Questions From Homework

(2) 2) $\int \frac{x+1}{x^2+2x-6}\,dx$

$u = x^2+2x-6$

$du = 2x+2\,dx$

$\frac{1}{2}du = x+1\,dx$

$= \int \frac{1}{u} \cdot \frac{1}{2}\,du$

$= \frac{1}{2}\int \frac{1}{u}\,du$

$= \frac{1}{2}\ln|u| + C$

$= \frac{1}{2}\ln|x^2+2x-6| + C$

Questions From Homework

(4) c) $$\int_{\pi/6}^{\pi/2} \frac{\cos\theta}{\sin^3\theta}\,d\theta = \int_{1/2}^{1} \frac{1}{u^3}\cdot du$$

$u = \sin\theta$

$\frac{du}{d\theta} = \cos\theta$

$du = \cos\theta\, d\theta$

θ	u
$\pi/2$	1
$\pi/6$	$1/2$

$$= \int_{1/2}^{1} u^{-3}\,du$$

$$= \frac{u^{-2}}{-2}\Big|_{1/2}^{1}$$

$$= -\frac{1}{2u^2}\Big|_{1/2}^{1}$$

$$= \frac{-1}{2(1)^2} - \frac{-1}{2(1/2)^2}$$

$$= \frac{-1}{2} - -2$$

$$= -\frac{1}{2} + \frac{4}{2} = \frac{3}{2}$$

Warm Up

$$\int 5x^2 \sin(4x^3+1)dx \qquad = -\frac{5}{12}\cos(4x^3-1)+C$$

$u = 4x^3+1$

$du = 12x^2\,dx$

$\frac{5}{12}du = 5x^2 dx$

$= \int \sin u \cdot \frac{5}{12} du$

$= \frac{5}{12}\int \sin u\, du$

$= \frac{5}{12}(-\cos u) + C = \frac{-5}{12}\cos u + C = \boxed{\frac{-5}{12}\cos(4x^3+1)+C}$

$$\int x\sqrt{2x^2-5}\,dx \qquad = \frac{(2x^2-5)^{3/2}}{6}+C$$

$u = 2x^2-5$

$du = 4x\,dx$

$\frac{1}{4}du = x\,dx$

$= \int u^{1/2} \cdot \frac{1}{4} du$

$= \frac{1}{4}\int u^{1/2} du$

$= \frac{1}{4}\left(\frac{2}{3}u^{3/2}\right) + C = \frac{2u^{3/2}}{12} + C = \boxed{\frac{1}{6}(2x^2-5)^{3/2} + C}$

$$\int \cot x\,dx \qquad = \ln|\sin x| + C$$

$= \int \frac{\cos x}{\sin x} dx$

$u = \sin x$

$du = \cos x\,dx$

$= \int \frac{1}{u} \cdot du$

$= \ln|u| + C$

$= \boxed{\ln|\sin x| + C}$

Differential and Integral Calculus 120

$\int$ Integration by Parts $\int$

$$f(x)g(x) = f'(x)g(x) + f(x)g'(x)$$

As we have discussed before, every differentiation rule has a corresponding integration rule.

The rule that corresponds to the Product Rule for differentiation is called the rule for integration by parts.

The product rule stated that if f and g are differentiable functions, then

$$\frac{d}{dx}[f(x)g(x)] = f(x)g'(x) + g(x)f'(x)$$

In the notation for indefinite integrals this equation becomes... $\int [f(x)g'(x)dx + g(x)f'(x)dx] = f(x)g(x)$

or $\int f(x)g'(x)dx + \int g(x)f'(x)dx = f(x)g(x)$

which can be rearranged as:

$$\boxed{\int f(x)g'(x)dx = f(x)g(x) - \int g(x)f'(x)dx}$$

this formulas above is called

the **formula for integration by parts**

It is perhaps easier to remember in the following notation..... Let $u = f(x)$ and $v = g(x)$

then the differentials are: $du = f'(x)dx$ $dv = g'(x)dx$

And by the Substitution Rule, the formulas becomes...

$$\int f(x)g'(x)dx = f(x)g(x) - \int g(x)f'(x)dx$$

Integration By Parts

$$\int udv = uv - \int vdu$$

Let's do an example.... Find: $\int \underbrace{x}_{u}\underbrace{\sin x dx}_{dv}$

It helps when you stick to this pattern:

we need to make an appropriate choice for u and dv

$u =$ x $dv =$ sinx dx

$du =$ 1dx $v =$ -cosx

Again, the goal in using integration by parts is to obtain a simpler integral than the one we started with... so we must decide on what u and dv are very carefully!

In general, when deciding on a choice for u and dv, we usually try to choose u = f(x) to be a function that becomes simpler when differentiated...

(or at least NOT more complicated)

as long as dv = g'(x)dx can be readily integrated to give v.

$$\int \underbrace{x}_{u}\underbrace{\sin x\, dx}_{dv} = x(-\cos x) - \int -\cos x\, dx$$

$$= -x\cos x + \int \cos x\, dx$$

$$\boxed{= -x\cos x + \sin x + C}$$

Find: $\int \underbrace{x}_{u}\underbrace{e^x dx}_{dv} = xe^x - \int e^x dx$

$$\boxed{= xe^x - e^x + C}$$

It helps when you stick to this pattern:

$u = $ x $\quad dv = $ $e^x dx$

$du = $ $1dx$ $\quad v = $ e^x

Find: $\int \underbrace{x}_{u} \underbrace{\cos(3x)dx}_{dv} = x\left(\frac{1}{3}\sin(3x)\right) - \int \frac{1}{3}\sin 3x\, dx$

$= \frac{1}{3}x\sin 3x - \frac{1}{3}\int \sin 3x\, dx$

$= \frac{1}{3}x\sin 3x - \frac{1}{3}\left(-\frac{1}{3}\cos 3x\right)$

$\boxed{= \frac{1}{3}x\sin 3x + \frac{1}{9}\cos 3x + C}$

It helps when you stick to this pattern:

$u =$ x $dv =$ $\cos(3x)dx$

$du =$ $1dx$ $v =$ $\frac{1}{3}\sin(3x)$

Find: $\int \underbrace{\ln x}_{u}\, \underbrace{dx}_{dv}$

It helps when you stick to this pattern:

$u = \ln x \qquad dv = 1\,dx$

$du = \frac{1}{x}dx \qquad v = 1x$

$= x\ln x - \int \cancel{x}\frac{1}{\cancel{x}}\,dx$

$= x\ln x - \int 1\,dx$

$= x\ln x - 1x + C$

$\boxed{= x\ln x - x + C}$

Find: $\int \underbrace{x^2}_{u} \underbrace{\sin(3x)dx}_{dv} = x^2\left(-\frac{1}{3}\cos 3x\right) - \int -\frac{1}{3}\cos 3x \, 2x\,dx$

It helps when you stick to this pattern:

(I) $u = x^2$ $dv = \sin 3x\,dx$

$du = 2x\,dx$ $v = -\frac{1}{3}\cos 3x$

$= -\frac{1}{3}x^2\cos 3x - \int\left(-\frac{2}{3}\right)x\cos 3x\,dx$

$= -\frac{1}{3}x^2\cos 3x + \frac{2}{3}\int \underbrace{x}_{u}\underbrace{\cos 3x\,dx}_{dv}$

(II) $u = x$ $dv = \cos 3x\,dx$

$du = 1dx$ $v = \frac{1}{3}\sin 3x$

$= -\frac{1}{3}x^2\cos 3x + \frac{2}{3}\left[x\left(\frac{1}{3}\sin 3x\right) - \int \frac{1}{3}\sin 3x\,dx\right]$

$= -\frac{1}{3}x^2\cos 3x + \frac{2}{3}\left[\frac{1}{3}x\sin 3x - \frac{1}{3}\int \sin 3x\,dx\right]$

$= -\frac{1}{3}x^2\cos 3x + \frac{2}{3}\left[\frac{1}{3}x\sin 3x + \frac{1}{9}\cos 3x\right]$

$$\boxed{= -\frac{1}{3}x^2\cos 3x + \frac{2}{9}x\sin 3x + \frac{2}{27}\cos 3x + C}$$

$$= -\frac{1}{3}x^2\cos 3x + \frac{2}{9}x\sin 3x + \frac{2}{27}\cos 3x + C$$

Find: $\int \underbrace{x^2}_{u}\underbrace{e^x dx}_{dv}$

It helps when you stick to this pattern:

(I) $u = x^2$ $dv = e^x dx$

$du = 2x\,dx$ $v = e^x$

(II) $u = x$ $dv = e^x dx$

$du = 1dx$ $v = e^x$

$$= x^2 e^x - \int e^x 2x\,dx$$

$$= x^2 e^x - 2\int \underbrace{x}_{u}\underbrace{e^x dx}_{dv}$$

$$= x^2 e^x - 2\left[x e^x - \int e^x dx\right]$$

$$= x^2 e^x - 2\left[x e^x - e^x\right]$$

$$\boxed{= x^2 e^x - 2xe^x + 2e^x + C}$$

$$= x^2 e^x - 2xe^x + 2e^x + C$$

Find: $\int x^2 \ln x dx$

It helps when you stick to this pattern:

$u =$ ______ $dv =$ ______

$du =$ ______ $v =$ ______

We've done this one already,
but let's do it again and evaluate: $\int_1^e \ln x\,dx$

It helps when you
stick to this pattern:

$u =$ ______ $dv =$ ______

$du =$ ______ $v =$ ______

Find: $$\int e^x \sin x dx$$

It helps when you stick to this pattern:

$u =$ ______ $dv =$ ______

$du =$ ______ $v =$ ______

Find: $\int e^x \cos x dx$

It helps when you stick to this pattern:

$u =$ ______ $dv =$ ______

$du =$ ______ $v =$ ______

Find: $\int \sin^{-1} x dx$

It helps when you stick to this pattern:

$u =$ ______ $dv =$ ______

$du =$ ______ $v =$ ______

may require substitution rule as well...

Find: $\int_0^{\frac{\pi}{3}} \sin x \ln(\cos x)dx$

It helps when you
stick to this pattern:

$u =$ ______ $dv =$ ______

$du =$ ______ $v =$ ______

Differential and Integral Calculus 120

Thursday, May 31, 2012

$\int$ Trigonometric Integrals! $\int$

WARM UP

Find: $$\int x^2 \sin x dx$$

It helps when you stick to this pattern:

$u =$ ______ $dv =$ ______

$du =$ ______ $v =$ ______

Trigonometric Identities

$$\boxed{\sin^2\theta + \cos^2\theta = 1}$$

$$1 - \sin^2\theta = \cos^2\theta$$

$$\tan^2\theta + 1 = \sec^2\theta$$

$$\sec^2\theta - 1 = \tan^2\theta$$

recall double angle identities:

$$\sin 2\theta = 2\sin\theta\cos\theta$$

$$\cos 2\theta = \cos^2\theta - \sin^2\theta$$

$$\cos 2\theta = 2\cos^2\theta - 1$$

$$\cos 2\theta = 1 - 2\sin^2\theta$$

also we will make use of the Half-angle identities:

$$\sin^2\theta = \frac{1-\cos 2\theta}{2} \qquad \cos^2\theta = \frac{1+\cos 2\theta}{2}$$

Let's start off by doing some integration of trigonometric functions that require direct substitution and practice using our identities:

$$\int \sin x \cos x dx$$

$$\int \cos^2 x dx$$

$$\int \sin^3 x dx$$

$$\int \cos^3 x dx$$

$$\int_0^{\pi/2} \cos^3 x \sin^4 x dx$$

$$\int \sin^2 x \cos x\,dx$$

$$\int \sin^5 x \cos^2 x\,dx$$

In the preceding examples, an odd power of sine or cosine enabled us to separate a single factor and convert the remaining even power. If the integrand contains even powers of both sine and cosine, this strategy fails. In this case, we can take advantage of the half-angle identities.

$$\int \sin^2 x\,dx$$

$$\int \sin^2 x \cos^2 x\,dx$$

$$\int \sin^4 x\,dx$$

may have to use the half-angle identity twice

Strategy for Evaluating $\int \sin^m x \cos^n x\,dx$

(a) If the power of cosine is odd ($n = 2k + 1$), save one cosine factor and use $\cos^2 x = 1 - \sin^2 x$ to express the remaining factors in terms of sine:

$$\int \sin^m x \cos^{2k+1} x\,dx = \int \sin^m x\,(\cos^2 x)^k \cos x\,dx$$
$$= \int \sin^m x\,(1 - \sin^2 x)^k \cos x\,dx$$

Then substitute $u = \sin x$.

(b) If the power of sine is odd ($m = 2k + 1$), save one sine factor and use $\sin^2 x = 1 - \cos^2 x$ to express the remaining factors in terms of cosine:

$$\int \sin^{2k+1} x \cos^n x\,dx = \int (\sin^2 x)^k \cos^n x \sin x\,dx$$
$$= \int (1 - \cos^2 x)^k \cos^n x \sin x\,dx$$

Then substitute $u = \cos x$. [Note that if the powers of both sine and cosine are odd, either (a) or (b) can be used.]

(c) If the powers of both sine and cosine are even, use the half-angle identities

$$\sin^2 x = \tfrac{1}{2}(1 - \cos 2x) \qquad \cos^2 x = \tfrac{1}{2}(1 + \cos 2x)$$

It is sometimes helpful to use the identity

$$\sin x \cos x = \tfrac{1}{2} \sin 2x$$

Evaluate:

$$\int \tan^6 x \sec^4 x dx$$

$$\int \tan^5 x \sec^7 x dx$$

Strate

Strategy for Evaluating $\int \tan^m x \sec^n x \, dx$

(a) If the power of secant is even ($n = 2k$), save a factor of $\sec^2 x$ and use $\sec^2 x = 1 + \tan^2 x$ to express the remaining factors in terms of $\tan x$:

$$\int \tan^m x \sec^{2k} x \, dx = \int \tan^m x \, (\sec^2 x)^{k-1} \sec^2 x \, dx$$

$$= \int \tan^m x \, (1 + \tan^2 x)^{k-1} \sec^2 x \, dx$$

Then substitute $u = \tan x$.

(b) If the power of tangent is odd ($m = 2k + 1$), save a factor of $\sec x \tan x$ and use $\tan^2 x = \sec^2 x - 1$ to express the remaining factors in terms of $\sec x$:

$$\int \tan^{2k+1} x \sec^n x \, dx = \int (\tan^2 x)^k \sec^{n-1} x \sec x \tan x \, dx$$

$$= \int (\sec^2 x - 1)^k \sec^{n-1} x \sec x \tan x \, dx$$

Then substitute $u = \sec x$.

We will also need to know the indefinite integrals of tan x (already found this one) and sec x when integrating these types of functions.

$$\boxed{\int \tan x dx = \ln\left|\sec x\right| + C}$$

$$\boxed{\int \sec x dx = \ln\left|\sec x + \tan x\right| + C}$$

$\int \tan x\, dx = \ln|\sec x| + C$

$\int \sec x\, dx = \ln|\sec x + \tan x| + C$

Find:

$$\int \tan^3 x dx$$

$$\int \sec^3 x dx$$

here, we can use integration by parts and rearrange for double the original integral.

Homework - Exercise 7.2

Differential and Integral Calculus 120

Friday, May 28, 2010

$\int$ Trigonometric Substitution! $\int$

Exercises 7.2 □ page 482

Exercises 7.2 □ page 482

1. $\frac{1}{5}\cos^5 x - \frac{1}{3}\cos^3 x + C$ **3.** $-\frac{11}{384}$

5. $\frac{1}{5}\sin^5 x - \frac{2}{7}\sin^7 x + \frac{1}{9}\sin^9 x + C$ **7.** $\pi/4$

9. $\frac{3}{8}x + \frac{1}{4}\sin 2x + \frac{1}{32}\sin 4x + C$

11. $(3x/2) + \cos 2x - \frac{1}{8}\sin 4x + C$ **13.** $(3\pi - 4)/192$

15. $\left[\frac{2}{7}\cos^3 x - \frac{2}{3}\cos x\right]\sqrt{\cos x} + C$

17. $\frac{1}{2}\cos^2 x - \ln|\cos x| + C$ **19.** $\ln(1 + \sin x) + C$

21. $\tan x - x + C$ **23.** $\tan x + \frac{1}{3}\tan^3 x + C$ **25.** $\frac{1}{5}$

27. $\frac{1}{3}\sec^3 x - \sec x + C$ **29.** $\frac{38}{15}$

31. $\frac{1}{4}\sec^4 x - \tan^2 x + \ln|\sec x| + C$ **33.** $\frac{1}{2}\tan^2 x + C$

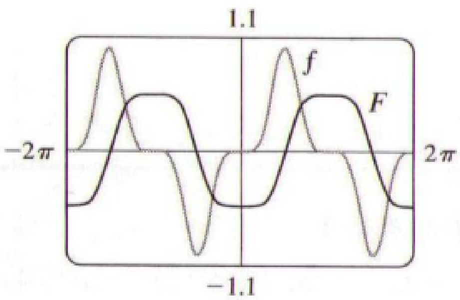
35. $\sqrt{3} - (\pi/3)$

37. $-\frac{1}{3}\cot^3 w - \frac{1}{5}\cot^5 w + C$ **39.** $\ln|\csc x - \cot x| + C$

41. $\frac{1}{2}\left[\frac{1}{3}\sin 3x - \frac{1}{7}\sin 7x\right] + C$ **43.** $\frac{1}{4}\sin 2\theta + \frac{1}{24}\sin 12\theta + C$

45. $\frac{1}{2}\sin 2x + C$

47. $-\frac{1}{5}\cos^5 x + \frac{2}{3}\cos^3 x - \cos x + C$



Trigonometric Substitution

This is an integration technique introduced as a means of evaluating integrals involving the radical forms:

$\boxed{\sqrt{a^2-x^2}}$ $\boxed{\sqrt{a^2+x^2}}$ $\boxed{\sqrt{x^2-a^2}}$

For integrals involving:	let	then
$\sqrt{a^2-x^2}$	$x=a\sin\theta$	$\sqrt{a^2-x^2}=a\cos\theta$
$\sqrt{a^2+x^2}$	$x=a\tan\theta$	$\sqrt{a^2+x^2}=a\sec\theta$
$\sqrt{x^2-a^2}$	$x=a\sec\theta$	$\sqrt{x^2-a^2}=a\tan\theta$

Just as with algebraic substitution, our objective with trigonometric substitution is to eliminate the radicals in the integrand. There are three substitutions that accomplish this objective for the three types of radicals outlined in the table above. $x \neq a$

To show that the radical is eliminated as indicated in each of the three cases, we need to use the following trigonometric identities: $\sin^2\theta+\cos^2\theta=1$

$$1-\sin^2\theta=\cos^2\theta$$
$$1+\tan^2\theta=\sec^2\theta$$
$$\sec^2\theta-1=\tan^2\theta$$

recall double angle identity:

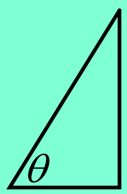
$\boxed{\sin 2\theta = 2\sin\theta\cos\theta}$

also we will make use of the Half-angle identities:

$\boxed{\sin^2\theta=\frac{1-\cos 2\theta}{2}}$ $\boxed{\cos^2\theta=\frac{1+\cos 2\theta}{2}}$

We will always manipulate the radical first so that we have the constant... a = 1

It also helps to build a triangle marking angle theta.



use the help of a triangle to define theta before substitution

$$\int \sqrt{1-x^2}\,dx$$

$$\int \sqrt{4-x^2}\,dx$$

use the help of a triangle to define theta before substitution

$$\int \frac{1}{\sqrt{1-x^2}}\,dx$$

$$\int \frac{1}{x^2\sqrt{1-x^2}}\,dx$$

$$\int \frac{dx}{x^2\sqrt{9-x^2}}$$

$$\int \frac{1}{\sqrt{9-49x^2}}dx$$

$$\int \sqrt{16+25x^2}dx$$

$$\int \frac{1}{\sqrt{16+36x^2}}\,dx$$

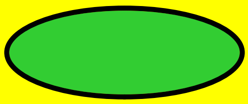
Use trigonometric substitution to find each of the following:

$$\int \sqrt{36 - 25x^2}\, dx$$

Find the area enclosed by the ellipse: $$\frac{x^2}{25}+\frac{y^2}{4}=1$$

sketch (find area in first quadrant from x = 0 to x =5)

y = ?



Homework - Exercise 7.3

Differential and Integral Calculus 120

Monday, May 31, 2010

$\int$ Partial Fractions! $\int$

Exercises 7.3 □ page 488

1. $\sqrt{x^2-9}/(9x)+C$ **3.** $\frac{1}{3}(x^2-18)\sqrt{x^2+9}+C$

5. $\pi/24+\sqrt{3}/8-\frac{1}{4}$ **7.** $-\sqrt{25-x^2}/(25x)+C$

9. $(1/\sqrt{3})\ln\left|\left(\sqrt{x^2+3}-\sqrt{3}\right)/x\right|+C$

11. $\frac{1}{4}\sin^{-1}(2x)+\frac{1}{2}x\sqrt{1-4x^2}+C$

13. $\sqrt{9x^2-4}-2\sec^{-1}(3x/2)+C$

15. $\left(x/\sqrt{a^2-x^2}\right)-\sin^{-1}(x/a)+C$ 17. $\sqrt{x^2-7}+C$

19. $\ln\left(1+\sqrt{2}\right)$ **21.** $\frac{64}{1215}$

23. $\frac{1}{2}\left[\sin^{-1}(x-1)+(x-1)\sqrt{2x-x^2}\right]+C$

25. $\frac{1}{3}\ln\left|3x+1+\sqrt{9x^2+6x-8}\right|+C$

27. $\frac{1}{2}\left[\tan^{-1}(x+1)+(x+1)/(x^2+2x+2)\right]+C$

29. $\frac{1}{2}\left[e^t\sqrt{9-e^{2t}}+9\sin^{-1}(e^t/3)\right]+C$

33. $3\pi/2$ **37.** 0.81, 2; 2.10

39. $r\sqrt{R^2-r^2}+\pi r^2/2-R^2\arcsin(r/R)$ **41.** $2\pi^2Rr^2$

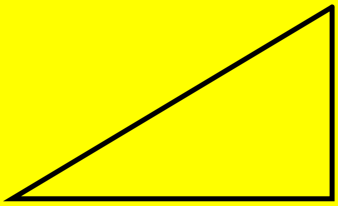
WARM UP

Find the area enclosed by the ellipse: $$\frac{x^2}{16}+\frac{y^2}{36}=1$$

sketch (find area in first quadrant from x = 0 to x =4)

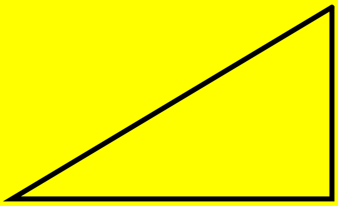
y = ?

$$\int \frac{1}{x^2\sqrt{x^2+4}}\,dx$$



recall: $\cot^2\theta + 1 = \csc^2\theta$

$$\int \frac{\sqrt{9-x^2}}{x^2}\,dx$$



Integration of Rational Functions by Partial Fractions

This technique allows us to integrate any rational function (a ratio of polynomials) $\boxed{f(x)=\frac{P(x)}{Q(x)}}$

by expressing it as a sum of simpler fractions, called partial fractions, that we already know how to integrate.

Let's do an example.... $\displaystyle\int \frac{x+5}{x^2+x-2}\,dx$

- rearrange
- collect & equate like terms
- and use system of equations

to solve for A and B

Find: $$\int \frac{x+7}{x^2-x-6}\,dx$$

Find: $$\int \frac{x^2+2x-1}{2x^3+3x^2-2x}\,dx$$

Find: $$\int \frac{1}{2x^2 + 5x + 2}\,dx$$

Find:

$$\int \frac{5x^2 + 15x - 36}{x^3 - 9x}\,dx$$

The previous examples dealt with the denominator being a product of distinct linear factors. **(CASE I)**

In the following example, we look at the case when the denominator is the product of linear factors, however some are repeated.

If we do it the same way as before, solving for A and B using a system of equations doesn't work as all terms in the elimination cancel.

(CASE II)

So....by inspecting the denominator...

for each factor of the form $(px + q)^m$, the partial fraction decomposition must include the following sum of m fractions:

$$\boxed{\frac{A}{px+q}+\frac{B}{(px+q)^2}+\frac{C}{(px+q)^3}+\ldots+\frac{const}{(px+q)^m}}$$

Let's do the example.... $\int \frac{6x+7}{(x+2)^2}dx$

multiply by common denominator $(x+2)^2$ .... and solve for A and B!

- then integrate!

Find: $$\int \frac{5x^2+20x+6}{x^3+2x^2+x}\,dx$$

simplify to...

repeating linear factor

(CASE III)

So....by inspecting the denominator Q(x)...
for each factor of the form $(ax^2 + bx + c)$, which is an irreducible quadratic factor, the partial fraction decomposition must take the form:

$$\boxed{\frac{Ax+B}{ax^2+bx+c}}$$

if the irreducible quadratic factor is repeating $(ax^2 + bx + c)^n$, the partial fraction decomposition must include the following sum of n fractions:

$$\boxed{\frac{Ax+B}{(ax^2+bx+c)}+\frac{Cx+D}{(ax^2+bx+c)^2}+\ldots+\frac{const(x)+const}{(ax^2+bx+c)^n}}$$

Here are some examples of how we would do the partial fraction decomposition for some rational functions:

(we will not solve these - they are a little tedious to say the least to find all the coefficients)

$$\frac{x^3-x+1}{x^2(x-1)^3}=$$

$$\frac{x^3+x^2+1}{x(x-1)(x^2+x+1)(x^2+1)^3}=$$

Find: $$\int \frac{2x^3 - 4x - 8}{(x^2 - x)(x^2 + 4)}\,dx$$

Evaluate: $$\int_0^1 \frac{3x+4}{x^3-2x-4}\,dx$$

Don't lose sight of easier methods when possible....

Sometimes partial fractions can be avoided when integrating a rational function.

example, Find:

$$\int \frac{x^2+1}{x(x^2+3)}\,dx$$

$$= \int \frac{x^2+1}{x^3+3x}\,dx$$

how about straight up substitution...

Find: $\int \frac{2x^2 - x + 4}{x^3 + 4x}\,dx$

Find: $$\int \frac{-2x+4}{(x^2+1)(x-1)^2}\,dx$$

Find: $$\int \frac{1 - x + 2x^2 - x^3}{x(x^2+1)^2}\,dx$$

(CASE IV)

For the case of a rational function: $f(x) = \dfrac{P(x)}{Q(x)}$

where the degree of the denominator Q(x) is **less than or equal to** the degree of the numerator P(x),

(this is called an improper rational function)

then divide P(x) by Q(x) by **long division** to obtain a quotient S(x) and the remainder R(x).

$$\boxed{f(x) = \frac{P(x)}{Q(x)} = S(x) + \frac{R(x)}{Q(x)}}$$

Example Find: $\displaystyle\int \frac{x^3 + x}{x - 1}\,dx$

Find:

$$= \int \frac{5x^3 - x^2}{x^2 - 1} dx$$

Find: $\displaystyle\int \frac{2x^5 - 5x}{(x^2+2)^2}dx$

long division case

$$= \int 2x - \frac{8x^3 + 13x}{(x^2+2)^2}dx$$

Evaluate: $$\int_3^4 \frac{x+4}{x-2}dx$$

Find: $$\int \frac{x^4 - 2x^2 + 4x + 1}{x^3 - x^2 - x + 1}\,dx$$

Rationalizing Substitutions:

Find: $\displaystyle\int \frac{\sqrt{x+4}}{x}\,dx$ $\displaystyle = 2\int \frac{u^2}{u^2-4}\,du$

Let:

- integrate by partial fractions using Case IV technique (long division + Remainder) as the degree of the numerator equals the degree of the denominator

$$= 2\int \left(1+\frac{4}{u^2-4}\right)du$$

integrate by partial fractions using Case I technique

Find: