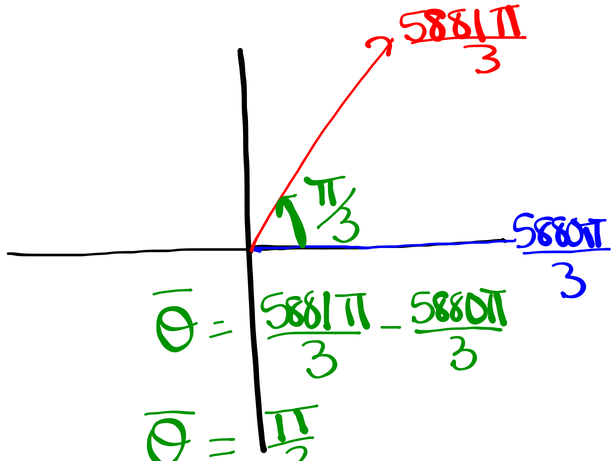
Sketch the following and determine a negative angle co-terminal with:

$(i)\ \frac{5881\pi}{3}$



$\frac{5880\pi}{3}, \frac{5881\pi}{3}, \frac{5882\pi}{3}$

1960π

$\bar{\theta} = \frac{5881\pi}{3} - \frac{5880\pi}{3}$

$\bar{\theta} = \frac{\pi}{3}$

Negative coterminal angle:

$\frac{5881\pi}{3} - \frac{1962\pi}{1}$

$\frac{5881\pi}{3} - \frac{5886\pi}{3}$

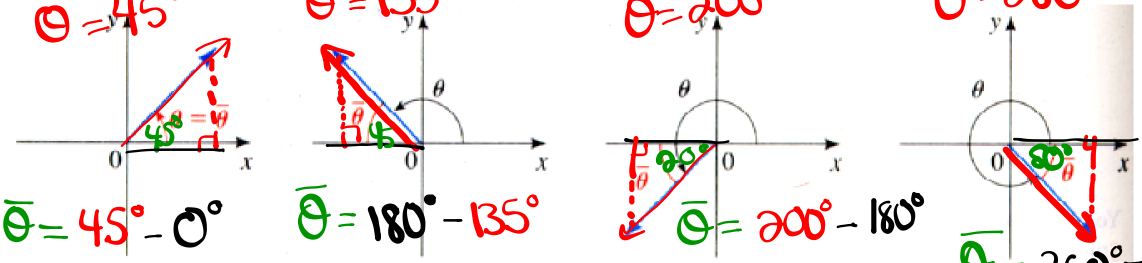
$\frac{-5\pi}{3}$

Reference Triangles:

Definition 17 *The reference angle $\bar{\theta}$ of an angle θ in standard position is the acute angle (between 0 and 90°) the terminal side makes with the x-axis.*

arm

The picture below illustrates this concept.



$\theta = 45°$ | $\bar{\theta} = 45° - 0°$ | $\bar{\theta} = 45°$

$\theta = 135°$ | $\bar{\theta} = 180° - 135°$ | $\bar{\theta} = 45°$

$\theta = 200°$ | $\bar{\theta} = 200° - 180°$ | $\bar{\theta} = 20°$

$\theta = 280°$ | $\bar{\theta} = 360° - 280°$ | $\bar{\theta} = 80°$

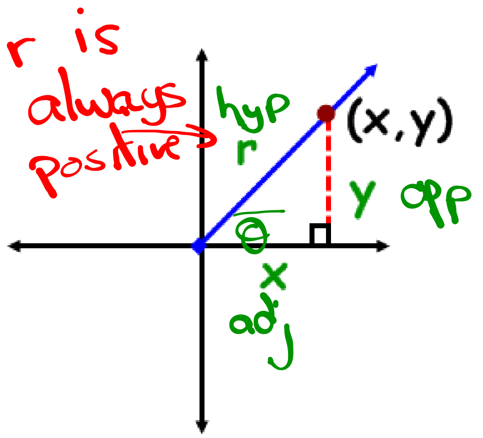
What is the significance of reference angles?

Angles on the Cartesian Plane

$\angle 90^\circ$ or $< \pi/2$ or < 1.57 rads

- **Reference Angle** (θ) - an acute angle formed between the terminal arm and the x-axis.

- **Reference Triangle** - a triangle formed by drawing a perpendicular line from a point on the terminal to the x-axis.



Notice what will happen if the rotation moves into other quadrants?

TRIG RATIOS on the CARTESIAN PLANE

$$\sin\theta = \frac{y}{r} = \frac{o}{h} \qquad \csc\theta = \frac{r}{y} = \frac{h}{o}$$

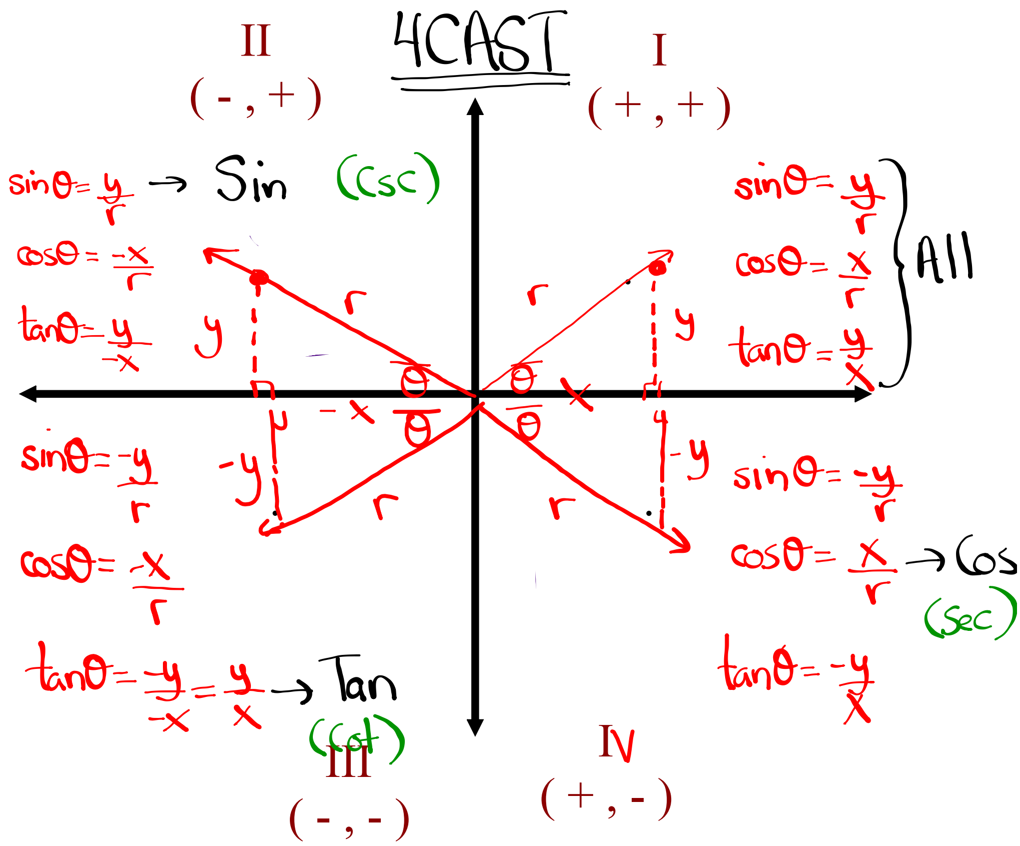
$$\cos\theta = \frac{x}{r} = \frac{a}{h} \qquad \sec\theta = \frac{r}{x} = \frac{h}{a}$$

$$\tan\theta = \frac{y}{x} = \frac{o}{a} \qquad \cot\theta = \frac{x}{y} = \frac{a}{o}$$

"Primary" "Reciprocal"

TRIG RATIOS IN ALL 4 QUADRANTS

What primary trig ratios are POSITIVE in...



Where is θ if...

$\csc\theta < 0$

$\sin\theta < 0$ & $\tan\theta < 0$

$\csc\theta > 0$ & $\cot\theta < 0$

Homework

If $\sec\theta = -\sqrt{10}$ *and* $\sin\theta > 0$, determine the value of $\csc\theta$

$\sec\theta = -\frac{\sqrt{10}}{1} = \frac{hyp}{adj} = \frac{r}{x}$

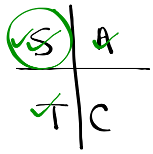
$r = \sqrt{10}$ (always +)

$x = -1$

① Determine what quadrant:

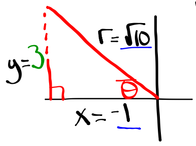
$\sec\theta < 0$ & $\sin\theta > 0$

or $\cos\theta < 0$ & $\sin\theta > 0$



θ is in Q2

② Draw a diagram



③ $x^2 + y^2 = r^2$

$(-1)^2 + y^2 = (\sqrt{10})^2$

$1 + y^2 = 10$

$y^2 = 9$

$y = \pm 3$

$\boxed{y = 3 \text{ (Q2)}}$

④ Find $\csc\theta$:

$\csc\theta = \frac{r}{y}$

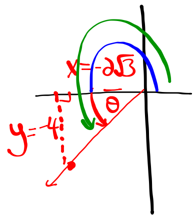
$\boxed{\csc\theta = \frac{\sqrt{10}}{3}}$

Determine the measure ~~(in radians)~~ of an angle whose terminal arm passes through the ordered pair $(-2\sqrt{3}, -4)$

$x = -2\sqrt{3}$

$y = -4$

① Draw a diagram



② Find $\bar{\theta}$

$\tan\bar{\theta} = \frac{y}{x}$

$\tan\bar{\theta} = \frac{\cancel{-4}^{2}}{\cancel{-2}\sqrt{3}} = \frac{2}{\sqrt{3}}$

$\tan\bar{\theta} = 1.1547$

$\bar{\theta} = \tan^{-1}(1.1547)$

convert calculator to radians

$\boxed{\bar{\theta} = 0.86 \text{ rads}}$

③ Find θ

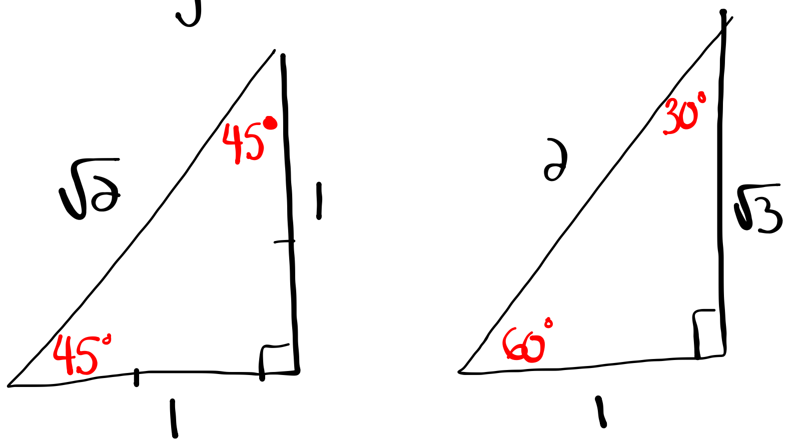
$\theta = \pi + \bar{\theta}$

$\theta = 3.14 + 0.86$

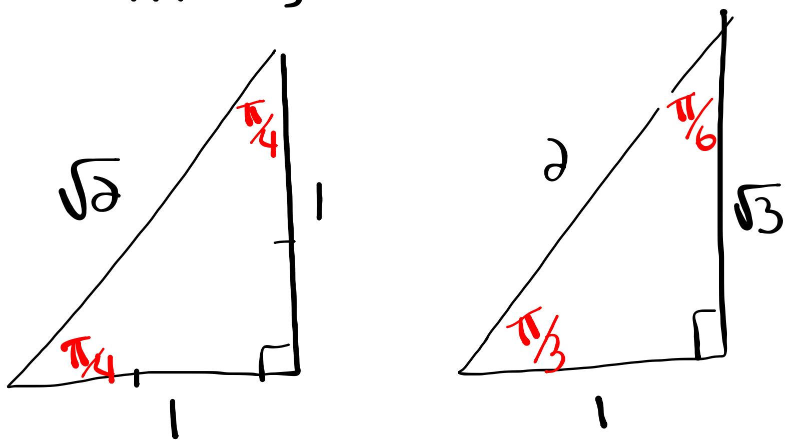
$\boxed{\theta = 4 \text{ rads}}$

$\theta = \pi - \bar{\theta}$ $\theta = 180° - \bar{\theta}$	$\theta = \bar{\theta}$
$\theta = 180° + \bar{\theta}$ $\theta = \pi + \bar{\theta}$	$\theta = 360° - \bar{\theta}$ $\theta = 2\pi - \bar{\theta}$

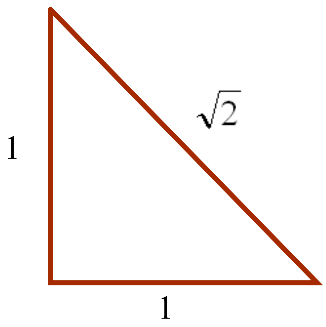
In Degrees

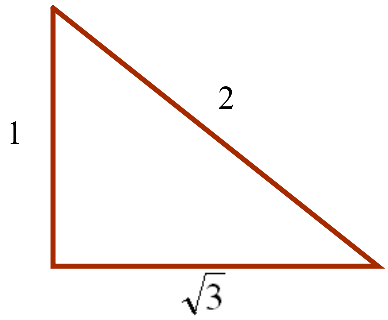


In Radians



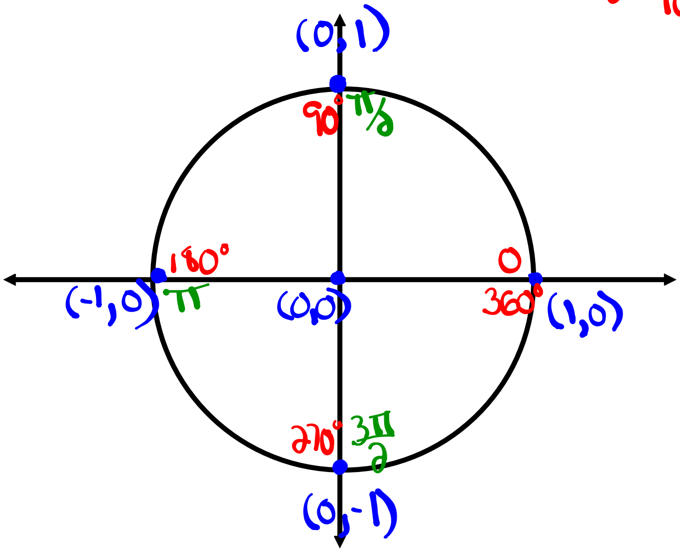
Special Angles (in radians)





Quadrantal Angles

(Multiples of 90°)



The Unit Circle
- Center is @ (0,0)
- radius is 1 unit

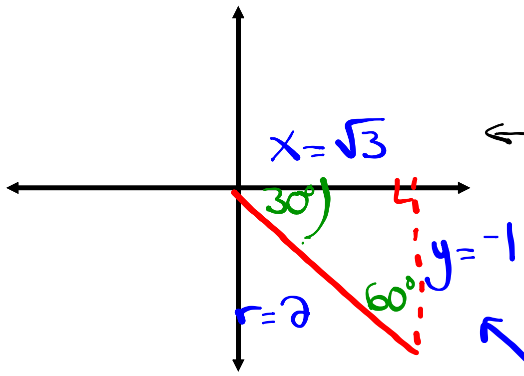
Solving ~~Trig Expressions~~ by Sketching Angles

Ex. Evaluate the $\sin 690°$

Optional

(i) Find principal angle:

$\theta = 690° - 360° = 330°$



(ii) Sketch (Q4)

(iii) Find $\bar{\theta}$

$\bar{\theta} = 360° - 330° = \boxed{30°}$

(iv) Label

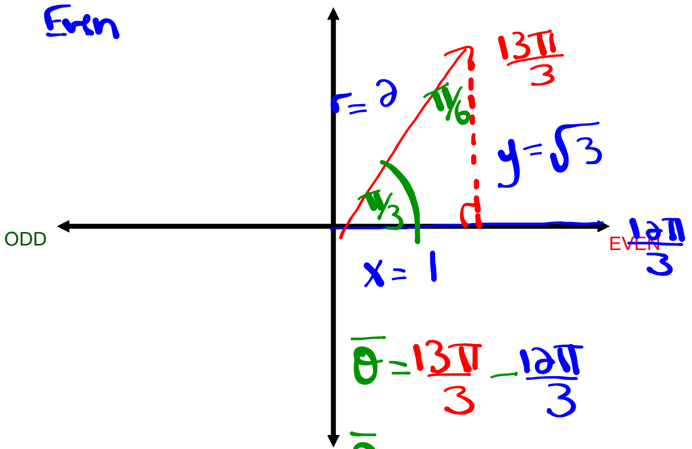
(v) Evaluate $\boxed{\sin 690° = -\frac{1}{2}}$

Ex. $\cos \frac{13\pi}{3} = \frac{a}{h} = \frac{x}{r} = \boxed{\frac{1}{2}}$

$\frac{12\pi}{3}$, $\boxed{\frac{13\pi}{3}}$, $\frac{14\pi}{3}$

4π

Even



$\bar{\theta} = \frac{13\pi}{3} - \frac{12\pi}{3}$

$\bar{\theta} = \frac{\pi}{3}$

Homework

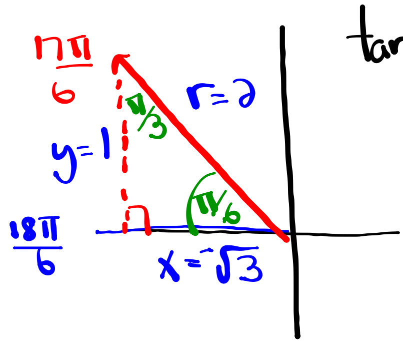
Evaluate each Trig Expression (provide a sketch of each angle)

1. $\tan\dfrac{17\pi}{6} = \dfrac{-1}{\sqrt{3}}$ 2. $\sin\dfrac{15\pi}{4} = \dfrac{-1}{\sqrt{2}}$ 3. $\cos\left(-\dfrac{21\pi}{4}\right) = \dfrac{-1}{\sqrt{2}}$

① $\dfrac{16\pi}{6}, \boxed{\dfrac{17\pi}{6}}, \dfrac{18\pi}{6}$

3π

(odd)



$\tan\dfrac{17\pi}{6} = \dfrac{o}{a} = \dfrac{y}{x} = \dfrac{1}{-\sqrt{3}}$

$= -\dfrac{1\cdot\sqrt{3}}{\sqrt{3}\cdot\sqrt{3}}$

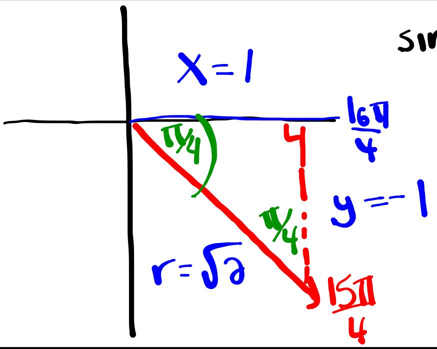
$\boxed{\tan\dfrac{17\pi}{6} = -\dfrac{\sqrt{3}}{3}}$

---

② $\dfrac{14\pi}{4}, \boxed{\dfrac{15\pi}{4}}, \dfrac{16\pi}{4}$

4π

(Even)



$\sin\dfrac{15\pi}{4} = \dfrac{o}{h} = \dfrac{y}{r} = -\dfrac{1\cdot\sqrt{2}}{\sqrt{2}\cdot\sqrt{2}}$

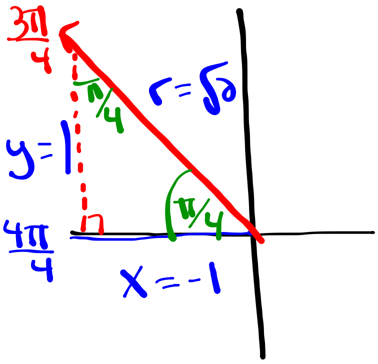
$\boxed{\sin\dfrac{15\pi}{4} = -\dfrac{\sqrt{2}}{2}}$

---

③ $-\dfrac{21\pi}{4} + \dfrac{6\pi}{1}$

$-\dfrac{21\pi}{4} + \dfrac{24\pi}{4}$

$\dfrac{3\pi}{4}$



$\cos\left(-\dfrac{21\pi}{4}\right) = \cos\dfrac{3\pi}{4} = \dfrac{a}{h} = \dfrac{x}{r}$

$= \dfrac{-1\cdot\sqrt{2}}{\sqrt{2}\cdot\sqrt{2}}$

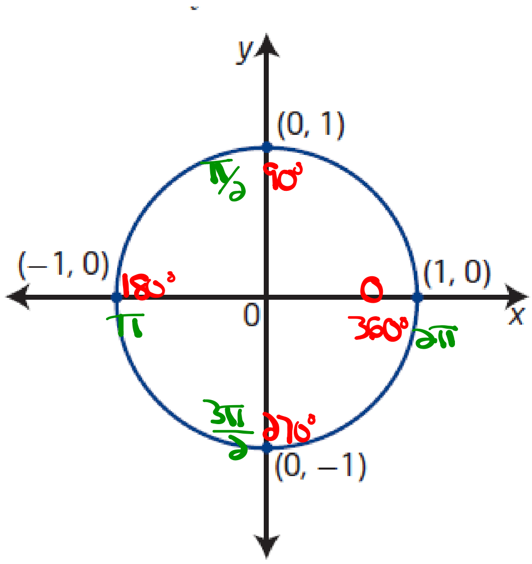
$\boxed{\cos\left(-\dfrac{21\pi}{4}\right) = -\dfrac{\sqrt{2}}{2}}$

$\dfrac{2\pi}{4}, \boxed{\dfrac{3\pi}{4}}, \dfrac{4\pi}{4}$

1π

(odd)

Unit Circle



unit circle

- a circle with radius 1 unit $(r = 1)$
- a circle of radius 1 unit with centre at the $(0,0)$ origin on the Cartesian plane is known as *the* unit circle

$\sin\theta = \frac{y}{r} = \frac{y}{1} = y \rightarrow$ Ex: $\sin 90° = 1$ $(0,1)$

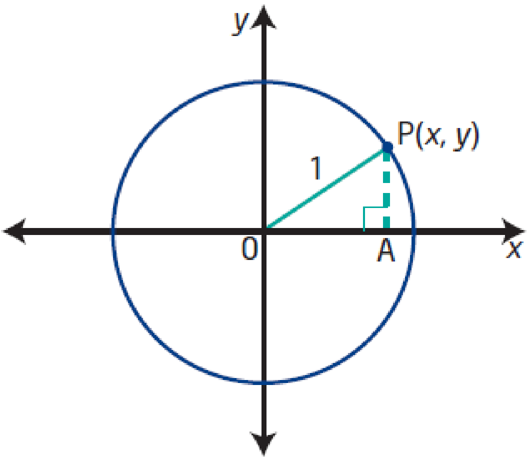
$\cos\theta = \frac{x}{r} = \frac{x}{1} = x \rightarrow$ Ex: $\cos\pi = -1$ $(-1,0)$

$\tan\theta = \frac{y}{x} \rightarrow$ Ex: $\tan 270° = \frac{-1}{0}$ = undefined $(0,-1)$

$\csc\theta = \frac{r}{y} = \frac{1}{y} \rightarrow$ Ex: $\csc 360° = \frac{1}{0}$ = undefined $(1,0)$

$\sec\theta = \frac{r}{x} = \frac{1}{x} \rightarrow$ Ex: $\sec 5\pi = \frac{1}{-1} = -1$ $(-1,0)$

$\cot\theta = \frac{x}{y} \rightarrow$ Ex: $\cot\frac{3\pi}{2} = \frac{0}{-1} = 0$ $(0,-1)$



$x^2 + y^2 = r^2$

$x^2 + y^2 = (1)^2$

The equation of the unit circle is $x^2 + y^2 = 1$.

$r = 1$

Determine the equation of a circle with centre at the origin and radius 6.

$r = 6$

$x^2 + y^2 = r^2$

$x^2 + y^2 = (6)^2$

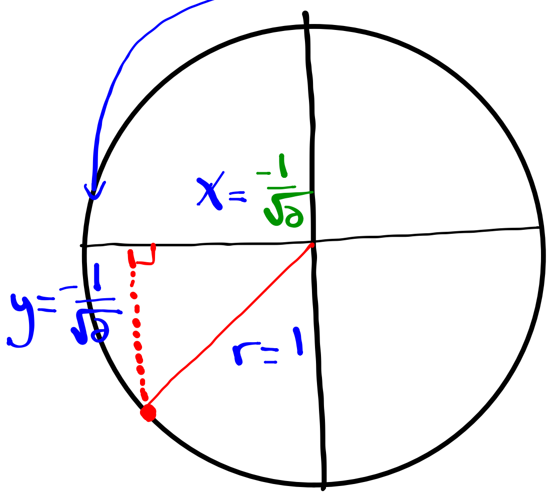
$\boxed{x^2 + y^2 = 36}$

Problems Involving the Unit Circle:

Determine Coordinates for Points of the Unit Circle

Determine the coordinates (x, y) for all points on the unit circle $r = 1$ that satisfy the conditions given. Draw a diagram in each case.

- the y-coordinate is $-\frac{1}{\sqrt{2}}$ and the point is in quadrant III



Find x

$x^2 + y^2 = r^2$

$x^2 + \left(-\frac{1}{\sqrt{2}}\right)^2 = (1)^2$

$x^2 + \frac{1}{2} = 1$

$x^2 = 1 - \frac{1}{2}$

$x^2 = \frac{2}{2} - \frac{1}{2}$

$x^2 = \frac{1}{2}$

$x = \pm \frac{\sqrt{1}}{\sqrt{2}}$

$x = \pm \frac{1}{\sqrt{2}}$

$\boxed{x = \frac{-1}{\sqrt{2}}}$ (Q3)

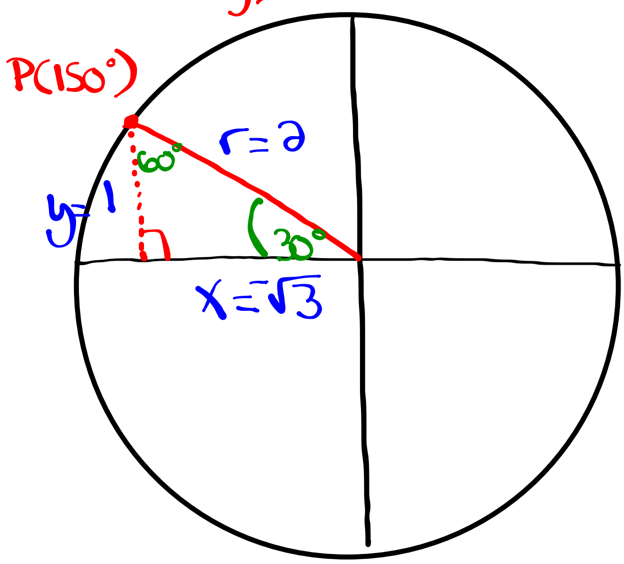
coordinates are $\left(-\frac{1}{\sqrt{2}}, -\frac{1}{\sqrt{2}}\right)$

Problems Involving the Unit Circle:

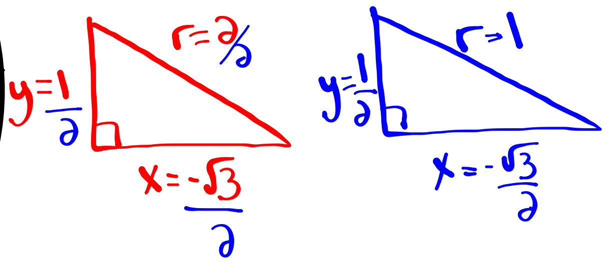
If $P(150^\circ)$ is the point at which the terminal arm of an angle θ in standard position intersects the unit circle, determine the exact coordinates of...

$r=1$

(x,y)

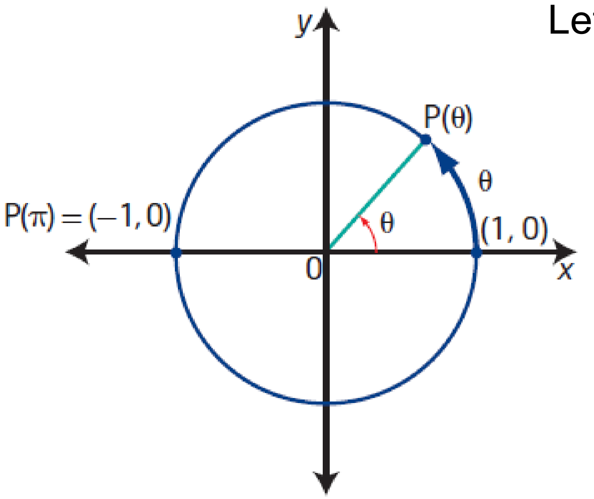


Scale the diagram so that r=1 (unit circle)



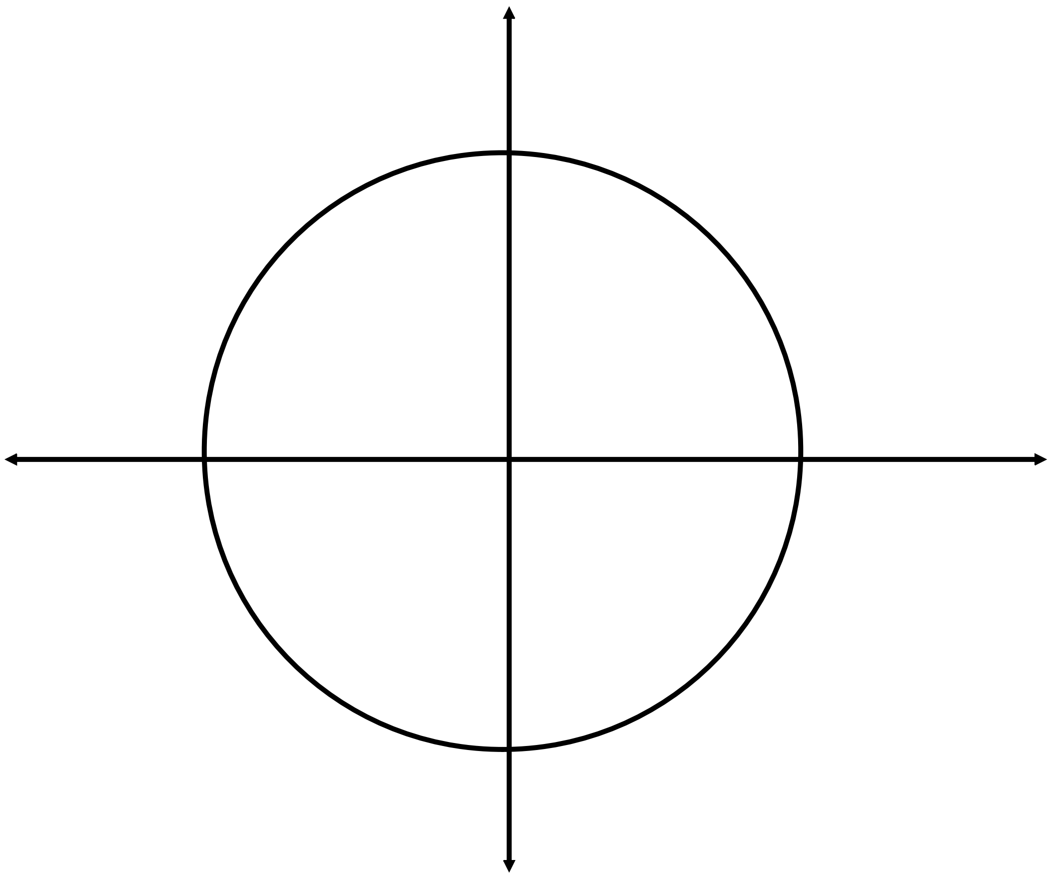
coordinates are $\left(-\frac{\sqrt{3}}{2}, \frac{1}{2}\right)$

Special Angles on the Unit Circle:

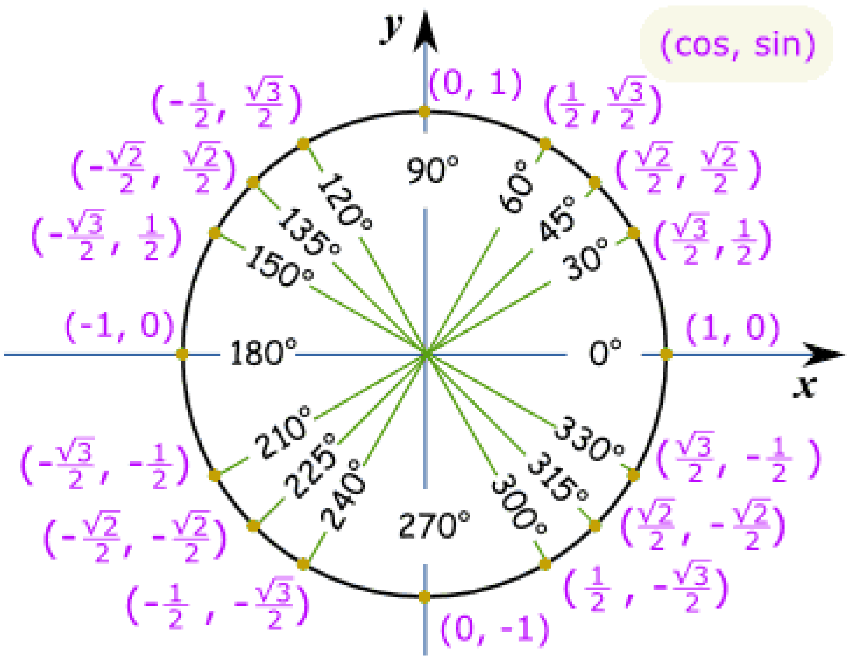


Let's use $\frac{\pi}{4}$ as our reference angle

Construct reference triangles
for all multiples of $\pi/4$
between 0 and 2π

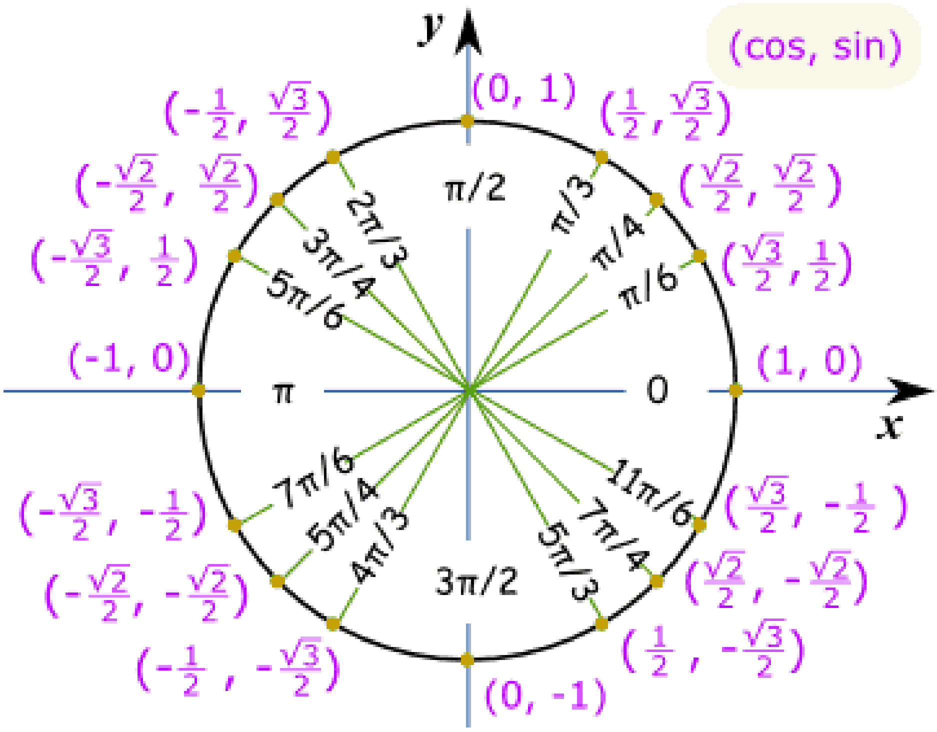


Unit Circle of Special Angles in Degrees



This is lovely...so what is it used for????

Unit Circle of Special Angles in Radians



Finish worksheet

Attachments

Worksheet - Sketching Angles in Radians.doc