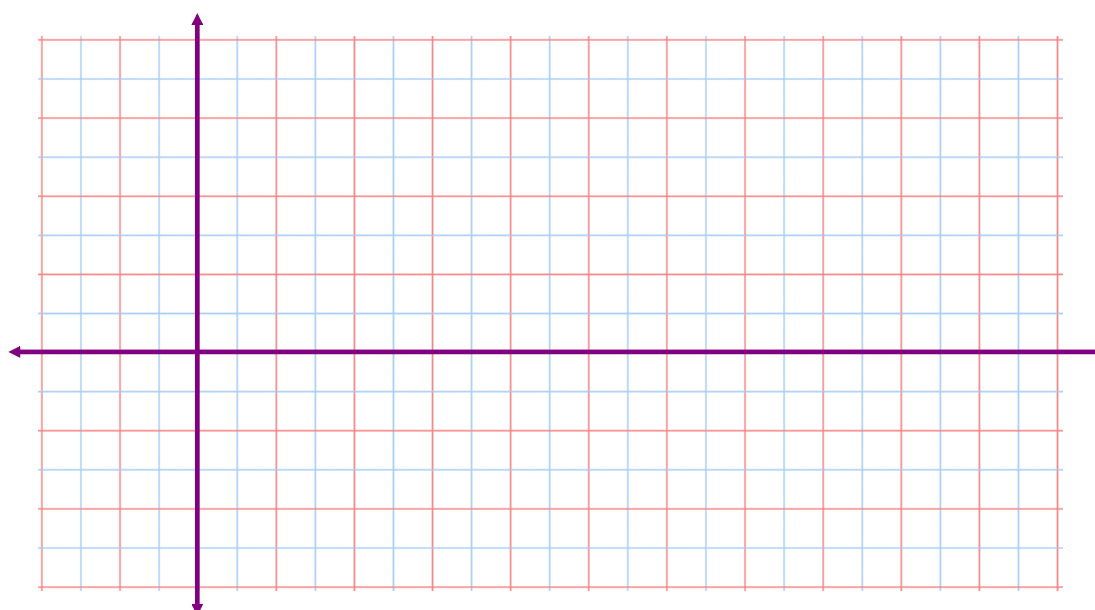
Mapping:

DOMAIN	
RANGE	
AMPLITUDE	
PERIOD	
PHASE SHIFT	
VERTICAL TRANSLATION	
EQUATION OF SINUSOIDAL AXIS	

$\theta$	$y$
0	
90	
180	
270	
360	

New points after mapping →

$\theta$	$y$



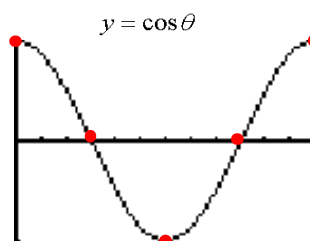


Hopefully you are not too puzzled for this one...

$$\frac{1}{2}(y+1) = 3 \cos\left(\frac{1}{2}\theta - 90^\circ\right) + 2$$

Remember...Put in standard form first!!

Remember what the graph of cosine looks like ??



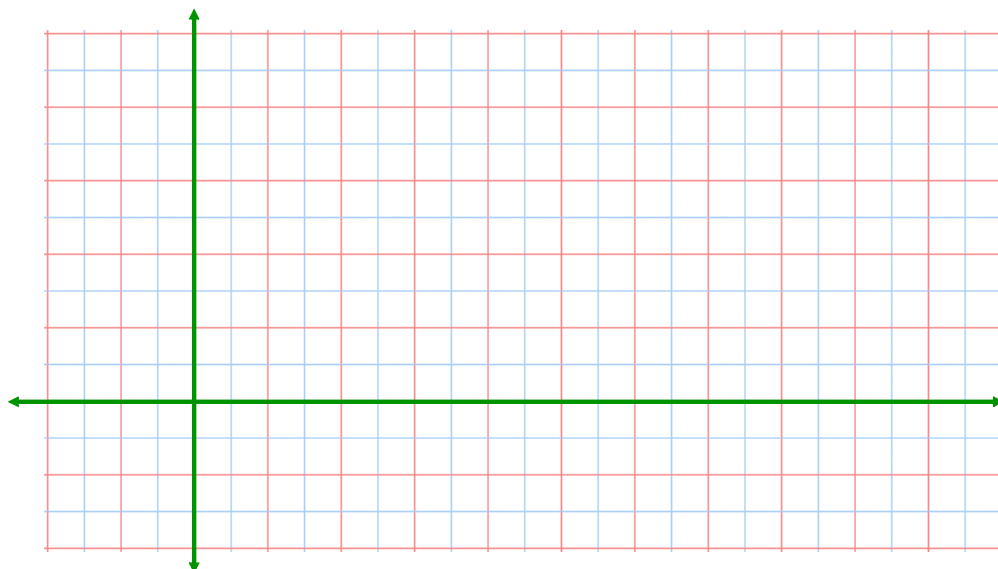
Mapping:

$\theta$	$y$
0	
90	
180	
270	
360	

New points after mapping

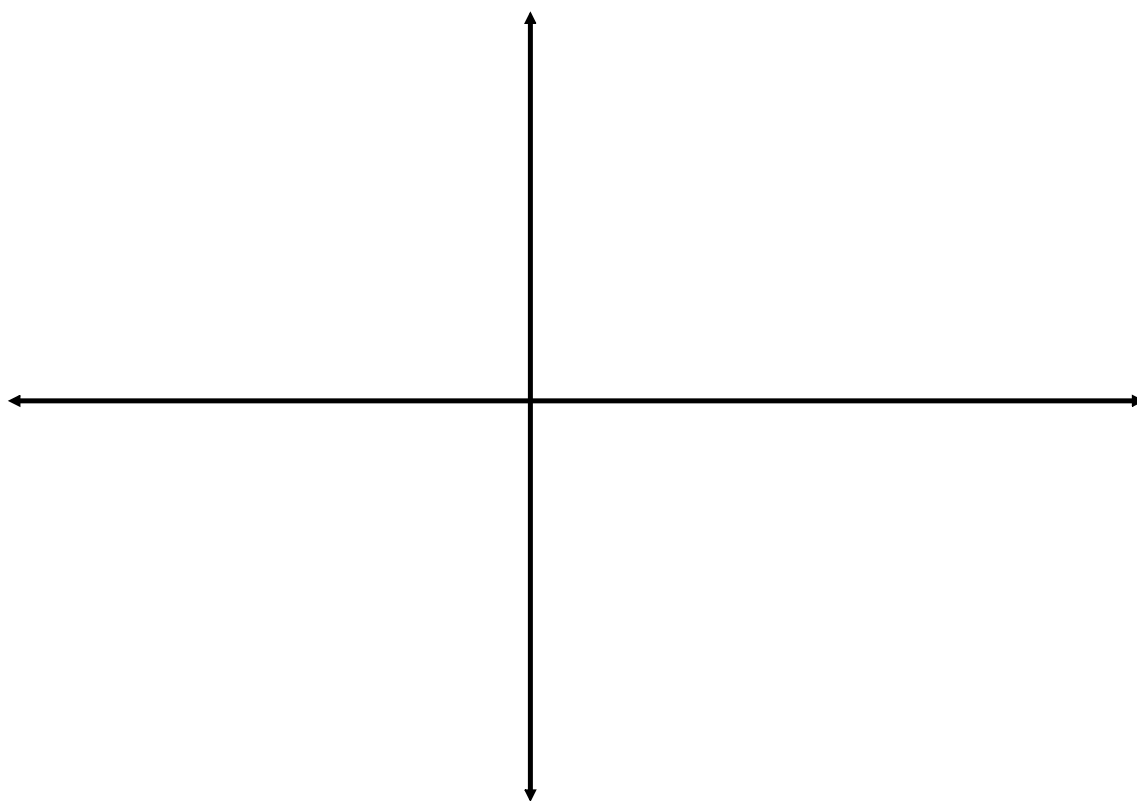
$\theta$	$y$

DOMAIN	
RANGE	
AMPLITUDE	
PERIOD	
PHASE SHIFT	
VERTICAL TRANSLATION	
EQUATION OF SINUSOIDAL AXIS	



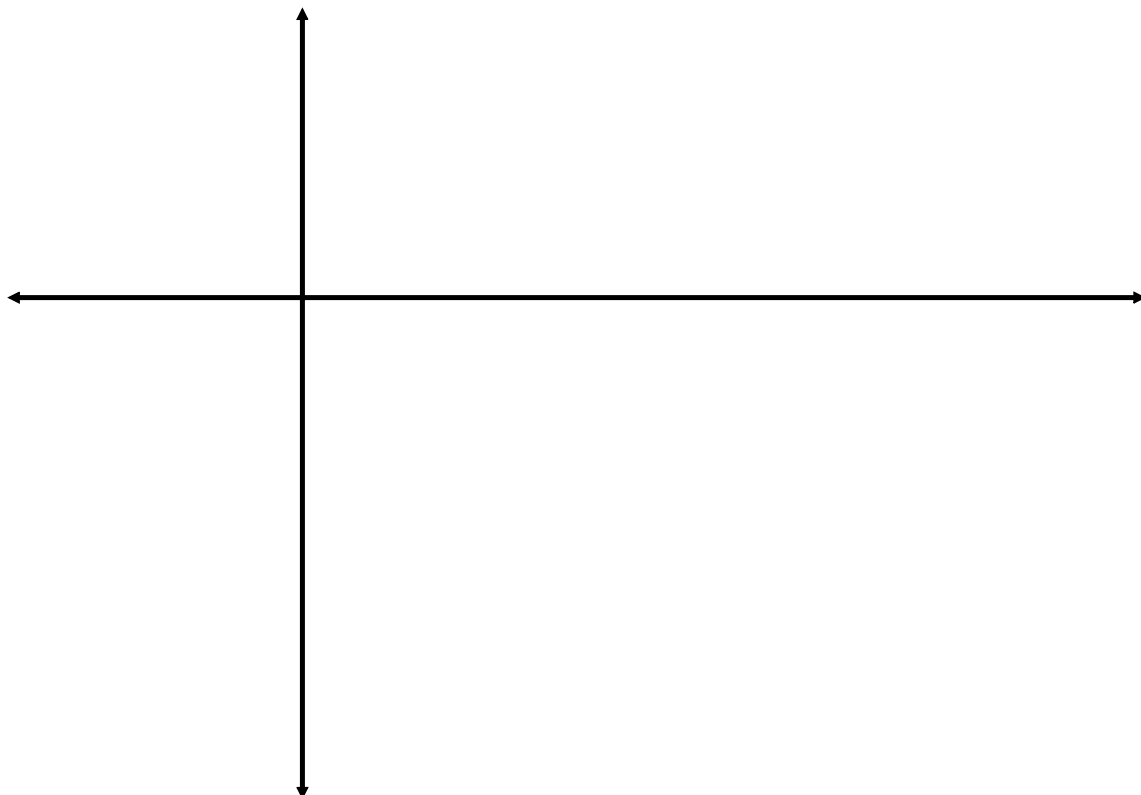
$$\text{EX. } y = \cos\left(2x - \frac{\pi}{3}\right) - 1$$

AMPLITUDE	
PERIOD	
PHASE SHIFT	
VERTICAL TRANSLATION	
EQUATION OF SINUSOIDAL AXIS	



## Example...

Graph the equation  $y = -3 \sin(2\theta + \pi) + 1$  using mapping notation.



# Homework

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## Worksheet # 1 - 8

*Worksheet - Sketching Sinusoidal Relations*

## Solutions to the homework

①  $y = 3\cos(x) - 2$

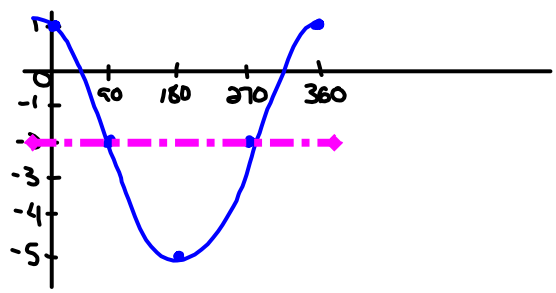
$A = 3 \quad b = 1 \quad C = 0 \quad D = -2 \quad P = 360^\circ$

$y = \cos x$

x	y
0	1
90	0
180	-1
270	0
360	1



x	y
0	1
90	-2
180	-5
270	-2
360	1



②  $y = -\sin(2x - \frac{\pi}{6})$

$y = -\sin[2(x - \frac{\pi}{12})]$

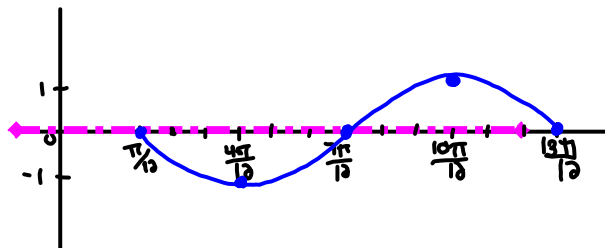
$A = 1 \quad b = 2 \quad C = \frac{\pi}{12} \quad D = 0 \quad P = \pi$

$y = \sin x$

x	y
0	0
$\frac{\pi}{2}$	1
$\pi$	0
$\frac{3\pi}{2}$	-1
$2\pi$	0



x	y
$\frac{\pi}{12}$	0
$\frac{\pi}{3}$	-1
$\frac{5\pi}{12}$	0
$\frac{2\pi}{3}$	1
$\frac{7\pi}{12}$	0



③  $y = 4\sin(3x - 180^\circ) + 2$

$y = 4\sin[3(x - 60^\circ)] + 2$

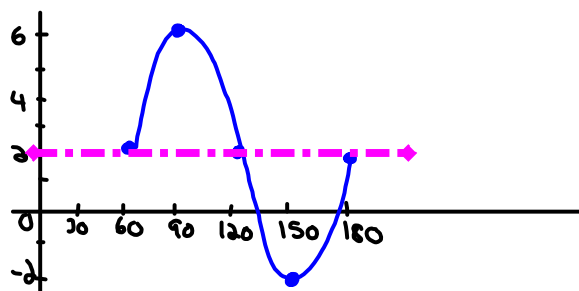
$A = 4 \quad b = 3 \quad C = 60 \quad D = 2 \quad P = 120^\circ$

$y = \sin x$

x	y
0	0
90	1
180	0
270	-1
360	0

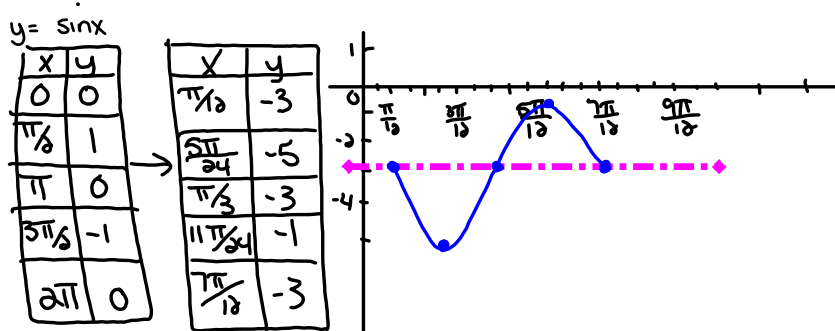


x	y
60	2
90	6
120	2
150	-2
180	2



$$\begin{aligned} \textcircled{5} \quad 2y+3 &= -4\sin\left(4x-\frac{\pi}{12}\right)-3 \\ 2y &= -4\sin\left[4\left(x-\frac{\pi}{12}\right)\right]-6 \\ y &= -2\sin\left[4\left(x-\frac{\pi}{12}\right)\right]-3 \end{aligned}$$

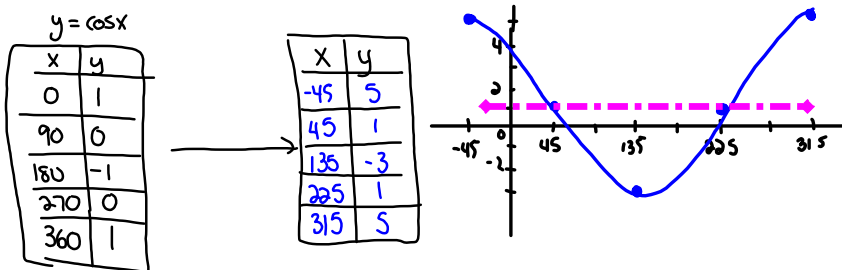
$$A=2 \quad b=4 \quad C=\frac{\pi}{12} \quad D=-3 \quad P=\frac{\pi}{2}$$



$$\textcircled{6} \quad y-1 = 2\cos(\theta+45^\circ)+0$$

$$\begin{aligned} y-1 &= 4\cos(\theta+45^\circ)+0+1 \\ y &= 4\cos(\theta+45^\circ)+1 \end{aligned}$$

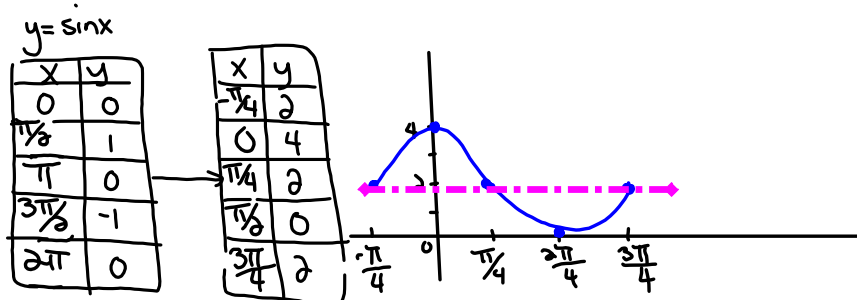
$$A=4 \quad b=1 \quad C=45 \quad D=1 \quad P=360$$



$$\begin{aligned} \textcircled{1} \quad \frac{1}{2}y-1 &= \sin\left[2\left(x+\frac{\pi}{4}\right)\right] \\ \frac{1}{2}y &= \sin\left[2\left(x+\frac{\pi}{4}\right)\right]+1 \end{aligned}$$

$$y = 2\sin\left[2\left(x+\frac{\pi}{4}\right)\right]+2$$

$$A=2 \quad b=2 \quad C=-\frac{\pi}{4} \quad D=2 \quad P=\pi$$





$$\textcircled{8} \quad y = -4 \cos(3x + 90^\circ) - 2$$

$$y = -4 \cos[3(x + 30)] - 2$$

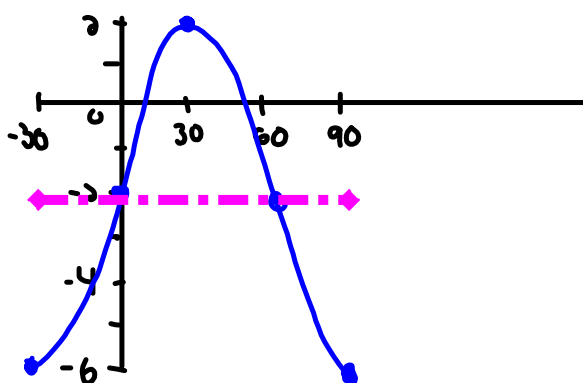
$$A = 4 \quad b = 3 \quad c = -30 \quad D = -2 \quad P = 120$$

$$y = \cos x$$

x	y
0	1
90	0
180	-1
270	0
360	1



x	y
-30	-6
0	-2
30	2
60	-2
90	-6



# Extra Practice

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 Worksheet - Sketching Trigonometric Functions.doc

## Questions from the homework???

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 Worksheet Solns - Sketching Sinusoidal Relations.doc

## Attachments

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worksheet-sketching in radian measure.doc

Worksheet - Finding the Equation.doc

Worksheet - Sketching Trigonometric Functions.doc

Worksheet Solns - Sketching Sinusoidal Relations.doc

Worksheet - Sketching Sinusoidal relations (sept06).pdf

Bonus Soln - Fox Population.doc

Worksheet Solns - Applications of Sinusoidal Relations.doc

Review - Practice Test for Sinusoidal Functions.doc

Review - Trigonometric Functions(3)(4).doc