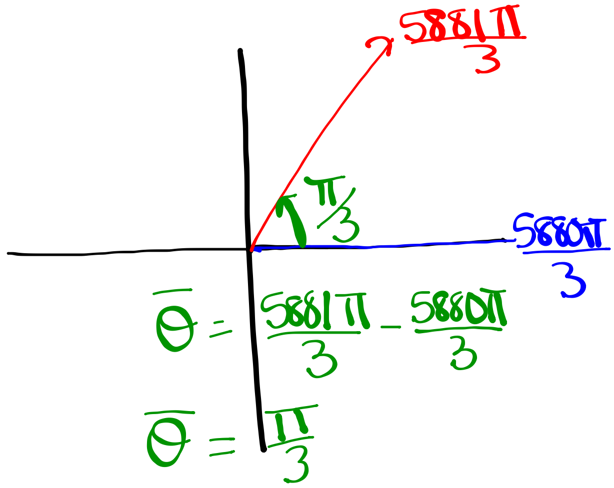
Sketch the following and determine a negative angle co-terminal with:

(i) $\frac{5881\pi}{3}$

$\frac{5880\pi}{3}, \frac{5881\pi}{3}, \frac{5882\pi}{3}$

1960π



Negative coterminal angle:

$\frac{5881\pi}{3} - \frac{1962\pi}{1}$

$\frac{5881\pi}{3} - \frac{5886\pi}{3}$

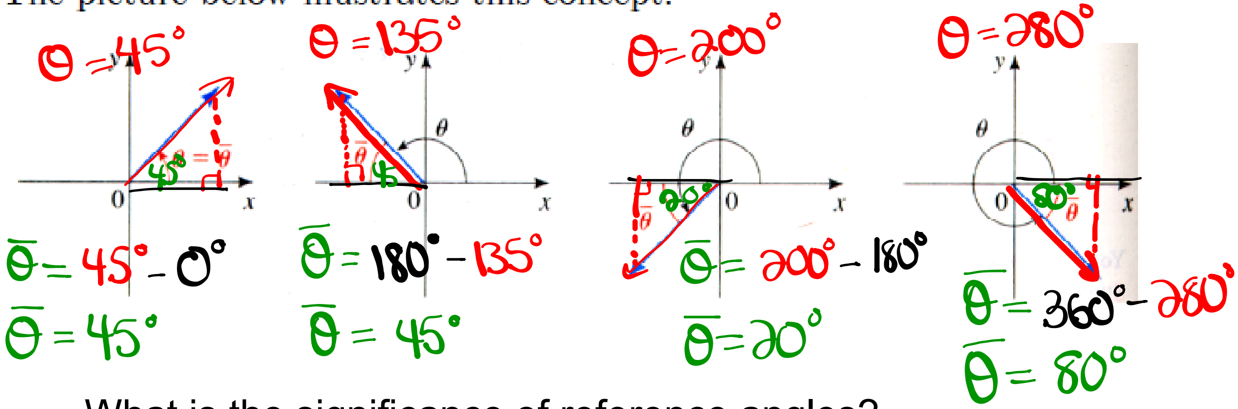
$\frac{-5\pi}{3}$

Reference Triangles:

Definition 17 *The reference angle $\bar{\theta}$ of an angle θ in standard position is the acute angle (between 0 and 90°) the terminal side makes with the x-axis.*

arm

The picture below illustrates this concept.



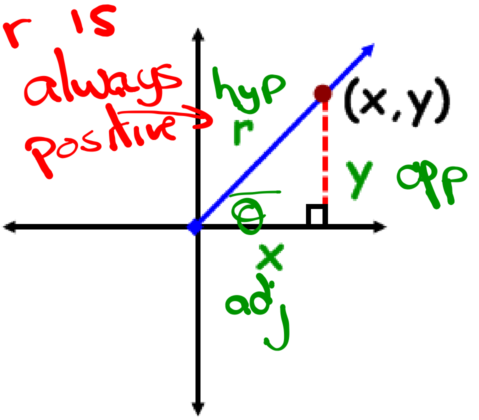
What is the significance of reference angles?

Angles on the Cartesian Plane

$\angle 90°$ or $< \pi/2$ or < 1.57 rads

- **Reference Angle** θ - an acute angle formed between the terminal arm and the x-axis.
- **Reference Triangle** - a triangle formed by drawing a perpendicular line from a point on the terminal to the x-axis.

r is always positive



Notice what will happen if the rotation moves into other quadrants?

TRIG RATIOS on the CARTESIAN PLANE

$$\sin\theta = \frac{y}{r} = \frac{o}{h} \qquad \csc\theta = \frac{r}{y} = \frac{h}{o}$$

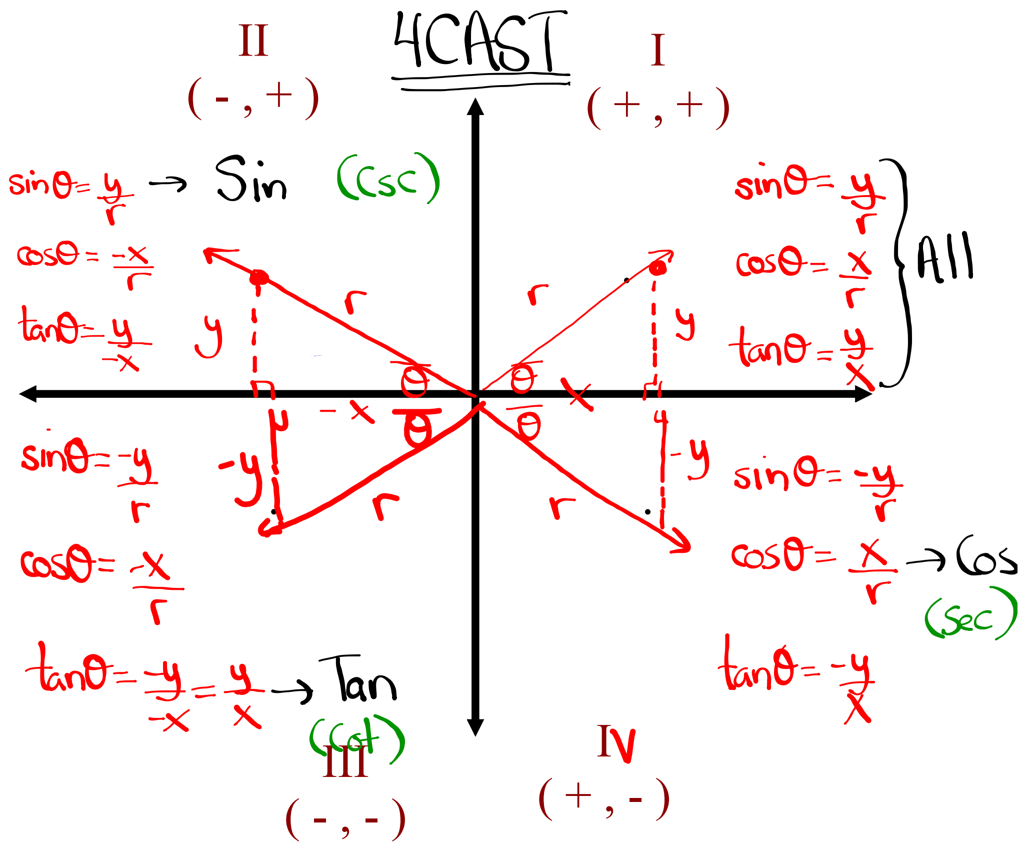
$$\cos\theta = \frac{x}{r} = \frac{a}{h} \qquad \sec\theta = \frac{r}{x} = \frac{h}{a}$$

$$* \tan\theta = \frac{y}{x} = \frac{o}{a} \qquad \cot\theta = \frac{x}{y} = \frac{a}{o}$$

"Primary" "Reciprocal"

TRIG RATIOS IN ALL 4 QUADRANTS

What primary trig ratios are POSITIVE in...



Where is θ if...

$\csc\theta < 0$

$\sin\theta < 0$ & $\tan\theta < 0$

$\csc\theta > 0$ & $\cot\theta < 0$

Homework

If $\sec\theta = -\sqrt{10}$ *and* $\sin\theta > 0$, determine the value of $\csc\theta$

$\sec\theta = -\frac{\sqrt{10}}{1} = \frac{hyp}{adj} = \frac{r}{x}$

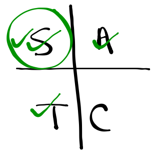
$r = \sqrt{10}$ (always +)

$x = -1$

① Determine what quadrant:

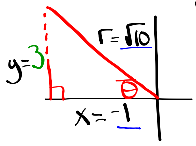
$\sec\theta < 0$ & $\sin\theta > 0$

or $\cos\theta < 0$ & $\sin\theta > 0$



θ is in Q2

② Draw a diagram



③ $x^2 + y^2 = r^2$

$(-1)^2 + y^2 = (\sqrt{10})^2$

$1 + y^2 = 10$

$y^2 = 9$

$y = \pm 3$

$\boxed{y = 3 \ (Q2)}$

④ Find $\csc\theta$:

$\csc\theta = \frac{r}{y}$

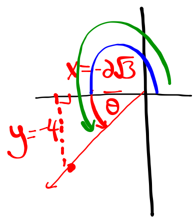
$\boxed{\csc\theta = \frac{\sqrt{10}}{3}}$

Determine the measure ~~(in radians)~~ of an angle whose terminal arm passes through the ordered pair $(-2\sqrt{3}, -4)$

$x = -2\sqrt{3}$

$y = -4$

① Draw a diagram



② Find $\bar{\theta}$

$\tan\bar{\theta} = \frac{y}{x}$

$\tan\bar{\theta} = \frac{\cancel{-4}^{2}}{\cancel{-2\sqrt{3}}_{1}} = \frac{2}{\sqrt{3}}$

$\tan\bar{\theta} = 1.1547$

$\bar{\theta} = \tan^{-1}(1.1547)$

convert calculator to radians

$\boxed{\bar{\theta} = 0.86 \text{ rads}}$

③ Find θ

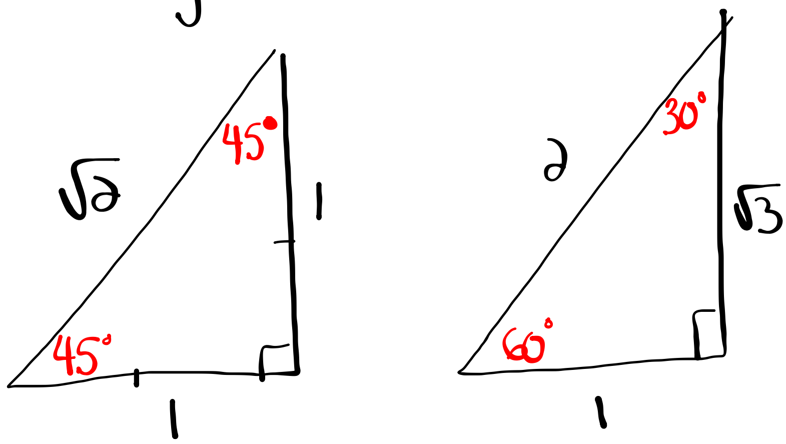
$\theta = \pi + \bar{\theta}$

$\theta = 3.14 + 0.86$

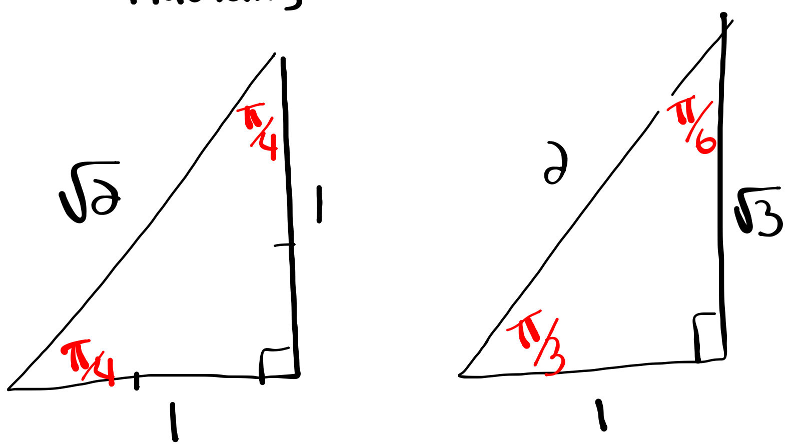
$\boxed{\theta = 4 \text{ rads}}$

$\theta = \pi - \bar{\theta}$ $\theta = 180° - \bar{\theta}$	$\theta = \bar{\theta}$
$\theta = 180° + \bar{\theta}$ $\theta = \pi + \bar{\theta}$	$\theta = 360° - \bar{\theta}$ $\theta = 2\pi - \bar{\theta}$

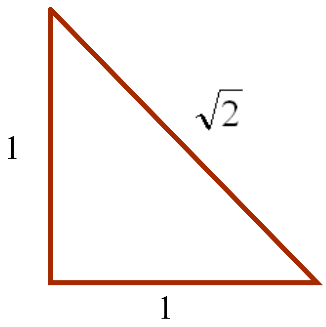
In Degrees

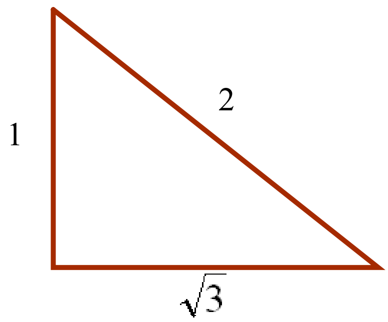


In Radians

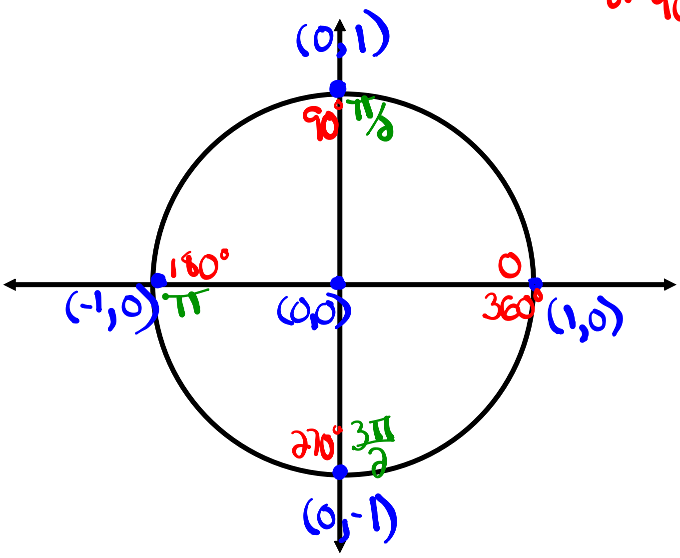


Special Angles (in radians)





Quadrantal Angles (Multiples of 90°)



The Unit Circle

- Center is @ (0,0)
- radius is 1 unit

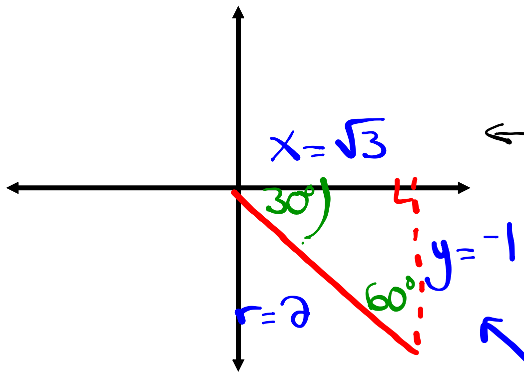
Solving Trig Expressions by Sketching Angles

Ex. Evaluate the $\sin 690°$

Optional

(i) Find principal angle:

$\theta = 690° - 360° = 330°$



(ii) Sketch (Q4)

(iii) Find $\bar{\theta}$

$\bar{\theta} = 360° - 330° = \boxed{30°}$

(iv) Label

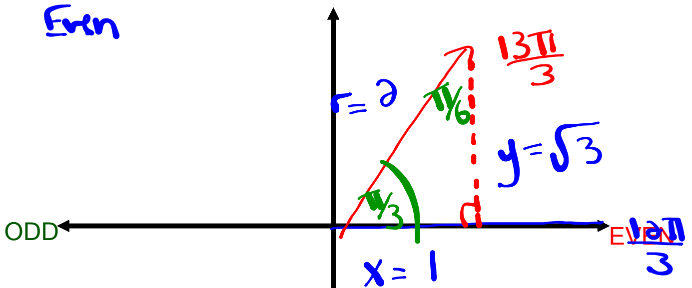
(v) Evaluate $\boxed{\sin 690° = -\frac{1}{2}}$

Ex. $\cos \frac{13\pi}{3} = \frac{a}{h} = \frac{x}{r} = \boxed{\frac{1}{2}}$

$\frac{12\pi}{3}$, $\boxed{\frac{13\pi}{3}}$, $\frac{14\pi}{3}$

4π

Even



$\bar{\theta} = \frac{13\pi}{3} - \frac{12\pi}{3}$

$\bar{\theta} = \frac{\pi}{3}$

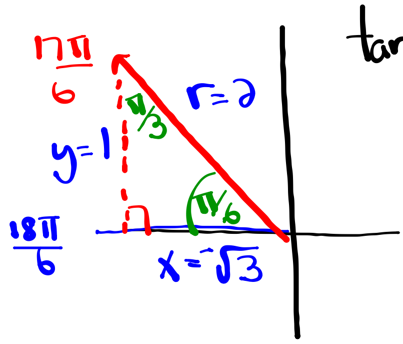
Homework

Evaluate each Trig Expression (provide a sketch of each angle)

1. $\tan\frac{17\pi}{6} = \frac{-1}{\sqrt{3}}$ 2. $\sin\frac{15\pi}{4} = \frac{-1}{\sqrt{2}}$ 3. $\cos\left(-\frac{21\pi}{4}\right) = \frac{-1}{\sqrt{2}}$

① $\frac{16\pi}{6}, \boxed{\frac{17\pi}{6}}, \frac{18\pi}{6}$

3π (odd)



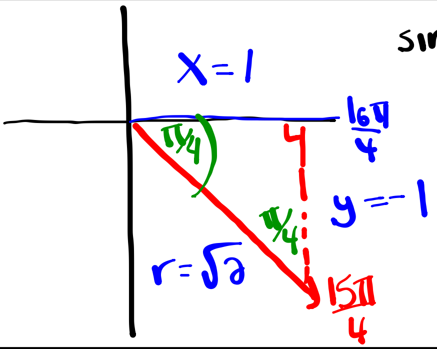
$\tan\frac{17\pi}{6} = \frac{o}{a} = \frac{y}{x} = \frac{1}{-\sqrt{3}}$

$= -\frac{1}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}}$

$\boxed{\tan\frac{17\pi}{6} = -\frac{\sqrt{3}}{3}}$

② $\frac{14\pi}{4}, \boxed{\frac{15\pi}{4}}, \frac{16\pi}{4}$

4π (Even)



$\sin\frac{15\pi}{4} = \frac{o}{h} = \frac{y}{r} = -\frac{1}{\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}}$

$\boxed{\sin\frac{15\pi}{4} = -\frac{\sqrt{2}}{2}}$

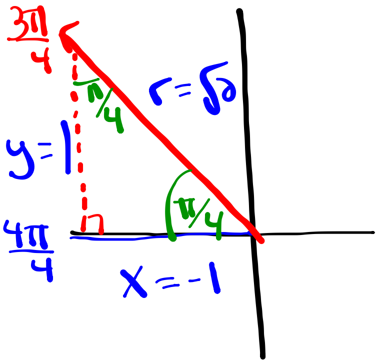
③ $-\frac{21\pi}{4} + \frac{6\pi}{1}$

$-\frac{21\pi}{4} + \frac{24\pi}{4}$

$\frac{3\pi}{4}$

$\frac{2\pi}{4}, \boxed{\frac{3\pi}{4}}, \frac{4\pi}{4}$

1π (odd)

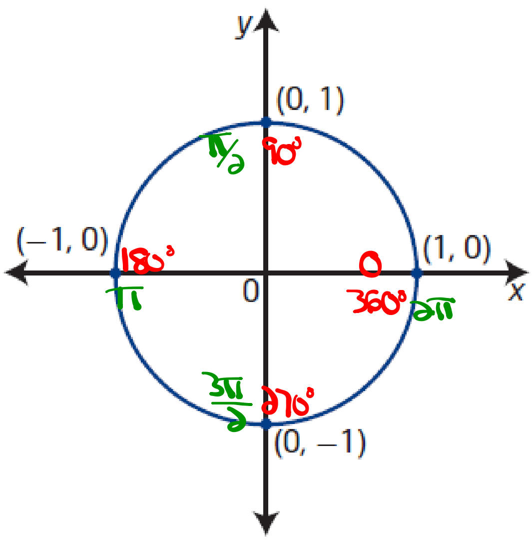


$\cos\left(-\frac{21\pi}{4}\right) = \cos\frac{3\pi}{4} = \frac{a}{h} = \frac{x}{r}$

$= \frac{-1}{\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}}$

$\boxed{\cos\left(-\frac{21\pi}{4}\right) = -\frac{\sqrt{2}}{2}}$

Unit Circle



unit circle

- a circle with radius 1 unit $(r = 1)$
- a circle of radius 1 unit with centre at the $(0,0)$ origin on the Cartesian plane is known as *the* unit circle

$\sin\theta = \frac{y}{r} = \frac{y}{1} = y \rightarrow$ Ex: $\sin 90° = 1$

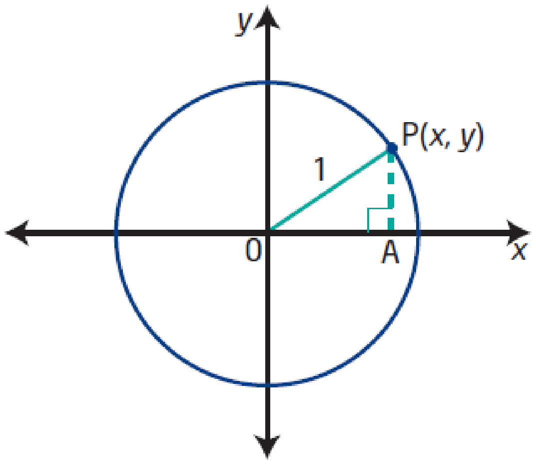
$\cos\theta = \frac{x}{r} = \frac{x}{1} = x \rightarrow$ Ex: $\cos\pi = -1$

$\tan\theta = \frac{y}{x} \longrightarrow$ Ex: $\tan 270° = \frac{-1}{0} =$ undefined

$\csc\theta = \frac{1}{y} \longrightarrow$ Ex: $\csc 360° = \frac{1}{0} =$ undefined

$\sec\theta = \frac{1}{x} \longrightarrow$ Ex: $\sec 5\pi = \frac{1}{-1} = -1$

$\cot\theta = \frac{x}{y} \longrightarrow$ Ex: $\cot\frac{3\pi}{2} = \frac{0}{-1} = 0$



$x^2+y^2=r^2$

$x^2+y^2=(1)^2$

The equation of the unit circle is $x^2 + y^2 = 1$.

$r=1$

Determine the equation of a circle with centre at the origin and radius 6.

$r=6$

$x^2+y^2=r^2$

$x^2+y^2=(6)^2$

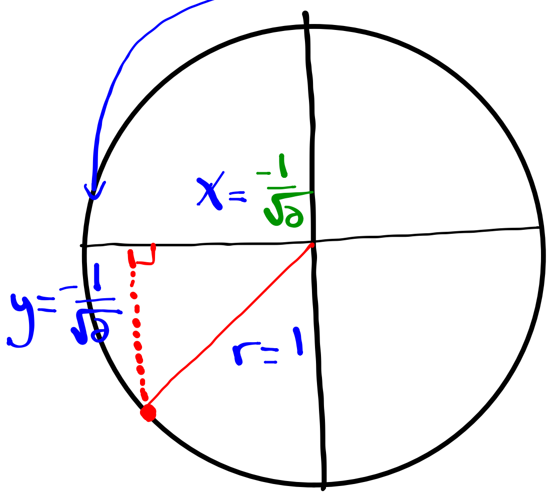
$\boxed{x^2+y^2=36}$

Problems Involving the Unit Circle:

Determine Coordinates for Points of the Unit Circle

Determine the coordinates (x, y) for all points on the unit circle $r=1$ that satisfy the conditions given. Draw a diagram in each case.

- the y-coordinate is $-\frac{1}{\sqrt{2}}$ and the point is in quadrant III



Find x

$x^2 + y^2 = r^2$

$x^2 + \left(-\frac{1}{\sqrt{2}}\right)^2 = (1)^2$

$x^2 + \frac{1}{2} = 1$

$x^2 = 1 - \frac{1}{2}$

$x^2 = \frac{2}{2} - \frac{1}{2}$

$x^2 = \frac{1}{2}$

$x = \pm \frac{\sqrt{1}}{\sqrt{2}}$

$x = \pm \frac{1}{\sqrt{2}}$

$\boxed{x = -\frac{1}{\sqrt{2}}}$ (Q3)

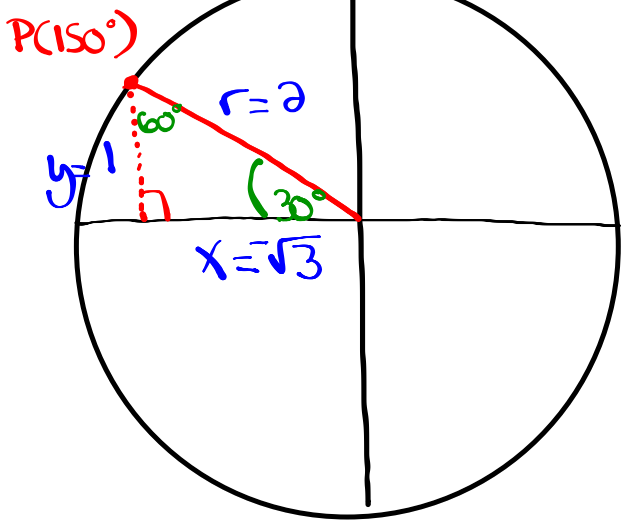
coordinates are $\left(-\frac{1}{\sqrt{2}}, -\frac{1}{\sqrt{2}}\right)$

Problems Involving the Unit Circle:

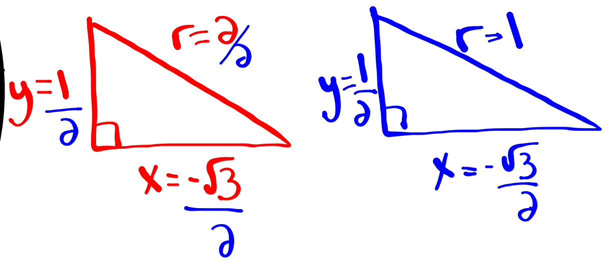
If $P(150^\circ)$ is the point at which the terminal arm of an angle θ in standard position intersects the unit circle, determine the exact coordinates of...

$r=1$

(x, y)

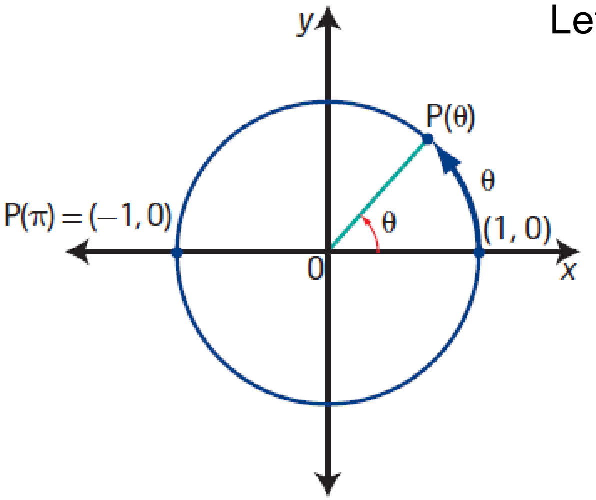


Scale the diagram so that r=1 (unit circle)



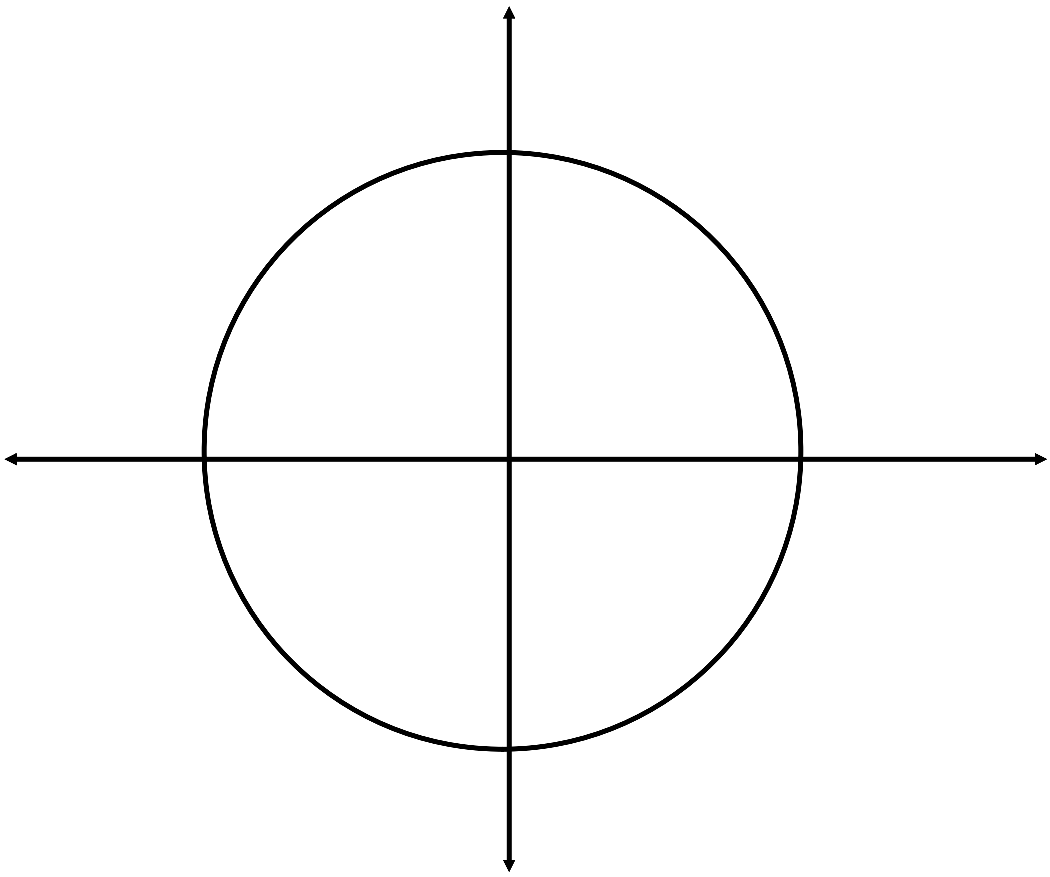
coordinates are $\left(-\frac{\sqrt{3}}{2}, \frac{1}{2}\right)$

Special Angles on the Unit Circle:

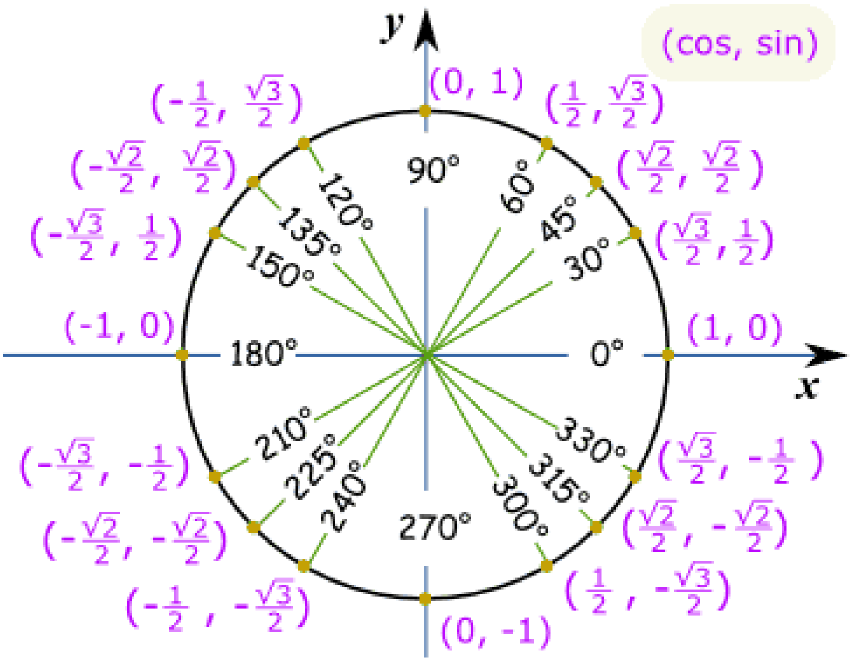


Let's use $\frac{\pi}{4}$ as our reference angle

Construct reference triangles
for all multiples of $\pi/4$
between 0 and 2π

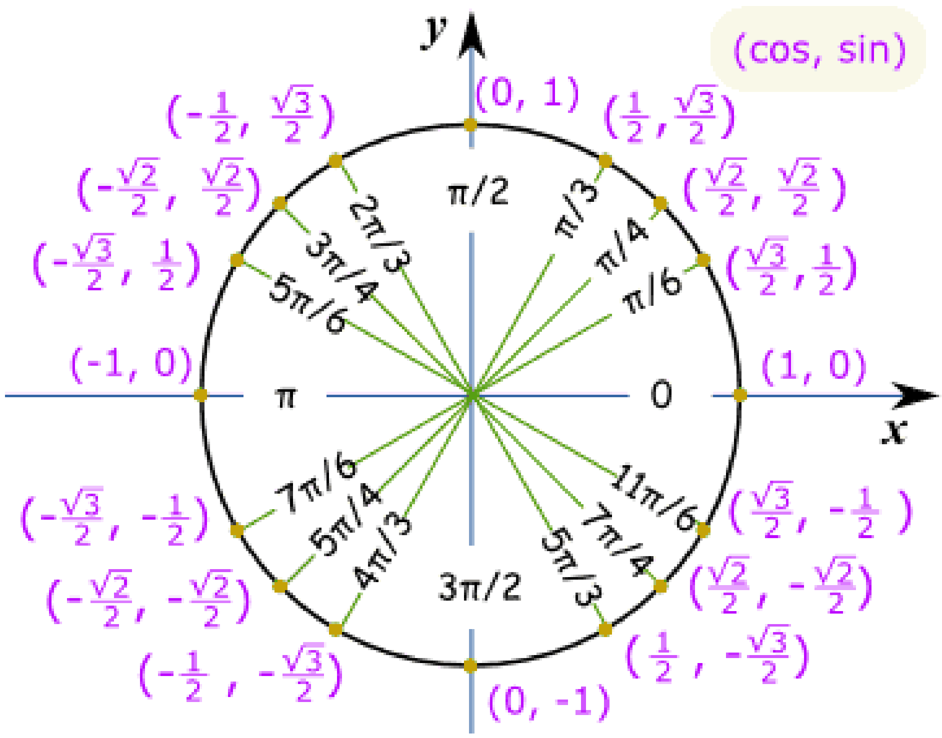


Unit Circle of Special Angles in Degrees



This is lovely...so what is it used for????

Unit Circle of Special Angles in Radians

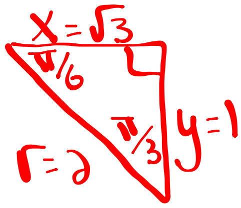


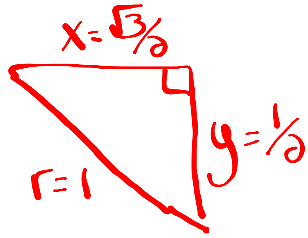
Questions from Homework

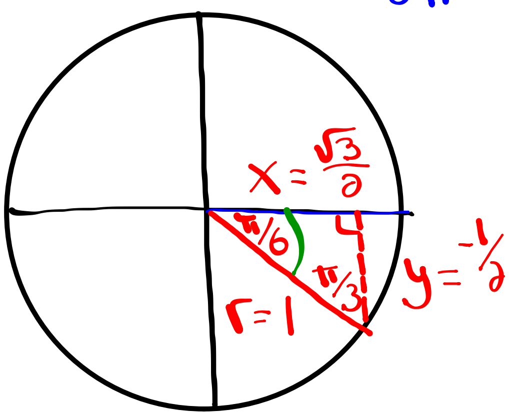
Scale to fit unit circle

① c) $\frac{10\pi}{6}, \frac{11\pi}{6}, \frac{12\pi}{6}$

2π







Coordinates are:

$\left(\frac{\sqrt{3}}{2}, -\frac{1}{2}\right)$

$\bar{\theta} = \frac{12\pi}{6} - \frac{11\pi}{6}$

$\bar{\theta} = \frac{\pi}{6}$

Questions from Homework

③ If $\csc\theta = -\frac{\sqrt{10}}{2}$ and $\tan\theta > 0$ determine the value of the 5 remaining trig ratios as radicals in simplest form.

Given:

$\csc\theta = -\frac{\sqrt{10}}{2} = \frac{h}{o} = \frac{r}{y}$

$r = \sqrt{10}$ (r is always positive)

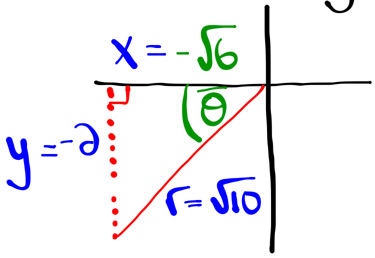
$y = -2$

① Determine what quadrant

$\csc < 0$ + $\tan\theta > 0$

Quad 3

② Draw a diagram:



④ Find the 5 trig ratios

$\sin\theta = \frac{-2}{\sqrt{10}} = \frac{-2\sqrt{10}}{10} = -\frac{\sqrt{10}}{5}$

$\cos\theta = \frac{-\sqrt{6}}{\sqrt{10}} = \frac{-\sqrt{60}}{10} = \frac{-2\sqrt{15}}{10} = \frac{-\sqrt{15}}{5}$

$\tan\theta = \frac{-2}{-\sqrt{6}} = \frac{2\sqrt{6}}{6} = \frac{\sqrt{6}}{3}$

$\sec\theta = \frac{\sqrt{10}}{-\sqrt{6}} = -\frac{\sqrt{60}}{6} = \frac{-2\sqrt{15}}{6} = -\frac{\sqrt{15}}{3}$

$\cot\theta = \frac{-\sqrt{6}}{-2} = \frac{\sqrt{6}}{2}$

③ Find the missing side:

$x^2 + y^2 = r^2$

$x^2 + (-2)^2 = (\sqrt{10})^2$

$x^2 + 4 = 10$

$x^2 = 6$

$x = \pm\sqrt{6}$

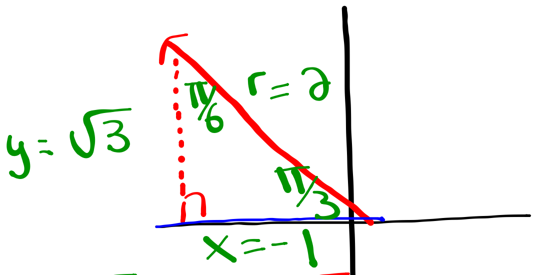
$x = -\sqrt{6}$ (Quad 3)

Questions from Homework

4 b) $\sec\frac{2\pi}{3} = \frac{2}{1} = -2$

$\frac{1\pi}{3}, \frac{2\pi}{3}, \frac{3\pi}{3}$

1π

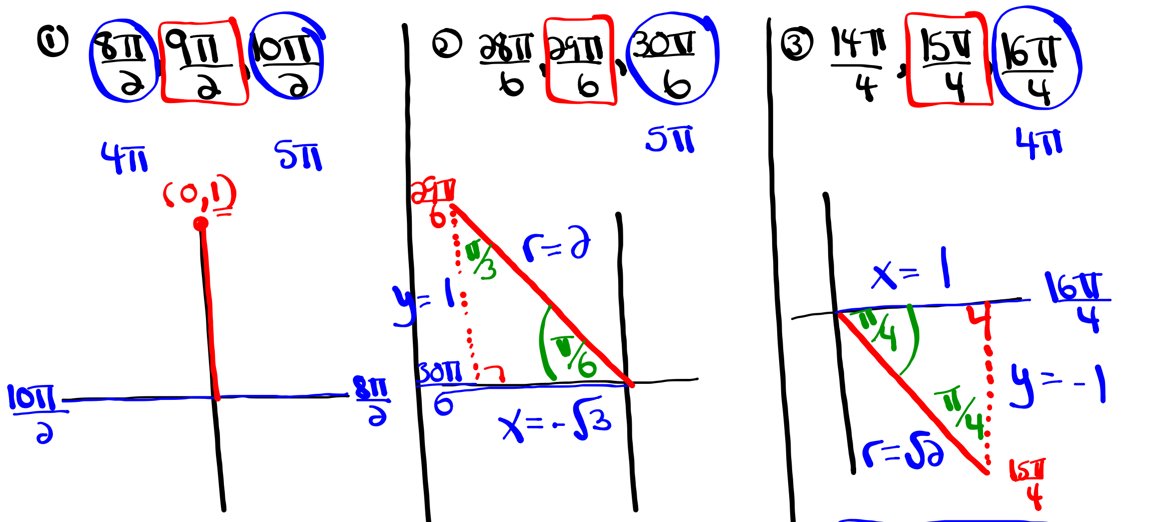


$\bar{\theta} = \frac{3\pi}{3} - \frac{2\pi}{3}$

$\bar{\theta} = \frac{\pi}{3}$

Evaluate without the use of a calculator:

$$\sin\frac{9\pi}{2} - \cos^2\left(\frac{29\pi}{6}\right)\tan\left(\frac{15\pi}{4}\right)$$



$\sin\frac{9\pi}{2} = 1$

$\cos\frac{29\pi}{6} = \frac{-\sqrt{3}}{2}$

$\tan\frac{15\pi}{4} = \frac{-1}{1} = -1$

$$\sin\frac{9\pi}{2} - \cos^2\left(\frac{29\pi}{6}\right)\tan\left(\frac{15\pi}{4}\right)$$

$$(1) - \left(\frac{-\sqrt{3}}{2}\right)^2(-1)$$

$$1 - \frac{3}{4}(-1)$$

$$1 + \frac{3}{4}$$

$$\frac{4}{4} + \frac{3}{4}$$

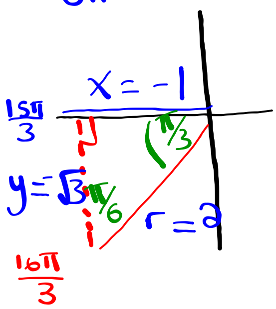
$$\frac{7}{4}$$

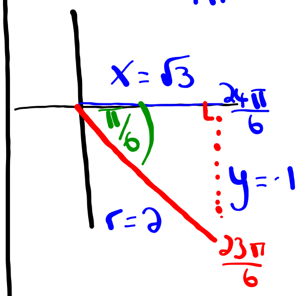
Evaluate without the use of a calculator:

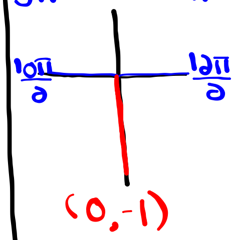
$$\cos\left(\frac{16\pi}{3}\right)\tan^2\left(\frac{23\pi}{6}\right)+\csc\left(\frac{11\pi}{2}\right)+\sin^2\left(\frac{27\pi}{4}\right)$$

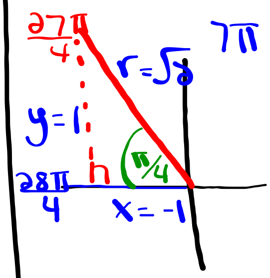
$\frac{15\pi}{3}, \frac{16\pi}{3}, \frac{17\pi}{3}$ | $\frac{22\pi}{6}, \frac{23\pi}{6}, \frac{24\pi}{6}$ | $\frac{10\pi}{2}, \frac{11\pi}{2}, \frac{12\pi}{2}$ | $\frac{26\pi}{4}, \frac{27\pi}{4}, \frac{28\pi}{4}$

5π | 4π | 5π, 6π | 7π









$\cos\frac{16\pi}{3} = \frac{-1}{2}\ \frac{x}{r}$

$\tan\frac{23\pi}{6} = \frac{-1}{\sqrt{3}}\ \frac{y}{x}$

$\csc\frac{11\pi}{2} = \frac{1}{-1}\ \frac{r}{y}$

$\csc\frac{11\pi}{2} = -1$

$\sin\frac{27\pi}{4} = \frac{1}{\sqrt{2}}\ \frac{y}{r}$

$$\cos\left(\frac{16\pi}{3}\right)\tan^2\left(\frac{23\pi}{6}\right)+\csc\left(\frac{11\pi}{2}\right)+\sin^2\left(\frac{27\pi}{4}\right)$$

$$\left(-\frac{1}{2}\right)\left(-\frac{1}{\sqrt{3}}\right)^2+(-1)+\left(\frac{1}{\sqrt{2}}\right)^2$$

$$\left(-\frac{1}{2}\right)\left(\frac{1}{3}\right)-1+\frac{1}{2}$$

$$-\frac{1}{6}-\frac{6}{6}+\frac{3}{6}$$

$$\frac{-4}{6}$$

$$-\frac{2}{3}$$

Homework: Worksheet - Sketching Angles in Radians.doc

Solutions...

1. $-\frac{5}{3}$

2. $\frac{-\sqrt{6}}{3}$

3. $-2-\sqrt{3}$

4. $\frac{-5}{3}$

5. $\frac{4+3\sqrt{3}}{6}$

6. $\frac{-10}{3}$

7. 0

8. $\frac{3+3\sqrt{3}}{-2}$

Attachments

Worksheet - Sketching Angles in Radians.doc