

Warm Up

Evaluate the following limits:

$$\lim_{x \rightarrow \frac{\pi}{4}} \frac{\sin x - \cos x}{x - \frac{\pi}{4}}$$

$$\lim_{x \rightarrow \frac{\pi}{4}} \frac{\cos x + \sin x}{1}$$

$$= \frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2}$$

$$= \frac{2\sqrt{2}}{2}$$

$$= \sqrt{2}$$

$$\lim_{x \rightarrow 0} \frac{\text{Arc sin } x}{x}$$

$$\lim_{x \rightarrow 0} \frac{1}{\sqrt{1-x^2}}$$

$$= 1$$

$$\lim_{x \rightarrow 0} \frac{e^x - 1}{\sin 2x}$$

$$\lim_{x \rightarrow 0} \frac{e^x}{2 \cos 2x}$$

$$= \frac{1}{2}$$

$$\lim_{x \rightarrow \infty} \frac{10x^2}{e^x}$$

$$\lim_{x \rightarrow \infty} \frac{20x}{e^x}$$

$$\lim_{x \rightarrow \infty} \frac{20}{e^x} = 0$$

Sigma Notation

A series is the sum of a sequence. We can write a series using sigma notation.

$$\sum_{i=1}^n t_i = t_1 + t_2 + t_3 + \dots + t_n$$

$$1 + 2 + 4 + \dots + 64 = \sum_{i=1}^7 2^{n-1}$$

the terms form a geometric sequence
with $a = 1$, $r = 2$, $t_n = 1(2)^{n-1}$

This symbol is read as "the sum of the terms
of the sequence given by $t_n = 2^{n-1}$ from $n = 1$
to $n = 7$ "

$$t_n = ar^{n-1} \text{ (geometric)}$$

$$t_n = (1)(2)^{n-1}$$

Example 1

Express the series 1 + 3 + 5 + 7 + 9 in sigma notation.

(arithmetic)

$$t_n = a + (n-1)d$$

$$t_n = 1 + (n-1)(2)$$

$$t_n = 1 + 2n - 2$$

$$t_n = 2n - 1$$

$$\sum_{i=1}^5 2n-1$$

$$d = 3-1 = 5-3 = 7-5$$

$$d = 2$$

$$a = 1$$

The properties of Sigma Notation that we use in this section are summarized below:

$$\sum_{i=1}^n c = c + c + c + \dots + c = nc \quad \left| \quad \sum_{i=1}^5 4 = 5(4) = 20$$

$$\sum_{i=1}^n ct_i = c \sum_{i=1}^n t_i, \quad c \text{ is a constant} \quad \left| \quad \sum_{i=1}^4 3t_i = 3 \sum_{i=1}^4 t_i$$

$$\sum_{i=1}^n (t_i + s_i) = \sum_{i=1}^n t_i + \sum_{i=1}^n s_i \quad \left| \quad \sum_{i=1}^n (4i^2 + 7i) = \sum_{i=1}^n 4i^2 + \sum_{i=1}^n 7i$$

$$= 4 \sum_{i=1}^n i^2 + 7 \sum_{i=1}^n i$$

Example 2

Use the basic properties of sigma notation to express
in terms of monomial summations.

$$\sum_{i=1}^n (3i-2)^2$$

$$\sum_{i=1}^n (3i-2)(3i-2)$$

$$\sum_{i=1}^n (9i^2 - 12i + 4)$$

$$\sum_{i=1}^n 9i^2 + \sum_{i=1}^n -12i + \sum_{i=1}^n 4$$

$$\left[9 \sum_{i=1}^n i^2 - 12 \sum_{i=1}^n i + 4n \right]$$

$$9 \left[\frac{n(n+1)(2n+1)}{6} \right] - 12 \left[\frac{n(n+1)}{2} \right] + 4n$$

The following sigma formulas will be extremely useful in the next few days when we are faced with the challenge of calculating the area under a curve.

$$\sum_{i=1}^n i = 1 + 2 + 3 + 4 + \dots + n = \frac{n(n+1)}{2} \quad \text{Linear}$$

$$\sum_{i=1}^n i^2 = 1^2 + 2^2 + 3^2 + 4^2 + \dots + n^2 = \frac{n(n+1)(2n+1)}{6} \quad \text{Quadratic}$$

$$\sum_{i=1}^n i^3 = 1^3 + 2^3 + 3^3 + 4^3 + \dots + n^3 = \left[\frac{n(n+1)}{2} \right]^2 \quad \text{Cubic}$$

Example 3

$$\sum_{i=1}^n (3i^2 - 2i)$$

$$= \sum_{i=1}^n 3i^2 + \sum_{i=1}^n -2i$$

$$= 3 \sum_{i=1}^n i^2 - 2 \sum_{i=1}^n i$$

$$= 3 \left[\frac{n(n+1)(2n+1)}{6} \right] - 2 \left[\frac{n(n+1)}{2} \right]$$

$$= \frac{n(n+1)(2n+1)}{2} - \frac{n(n+1)}{1}$$

$$= \frac{n(n+1)(2n+1)}{2} - \frac{2n(n+1)}{2}$$

$$= \frac{n(n+1)(2n+1) - 2n(n+1)}{2} \quad \text{factor}$$

$$= \frac{n(n+1)[(2n+1) - 2]}{2}$$

$$= \frac{n(n+1)(2n-1)}{2}$$

Example 4

$$\sum_{i=11}^{20} (2i^2 - 3i) = \sum_{i=1}^{20} (2i^2 - 3i) - \sum_{i=1}^{10} (2i^2 - 3i)$$

$$= \sum_{i=1}^{20} 2i^2 - \sum_{i=1}^{20} 3i - \left(\sum_{i=1}^{10} 2i^2 - \sum_{i=1}^{10} 3i \right)$$

$$= \underbrace{2 \sum_{i=1}^{20} i^2}_{\text{Quad}} - \underbrace{3 \sum_{i=1}^{20} i}_{\text{Lin}} - \underbrace{2 \sum_{i=1}^{10} i^2}_{\text{Quad}} + \underbrace{3 \sum_{i=1}^{10} i}_{\text{Lin}}$$

$$= 2 \left[\frac{n(n+1)(2n+1)}{3 \cancel{6}} \right] - 3 \left[\frac{n(n+1)}{2} \right] - 2 \left[\frac{n(n+1)(2n+1)}{3 \cancel{6}} \right] + 3 \left[\frac{n(n+1)}{2} \right]$$

$$= \frac{20(21)(41)}{3} - \frac{3(20)(21)}{2} - \frac{(10)(11)(21)}{3} + \frac{3(10)(11)}{2}$$

$$= 5740 - 630 - 770 + 165$$

$$= \boxed{4505}$$

$$\sum_{i=11}^{20} (2i^2 - 3i) = [2(11)^2 - 3(11)] + [2(12)^2 - 3(12)] + \dots + [2(20)^2 - 3(20)]$$
$$= 209 + 252 + 299 + 350 + 405 + 464 + 527 + 594 + 665 + 740$$
$$= 4505$$

Homework

Page 447 #1-3 omit ~~3-d, e~~ a, d, e
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