Correct Homework Sheet

① a) $f(x) = 3x^{-2} + 3x^3 + 1$

$f'(x) = -6x^{-3} + 9x^2$

b) $y = \dfrac{4x+1}{6-2x^5}$

$$y' = \frac{\overset{f'}{4}\,\overset{g}{(6-2x^5)} - \overset{g'}{(-10x^4)}\,\overset{f}{(4x+1)}}{(6-2x^5)^2}$$

c) $f(x) = 3(2x^5 + x - 5)^{10}$

$f'(x) = 30(2x^5 + x - 5)^9 (10x^4 + 1)$

d) $h(x) = (x^2 - x)\sqrt{4-9x} = (x^2-x)(4-9x)^{1/2}$

$$h'(x) = \overset{f'}{(2x-1)}\,\overset{g}{(4-9x)^{1/2}} + \overset{f}{(x^2-x)}\,\overbrace{\left(\frac{1}{2}\right)(4-9x)^{-1/2}(-9)}^{g'}$$

② a) $y = \dfrac{2}{x} + \dfrac{3}{5x^3} - 6\sqrt{x} + \sqrt[3]{9x^8} - 8\pi$

$y = 2x^{-1} + \dfrac{3}{5}x^{-3} - 6x^{1/2} + (9x^8)^{1/3} - 8\pi$

$y' = -2x^{-2} - \dfrac{9}{5}x^{-4} - 3x^{-1/2} + \dfrac{1}{3}(9x^8)^{-2/3}(72x^7) - 0$

b) $y = \sqrt[3]{\dfrac{1-x^6}{2+(5x-1)^4}} = \left(\dfrac{1-x^6}{2+(5x-1)^4}\right)^{1/3}$

$$y' = \frac{1}{3}\left(\frac{1-x^6}{2+(5x-1)^4}\right)^{-2/3}\left[\frac{-6x^5(2+(5x-1)^4) - (1-x^6)(4)(5x-1)^3(5)}{[2+(5x-1)^4]^2}\right]$$

c) $g(x) = (x-5)^3(7x^5+2x)^9(4-2x^3)^5$

$$g'(x) = 3(x-5)^2(1)(7x^5+2x)^9(4-2x^3)^5 + 9(7x^5+2x)^8(35x^4+2)(x-5)^3(4-2x^3)^5 + 5(4-2x^3)^4(-6x^2)(x-5)^3(7x^5+2x)^9$$

② d) $f(x) = \sqrt{25 + 4(2x-1)^4} = (25 + 4(2x-1)^4)^{1/2}$

$f'(x) = \dfrac{1}{2}(25 + 4(2x-1)^4)^{-1/2}(16(2x-1)^3(2))$

Correct Homework Sheet

③ a) $y=\sqrt{x^2-5x\sqrt{2x^3+3\sqrt{x}}}=\left[x^2-5x\left(2x^3+3x^{\frac{1}{2}}\right)^{\frac{1}{2}}\right]^{\frac{1}{2}}$

$y'=\frac{1}{2}\left[x^2-5x\left(2x^3+3x^{\frac{1}{2}}\right)^{\frac{1}{2}}\right]^{-\frac{1}{2}}\left(2x-\left[5\left(2x^3+3x^{\frac{1}{2}}\right)^{\frac{1}{2}}+5x\left(\frac{1}{2}\right)\left(2x^3+3x^{\frac{1}{2}}\right)^{-\frac{1}{2}}\left(6x^2+\frac{3}{2}x^{-\frac{1}{2}}\right)\right]\right)$

Correct Homework Sheet

③ b) $f(x) = \dfrac{8x^3(12x^2-5x)^8}{2-3\sqrt[5]{(1-32x^{10})}} = \dfrac{8x^3(12x^2-5x)^8}{2-3(1-32x^{10})^{1/5}}$

$$f'(x) = \frac{\left[24x^2(12x^2-5x)^8 + 8x^3(8)(12x^2-5x)^7(24x-5)\right]\left[2-3(1-32x^{10})^{1/5}\right] - \left[8x^3(12x^2-5x)^8\right]\left[-\frac{3}{5}(1-32x^{10})^{-4/5}(-320x^9)\right]}{\left[2-3(1-32x^{10})^{1/5}\right]^2}$$

Correct Homework Sheet

③ c) $$f(x) = \frac{\left[x^5 - x\sqrt{4-x^2}\right]^6}{12\sqrt{x}\,(5x^3-8)^7} = \frac{\overbrace{\left[x^5 - x(4-x^2)^{1/2}\right]^6}^{f(x)}}{\underbrace{12x^{1/2}(5x^3-8)^7}_{g(x)}}$$

$$f'(x) = \frac{\overbrace{6\left[x^5 - x(4-x^2)^{1/2}\right]^5\left[5x^4 - \left(1(4-x^2)^{1/2} + x\left(\tfrac{1}{2}\right)(4-x^2)^{-1/2}(-2x)\right)\right]}^{f'(x)}\overbrace{\left[12x^{1/2}(5x^3-8)^7\right]}^{g(x)} - \overbrace{\left[x^5 - x(4-x^2)^{1/2}\right]^6}^{f(x)}\overbrace{\left[6x^{-1/2}(5x^3-8)^7 + 12x^{1/2}(7)(5x^3-8)^6(15x^2)\right]}^{g'(x)}}{\left[12x^{1/2}(5x^3-8)^7\right]^2}$$

To be handed in today

$$f(x) = \sqrt[7]{\frac{9+16x^4}{\left[4x^5(3x^8+8x-2)\right]^5}} = \left[\frac{9+16x^4}{\left[4x^5(3x^8+8x-2)\right]^5}\right]^{1/7}$$

$$f'(x) = \frac{1}{7}\left[\frac{9+16x^4}{\left[4x^5(3x^8+8x-2)\right]^5}\right]^{-6/7}\left[\frac{64x^3\left[4x^5(3x^8+8x-2)\right]^5-(9+16x^4)(5)\left[4x^5(3x^8+8x-2)\right]^4\cdot\left[(20x^4)(3x^8+8x-2)+4x^5(24x^7+8)\right]}{\left[4x^5(3x^8+8x-2)\right]^{10}}\right]$$

Review

① a) $f(x) = \sqrt{x-5}$ $\quad f(x+h) = \sqrt{x+h-5}$

$$f'(x) = \lim_{h \to 0} \left(\frac{\sqrt{x+h-5} - \sqrt{x-5}}{h}\right) \frac{\left(\sqrt{x+h-5} + \sqrt{x-5}\right)}{\left(\sqrt{x+h-5} + \sqrt{x-5}\right)}$$

$$f'(x) = \lim_{h \to 0} \frac{\cancel{x}+h-\cancel{5} - (\cancel{x}-\cancel{5})}{h\left(\sqrt{x+h-5} + \sqrt{x-5}\right)}$$

$$f'(x) = \lim_{h \to 0} \frac{\cancel{h}}{\cancel{h}\left(\sqrt{x+h-5} + \sqrt{x-5}\right)} = \boxed{\frac{1}{2\sqrt{x-5}}}$$

② b) $f(x) = \frac{3}{\sqrt{x}} = \frac{3}{x^{1/2}} = 3x^{-1/2}$

$$f'(x) = -\frac{3}{2}x^{-3/2} = \frac{-3}{2x^{3/2}} = \frac{-3}{2\sqrt{x^3}}$$

Review

① b) $f(x) = \dfrac{2x-2}{x+3}$ $\qquad f(x+h) = \dfrac{2(x+h)-2}{(x+h)+3} = \dfrac{2x+2h-2}{x+h+3}$

CD: $(x+3)(x+h+3)$

$$f'(x) = \lim_{h\to 0} \frac{\dfrac{2x+2h-2}{x+h+3} - \dfrac{2x-2}{x+3}}{h}$$

$$f'(x) = \lim_{h\to 0} \frac{(x+3)(2x+2h-2) - (2x-2)(x+h+3)}{h(x+3)(x+h+3)}$$

$$f'(x) = \lim_{h\to 0} \frac{\cancel{2x^2} \cancel{+2xh} \cancel{-2x} \cancel{+6x} + 6h \cancel{-6} - (\cancel{2x^2} \cancel{+2xh} \cancel{+6x} \cancel{-2x} - 2h \cancel{-6})}{h(x+3)(x+h+3)}$$

$$f'(x) = \lim_{h\to 0} \frac{8\cancel{h}}{\cancel{h}(x+3)(x+h+3)} = \boxed{\frac{8}{(x+3)^2}}$$

Review:

② a) $f(x) = 3x^2 + 5x - 2$

$f'(x) = 6x + 5$

c) $f(x) = 2x^4 + \sqrt{x} = 2x^4 + x^{1/2}$

$f'(x) = 8x^3 + \frac{1}{2}x^{-1/2}$

$f'(x) = 8x^3 + \frac{1}{2x^{1/2}}$

d) $f(x) = \sqrt[3]{x^2} = x^{2/3}$

$f'(x) = \frac{2}{3}x^{-1/3}$

$f'(x) = \frac{2}{3x^{1/3}}$

Review:

③ a) $y = (3x^2 - 2)(4x+5)$

$y' = (6x)(4x+5) + 4(3x^2-2)$

$y' = 24x^2 + 30x + 12x^2 - 8$

$y' = 36x^2 + 30x - 8$

③ b) $g(x) = (x^2 - 5x + 2)(4x+1)$

$g'(x) = (2x - 5)(4x+1) + 4(x^2 - 5x + 2)$

$g'(x) = \underline{8x^2} + \underline{2x} \; \underline{-20x} - 5 \; \underline{+4x^2} \; \underline{-20x} + 8$

$g'(x) = 12x^2 - 38x + 3$

Review:

④ a) $f(x) = \dfrac{2x^2+3}{3x-2}$

$$f'(x) = \frac{4x(3x-2) - 3(2x^2+3)}{(3x-2)^2}$$

$$f'(x) = \frac{12x^2 - 8x - 6x^2 - 9}{(3x-2)^2}$$

$$f'(x) = \frac{6x^2 - 8x - 9}{(3x-2)^2}$$

④ b) $y = \dfrac{\sqrt{x}}{3+x^2} = \dfrac{x^{1/2}}{3+x^2}$ $\begin{array}{l} f(x) \\ g(x) \end{array}$

$$y' = \frac{\frac{1}{2}x^{-1/2}(3+x^2) - (2x)(x^{1/2})}{(3+x^2)^2}$$

$$y' = \frac{\frac{(3+x^2)}{\cancel{2x^{1/2}}}\cancel{2x^{1/2}} - \frac{2x^{3/2}}{1} \cdot 2x^{1/2}}{(3+x^2)^2 \; 2x^{1/2}}$$

CD: $2x^{1/2}$

$$y' = \frac{3+x^2-4x^2}{2\sqrt{x}\,(3+x^2)^2} = \frac{3-3x^2}{2\sqrt{x}\,(3+x^2)^2}$$

Review

⑤ Find the equation of the tangent line to the curve $y=(x^2-3)^8$ at $x=2 \leftarrow x_1$

(i) Find y:

$y=(2^2-3)^8$

$y=(4-3)^8$

$y=1^8$

$y=1 \leftarrow y_1$

(ii) Find y'

$y=(x^2-3)^8$

$y'=8(x^2-3)^7(2x)$

$y'=16x(x^2-3)^7$

(iii) Find m $(y'(2))$

$y'=16(2)[(2)^2-3]^7$

$y'=32(1)$

$y'=32 \leftarrow m$

(iv) Find equation:

$y-y_1=m(x-x_1)$

$y-1=32(x-2)$

$y-1=32x-64$

$\boxed{y=32x-63}$ or $32x-y-63=0$

Review:

⑥ a) $f(x) = 3(2x^2-4)^4$

$f'(x) = 12(2x^2-4)^3(4x)$

$f'(x) = 48x(2x^2-4)^3$

b) $y = \frac{16}{\sqrt{x-1}} = \frac{16}{(x-1)^{1/2}} = 16(x-1)^{-1/2}$

$y' = -8(x-1)^{-3/2}(1)$

$y' = -8(x-1)^{-3/2}$

$y' = \frac{-8}{(x-1)^{3/2}}$

Review:

① a) $f(x) = \left(\dfrac{2x+1}{x-1}\right)^5 = \dfrac{(2x+1)^5}{(x-1)^5}$

$f'(x) = 5\left(\dfrac{2x+1}{x-1}\right)^4 \left[\dfrac{2(x-1) - 1(2x+1)}{(x-1)^2}\right]$

$f'(x) = 5\,\dfrac{(2x+1)^4}{(x-1)^4}\left[\dfrac{\cancel{2x} - 2 - \cancel{2x} - 1}{(x-1)^2}\right]$

$f'(x) = 5 \cdot \dfrac{(2x+1)^4}{(x-1)^4} \cdot \dfrac{-3}{(x-1)^2} = \boxed{\dfrac{-15(2x+1)^4}{(x-1)^6}}$

Review

⑦ b) $y = (x^2-1)^3(3x-2)^2$

$y' = 3(x^2-1)^2(2x)(3x-2)^2 + (x^2-1)^3(2)(3x-2)(3)$

$y' = 6x(x^2-1)^2(3x-2)^2 + 6(x^2-1)^3(3x-2)$

$y' = 6(x^2-1)^2(3x-2)[\overbrace{x(3x-2) + (x^2-1)}^{3x^2-2x+x^2-1}]$

$y' = 6(x^2-1)^2(3x-2)(4x^2-2x-1)$

Review

① c) $y = \dfrac{(2x+1)^2}{(x^4-x+1)^2}$ $f(x)$ $g(x)$

$$\frac{f'(x)g(x) - f(x)g'(x)}{[g(x)]^2}$$

$$y' = \frac{2(2x+1)^1(2)(x^4-x+1)^2 - (2x+1)^2(2)(x^4-x+1)^1(4x^3-1)}{(x^4-x+1)^4}$$

$$y' = \frac{4(2x+1)(x^4-x+1)^2 - 2(4x^3-1)(2x+1)^2(x^4-x+1)}{(x^4-x+1)^4}$$

$$y' = \frac{2(2x+1)(x^4-x+1)\left[\overset{2x^4-2x+2}{2(x^4-x+1)} - \overset{(8x^4+4x^3-2x-1)}{(4x^3-1)(2x+1)}\right]}{(x^4-x+1)^4}$$

$$y' = \frac{2(2x+1)\cancel{(x^4-x+1)}(-6x^4-4x^3+3)}{(x^4-x+1)^{\cancel{4}\,3}}$$

$$y' = \frac{-2(2x+1)(6x^4+4x^3-3)}{(x^4-x+1)^3}$$

product quotient

Example 1 (See #6 on 2.4 and #4 on 2.5)

Let $F(x) = f(g(x))$ ← chain

If $f(2) = 3$, $\boxed{f'(2) = 5}$, $\boxed{g(1) = 2}$ and $\boxed{g'(1) = 4}$ $\boxed{\text{find } F'(1).}$

$$F'(x) = f'(g(x))g'(x)$$

$$F'(1) = f'(g(1)) \cdot g'(1)$$

$$F'(1) = f'(2) \cdot g'(1)$$

$$F'(1) = 5 \cdot 4$$

$$\boxed{F'(1) = 20}$$

Example 2

If $y = u^{10} + u^5 + 2$, where $u = 1 - 3x^2$, find $\left.\frac{dy}{dx}\right]_{x=1}$

(i) $\frac{dy}{du} = 10u^9 + 5u^4$

(ii) $\frac{du}{dx} = -6x$

(iv) when $x = 1$

$u = 1 - 3x^2$

$u = 1 - 3(1)^2$

$u = 1 - 3$

$u = -2$

(iii) $\left.\frac{dy}{dx}\right]_{x=1} = \frac{dy}{\cancel{du}} \cdot \frac{\cancel{du}}{dx}$

$\left.\frac{dy}{dx}\right]_{x=1} = [10u^9 + 5u^4][-6x]$

$\left.\frac{dy}{dx}\right]_{x=1} = [10(-2)^9 + 5(-2)^4][-6(1)]$

$\left.\frac{dy}{dx}\right]_{x=1} = [-5120 + 80][-6]$

$\left.\frac{dy}{dx}\right]_{x=1} = [-5040][-6]$

$$\boxed{\left.\frac{dy}{dx}\right|_{x=1} = 30240}$$

Option 2

If $y = u^{10} + u^5 + 2$, where $u = 1 - 3x^2$, find $\left.\frac{dy}{dx}\right]_{x=1}$

$$y = (1-3x^2)^{10} + (1-3x^2)^5 + 2$$

$$\left.\frac{dy}{dx}\right|_{x=1} = 10(1-3x^2)^9(-6x) + 5(1-3x^2)^4(-6x)$$

$$\left.\frac{dy}{dx}\right|_{x=1} = -60x(1-3x^2)^9 - 30x(1-3x^2)^4$$

$$\left.\frac{dy}{dx}\right|_{x=1} = -60(-512) - 30(16)$$

$$\left.\frac{dy}{dx}\right|_{x=1} = 30720 - 480$$

$$\boxed{\left.\frac{dy}{dx}\right|_{x=1} = 30240}$$

Page 103

(4) Find $\frac{dy}{dt}\Big]_{t=1} \longrightarrow$ when $t=1$: $r=\frac{1+1}{2(1)+1}=\frac{2}{3}$

$y=\sqrt{1+r^2}=(1+r^2)^{1/2}$

$\frac{dy}{dr}=\frac{1}{2}(1+r^2)^{-1/2}(2r)$

$\frac{dy}{dr}=\frac{\cancel{2}r}{\cancel{2}(1+r^2)^{1/2}}=\boxed{\frac{r}{\sqrt{1+r^2}}}$

$r=\frac{t+1}{2t+1}$

$\frac{dr}{dt}=\frac{1(2t+1)-2(t+1)}{(2t+1)^2}$

$\frac{dr}{dt}=\frac{2t+1-2t-2}{(2t+1)^2}=\boxed{\frac{-1}{(2t+1)^2}}$

$\frac{dy}{dt}\Big]_{t=1}=\left[\frac{r}{\sqrt{1+r^2}}\right]\left[\frac{-1}{(2t+1)^2}\right]$

$=\left[\frac{2/3}{\sqrt{1+(\frac{2}{3})^2}}\right]\left[\frac{-1}{(2(1)+1)^2}\right]$

$=\left[\frac{2/3}{\sqrt{1+\frac{4}{9}}}\right]\left[\frac{-1}{9}\right]$

$=\left[\frac{2/3}{\sqrt{13/9}}\right]\left[-\frac{1}{9}\right]$

$=\left[\frac{2/3}{\sqrt{13}/\sqrt{9}}\right]\left[-\frac{1}{9}\right]$

$=\left[\frac{2}{\cancel{3}}\cdot\frac{\cancel{3}}{\sqrt{13}}\right]\left[-\frac{1}{9}\right]$

$=\boxed{\frac{-2}{9\sqrt{13}}}$

Page 103

6 e) $F(x) = \frac{x}{\sqrt{2x+3}} = x(2x+3)^{-1/2}$

$F'(x) = 1(2x+3)^{-1/2} + x\left(-\frac{1}{2}\right)(2x+3)^{-3/2}(2)$

$F'(x) = (2x+3)^{-1/2} - x(2x+3)^{-3/2}$

$F'(x) = (2x+3)^{-3/2}\left[(2x+3) - x\right]$

$F'(x) = \frac{x+3}{(2x+3)^{3/2}} = \frac{x+3}{\sqrt{(2x+3)^3}}$

Page 103

(9) $F(x) = f(g(x))$ ← Composite Function

Given:

$g(2) = 4$

$g'(2) = 3$

$f'(4) = 5$

$f(2) = -1$

$f'(2) = 2$

find $F'(2)$

$F'(x) = f'(g(x)) \cdot g'(x)$

$F'(2) = f'(g(2)) \cdot g'(2)$

$F'(2) = f'(4) \cdot g'(2)$

$F'(2) = 5 \cdot 3$

$F'(2) = 15$