Questions From Homework

② d) $\int \frac{x+1}{x^2+2x-6}\,dx$

$u = x^2+2x-6$

$du = 2x+2\,dx$

$\frac{1}{2}du = x+1\,dx$

$= \int \frac{1}{u} \cdot \left(\frac{1}{2}du\right)$

$= \frac{1}{2}\int \frac{1}{u}\,du$

$= \frac{1}{2}\ln|u| + C$

Questions From Homework

(4) c) $\displaystyle\int_{\pi/6}^{\pi/2} \frac{\cos\theta}{\sin^3\theta}\,d\theta = \int_{1/2}^{1} \frac{1}{u^3}\cdot du$

$u = \sin\theta$

$\frac{du}{d\theta} = \cos\theta$

$du = \cos\theta\, d\theta$

θ	u
$\pi/2$	1
$\pi/6$	$1/2$

$= \displaystyle\int_{1/2}^{1} u^{-3}\,du$

$= \dfrac{u^{-2}}{-2}\Big|_{1/2}^{1}$

$= -\dfrac{1}{2u^2}\Big|_{1/2}^{1}$

$= \dfrac{-1}{2(1)^2} - \dfrac{-1}{2(\frac{1}{2})^2}$

$= \dfrac{-1}{2} - -2$

$= -\dfrac{1}{2} + \dfrac{4}{2} = \dfrac{3}{2}$

Warm Up

Find: $\int 5x^2 \sin(4x^3+1)dx$

$$= -\frac{5}{12}\cos(4x^3-1)+C$$

$\int x\sqrt{2x^2-5}\,dx$

$$= \frac{(2x^2-5)^{3/2}}{6}+C$$

$\int \cot x\,dx$

$$= \ln\left|\sin x\right| + C$$

Differential and Integral Calculus 120

$\int$ Integration by Parts $\int$

$$f(x)g(x) = f'(x)g(x) + f(x)g'(x)$$

As we have discussed before, every differentiation rule has a corresponding integration rule.

The rule that corresponds to the Product Rule for differentiation is called the rule for integration by parts.

The product rule stated that if f and g are differentiable functions, then

$$\frac{d}{dx}[f(x)g(x)] = f(x)g'(x) + g(x)f'(x)$$

In the notation for indefinite integrals this equation becomes...

$$\int [f(x)g'(x)dx + g(x)f'(x)dx] = f(x)g(x)$$

or

$$\int f(x)g'(x)dx + \int g(x)f'(x)dx = f(x)g(x)$$

which can be rearranged as:

$$\boxed{\int f(x)g'(x)dx = f(x)g(x) - \int g(x)f'(x)dx}$$

this formulas above is called
the **formula for integration by parts**

It is perhaps easier to remember in the following notation..... Let $u = f(x)$ and $v = g(x)$

then the differentials are: $du = f'(x)dx$ $dv = g'(x)dx$

And by the Substitution Rule, the formulas becomes...

$$\int f(x)g'(x)dx = f(x)g(x) - \int g(x)f'(x)dx$$

Integration By Parts

$$\int u\,dv = uv - \int v\,du$$

Let's do an example.... Find: $\int \underbrace{x}_{u} \underbrace{\sin x\,dx}_{dv}$

It helps when you stick to this pattern:

we need to make an appropriate choice for u and dv

$u =$ x $dv =$ sinx dx

$du =$ 1dx $v =$ -cosx

Again, the goal in using integration by parts is to obtain a simpler integral than the one we started with... so we must decide on what u and dv are very carefully!

In general, when deciding on a choice for u and dv, we usually try to choose u = f(x) to be a function that becomes simpler when differentiated...

(or at least NOT more complicated)

as long as dv = g'(x)dx can be readily integrated to give v.

$$\int \underbrace{x}_{u}\underbrace{\sin x\,dx}_{dv} = x(-\cos x) - \int -\cos x\,dx$$

$$= -x\cos x + \int \cos x\,dx$$

$$\boxed{= -x\cos x + \sin x + C}$$

Find: $\int \underbrace{x}_{u}\underbrace{e^{x}dx}_{dv}$ $= xe^{x} - \int e^{x}dx$

$\boxed{= xe^{x} - e^{x} + C}$

It helps when you stick to this pattern:

$u = \underline{x}$ $dv = \underline{e^{x}dx}$

$du = \underline{1dx}$ $v = \underline{e^{x}}$

Find: $\int \underbrace{x}_{u} \underbrace{\cos(3x)dx}_{dv} = x\left(\frac{1}{3}\sin(3x)\right) - \int \frac{1}{3}\sin 3x \, dx$

$= \frac{1}{3}x\sin 3x - \frac{1}{3}\int \sin 3x \, dx$

$= \frac{1}{3}x\sin 3x - \frac{1}{3}\left(-\frac{1}{3}\cos 3x\right)$

$\boxed{= \frac{1}{3}x\sin 3x + \frac{1}{9}\cos 3x + C}$

It helps when you stick to this pattern:

$u =$ x $\quad dv =$ $\cos(3x)dx$

$du =$ $1dx$ $\quad v =$ $\frac{1}{3}\sin(3x)$

Find: $$\int \underbrace{\ln x}_{u}\,\underbrace{dx}_{dv}$$

It helps when you stick to this pattern:

$u = \ln x$ $dv = 1\,dx$

$du = \frac{1}{x}dx$ $v = 1x$

$$= x\ln x - \int \cancel{x}\frac{1}{\cancel{x}}\,dx$$

$$= x\ln x - \int 1\,dx$$

$$= x\ln x - 1x + C$$

$$\boxed{= x\ln x - x + C}$$

Find: $\int \underbrace{x^2}_{u} \underbrace{\sin(3x)dx}_{dv} = x^2\left(-\frac{1}{3}\cos 3x\right) - \int -\frac{1}{3}\cos 3x \, 2x\,dx$

$= -\frac{1}{3}x^2\cos 3x - \int \left(-\frac{2}{3}\right) x\cos 3x\,dx$

$= -\frac{1}{3}x^2\cos 3x + \frac{2}{3}\int \underbrace{x}_{u}\underbrace{\cos 3x\,dx}_{dv}$

It helps when you stick to this pattern:

(I) $u = x^2$ $\quad dv = \sin 3x\,dx$

$du = 2x\,dx$ $\quad v = -\frac{1}{3}\cos 3x$

(II) $u = x$ $\quad dv = \cos 3x\,dx$

$du = 1dx$ $\quad v = \frac{1}{3}\sin 3x$

$= -\frac{1}{3}x^2\cos 3x + \frac{2}{3}\left[x\left(\frac{1}{3}\sin 3x\right) - \int \frac{1}{3}\sin 3x\,dx\right]$

$= -\frac{1}{3}x^2\cos 3x + \frac{2}{3}\left[\frac{1}{3}x\sin 3x - \frac{1}{3}\int \underbrace{\sin 3x\,dx}_{-\frac{1}{3}\cos 3x}\right]$

$= -\frac{1}{3}x^2\cos 3x + \frac{2}{3}\left[\frac{1}{3}x\sin 3x + \frac{1}{9}\cos 3x\right]$

$$\boxed{= -\frac{1}{3}x^2\cos 3x + \frac{2}{9}x\sin 3x + \frac{2}{27}\cos 3x + C}$$

$$= -\frac{1}{3}x^2\cos 3x + \frac{2}{9}x\sin 3x + \frac{2}{27}\cos 3x + C$$

Find: $\int \underbrace{x^2}_{u} \underbrace{e^x dx}_{dv}$

It helps when you stick to this pattern:

(I) $u = x^2$ $dv = e^x dx$

$du = 2x\,dx$ $v = e^x$

(II) $u = x$ $dv = e^x dx$

$du = 1dx$ $v = e^x$

$= x^2 e^x - \int e^x 2x\,dx$

$= x^2 e^x - 2\int \underbrace{x}_{u} \underbrace{e^x dx}_{dv}$

$= x^2 e^x - 2\left[xe^x - \int e^x dx\right]$

$= x^2 e^x - 2\left[xe^x - e^x\right]$

$\boxed{= x^2 e^x - 2xe^x + 2e^x + C}$

$$= x^2 e^x - 2xe^x + 2e^x + C$$

Find: $\int x^2 \ln x\,dx$

$$= \frac{1}{3}x^3\ln x - \int \frac{1}{3}x^3 \cdot \frac{1}{x}\,dx$$

$$= \frac{1}{3}x^3\ln x - \int \frac{1}{3}x^2\,dx$$

$$= \frac{1}{3}x^3\ln x - \frac{1}{3}\int x^2\,dx$$

$$= \frac{1}{3}x^3\ln x - \frac{1}{3}\left(\frac{1}{3}x^3\right)$$

$$\boxed{= \frac{1}{3}x^3\ln x - \frac{1}{9}x^3 + C}$$

It helps when you stick to this pattern:

$u = \ln x$ $\quad dv = x^2\,dx$

$du = \frac{1}{x}\,dx$ $\quad v = \frac{1}{3}x^3$

Find:

$$\int \underbrace{e^x}_{u} \underbrace{\sin x dx}_{dv} = e^x(-\cos x) - \int (-\cos x) e^x dx$$

$$\int e^x \sin x dx = -e^x \cos x + \int \underbrace{e^x}_{u} \underbrace{\cos x dx}_{dv}$$

$$\int e^x \sin x dx = -e^x \cos x + e^x \sin x - \int \sin x e^x dx$$

$$\int e^x \sin x dx = -e^x \cos x + e^x \sin x - \int e^x \sin x dx$$

$$\frac{2\int e^x \sin x dx}{2} = \frac{e^x \sin x - e^x \cos x}{2}$$

$$\int e^x \sin x d = \frac{1}{2}(e^x \sin x - e^x \cos x) + C$$

It helps when you stick to this pattern:

$u = e^x$ $dv = \sin x dx$
$du = e^x dx$ $v = -\cos x$

$u = e^x$ $dv = \cos x$
$du = e^x dx$ $v = \sin x$

this one as a little twist,
because you cannot get to a simpler integral
- rearrange for double the initial integral and
divide by two!

$$\boxed{= \frac{1}{2} e^x (\sin x - \cos x) + C}$$

Find: $\int e^{x} \cos x dx$

It helps when you
stick to this pattern:

$u =$ ______ $dv =$ ______

$du =$ ______ $v =$ ______

this one as a little twist,
because you cannot get to a simpler integral
- rearrange for double the initial integral and
divide by two!

$$= \frac{1}{2} e^{x} (\cos x + \sin x) + C$$

Find: $\int \underbrace{\sin^{-1} x}_{u}\, \underbrace{dx}_{dv}$

$$= x\sin^{-1}x - \int x \cdot \frac{1}{\sqrt{1-x^2}}\,dx$$

$$= x\sin^{-1}x - \int \frac{1}{(u)^{1/2}} \cdot -\frac{1}{2}du$$

$$= x\sin^{-1}x + \frac{1}{2}\int u^{-1/2}\,du$$

$$= x\sin^{-1}x + \frac{1}{2}\left(2u^{1/2}\right) + C$$

$$= x\sin^{-1}x + u^{1/2} + C$$

$$\boxed{= x\sin^{-1}x + \sqrt{1-x^2} + C}$$

It helps when you stick to this pattern:

$u =$ $\sin^{-1}x$ $\quad dv =$ $1\,dx$

$du =$ $\frac{1}{\sqrt{1-x^2}}dx$ $\quad v =$ x

may require substitution rule as well...

$u = 1-x^2$

$du = -2x\,dx$

$-\frac{1}{2}du = x\,dx$

<u>Homework</u> - Exercise 11.4 - pp. 515 - Q. 1,2,4

We've done this one already,
but let's do it again and evaluate: $\int_1^e \ln x\,dx$

It helps when you
stick to this pattern:

$u =$ ______ $dv =$ ______

$du =$ ______ $v =$ ______

Find: $$\int_0^{\frac{\pi}{3}} \sin x \ln(\cos x)dx$$

It helps when you
stick to this pattern:

$u = \underline{\qquad\quad}$ $dv = \underline{\qquad\quad}$

$du = \underline{\qquad\quad}$ $v = \underline{\qquad\quad}$