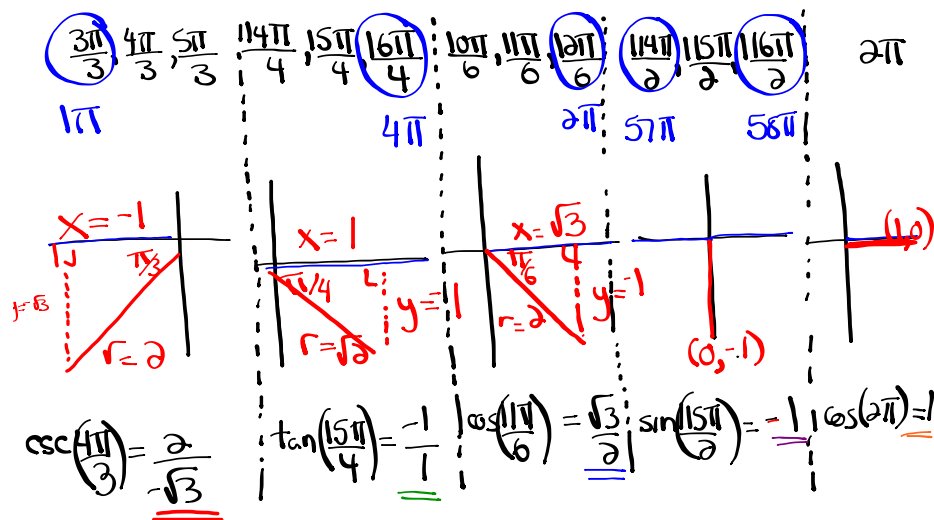


## Questions from Homework

$$\textcircled{5} \csc^2\left(\frac{4\pi}{3}\right) \tan\left(\frac{15\pi}{4}\right) + \cos\left(\frac{-13\pi}{6}\right) - \sin\left(\frac{115\pi}{2}\right) + \cos(-14\pi)$$

$\frac{-13\pi}{6} + \frac{24\pi}{6} = \frac{11\pi}{6}$        $-14\pi + 6\pi = -8\pi$

$$\csc^2\left(\frac{4\pi}{3}\right) \tan\left(\frac{15\pi}{4}\right) + \cos\left(\frac{11\pi}{6}\right) - \sin\left(\frac{115\pi}{2}\right) + \cos(2\pi)$$



$$\csc^2\left(\frac{4\pi}{3}\right) \tan\left(\frac{15\pi}{4}\right) + \cos\left(\frac{11\pi}{6}\right) - \sin\left(\frac{115\pi}{2}\right) + \cos(2\pi)$$

$$\left(\frac{2}{\sqrt{3}}\right)^2 (-1) + \left(\frac{\sqrt{3}}{2}\right) - (-1) + (1)$$

$$\left(\frac{4}{3}\right)(-1) + \frac{\sqrt{3}}{2} + 1 + 1$$

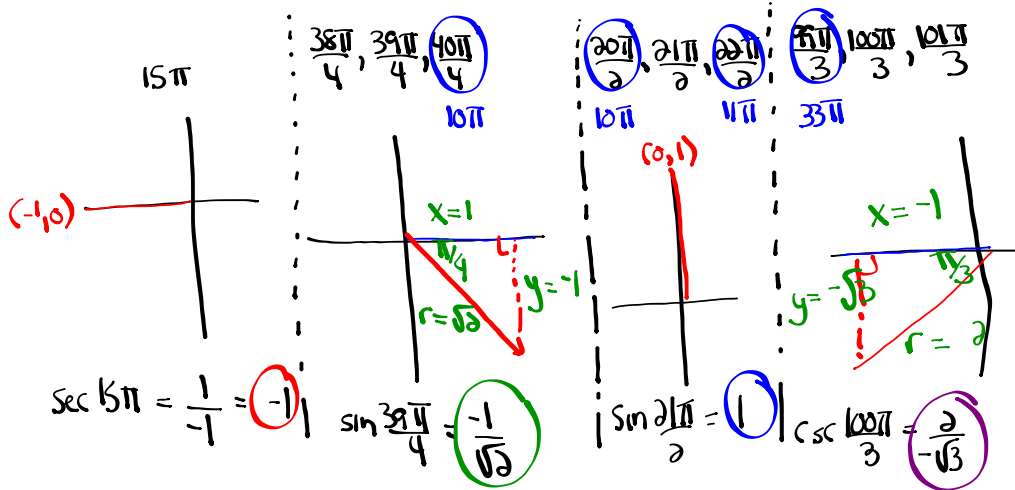
$$-\frac{4}{3} + \frac{\sqrt{3}}{2} + \frac{2}{1}$$

$$-\frac{8}{6} + \frac{3\sqrt{3}}{6} + \frac{12}{6}$$

$$\boxed{\frac{4 + 3\sqrt{3}}{6}} \quad \text{or} \quad \frac{3\sqrt{3} + 4}{6}$$

## Questions from Homework

⑥  $\sec 15\pi + \sqrt{2} \sin \frac{39\pi}{4} \sin \frac{21\pi}{2} - \csc^2 \frac{100\pi}{3}$



$$\frac{\sec 15\pi}{-1} + \sqrt{2} \frac{\sin \frac{39\pi}{4}}{\frac{-1}{\sqrt{2}}} \frac{\sin \frac{21\pi}{2}}{1} - \frac{\csc^2 \frac{100\pi}{3}}{\left(\frac{2}{-\sqrt{3}}\right)^2}$$

$$-1 + \sqrt{2} \left(\frac{-1}{\sqrt{2}}\right)(1) - \frac{4}{3}$$

$$-1 - \frac{\sqrt{2}}{\sqrt{2}} - \frac{4}{3}$$

$$-1 - 1 - \frac{4}{3}$$

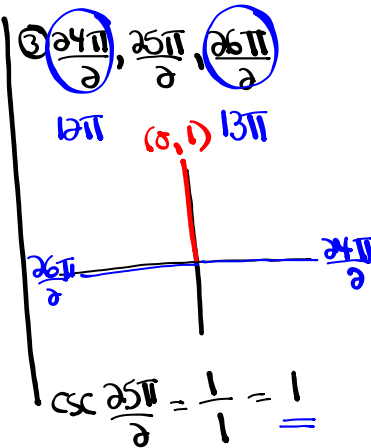
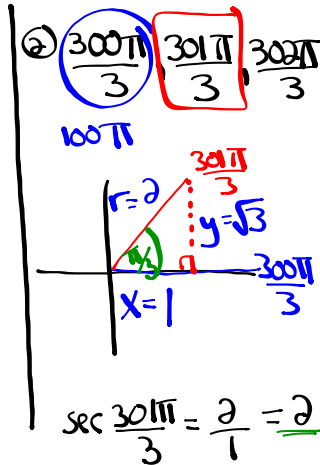
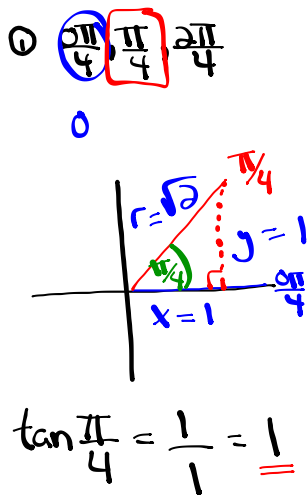
$$-\frac{3}{3} - \frac{3}{3} - \frac{4}{3}$$

$$\frac{-10}{3}$$

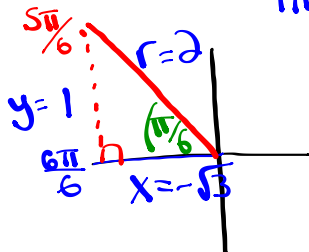
# Questions from Homework

$$\textcircled{1} \frac{\tan\left(\frac{\pi}{4}\right) + \sec\left(\frac{301\pi}{3}\right)}{\csc\left(\frac{25\pi}{2}\right) + \cot\left(\frac{5\pi}{6}\right)}$$

$$\begin{array}{l|l} * \frac{-15\pi}{4} + \frac{4\pi}{1} & \frac{-31\pi}{6} + \frac{6\pi}{1} \\ \hline \frac{-15\pi}{4} + \frac{16\pi}{4} & \frac{-31\pi}{6} + \frac{36\pi}{6} \\ \frac{\pi}{4} & \frac{5\pi}{6} \end{array}$$



④  $\frac{4\pi}{6}, \frac{5\pi}{6}, \frac{6\pi}{6}$   
π



$\cot \frac{5\pi}{6} = \frac{-\sqrt{3}}{1} = -\sqrt{3}$

$$\frac{\tan\left(\frac{\pi}{4}\right) + \sec\left(\frac{301\pi}{3}\right)}{\csc\left(\frac{25\pi}{2}\right) + \cot\left(\frac{5\pi}{6}\right)}$$

$$\frac{1 + 2}{1 - \sqrt{3}}$$

$$\frac{3(1 + \sqrt{3})}{(1 - \sqrt{3})(1 + \sqrt{3})}$$

$$\frac{3 + 3\sqrt{3}}{1 + \sqrt{3} - \sqrt{3} - 3}$$

$$\frac{3 + 3\sqrt{3}}{-2} \text{ or } \boxed{\frac{-3 - 3\sqrt{3}}{2}}$$

# Introduction to Trigonometric Equations

## trigonometric equation

- an equation involving trigonometric ratios

### Focus on...

---

- algebraically solving first-degree and second-degree trigonometric equations in radians and in degrees
- verifying that a specific value is a solution to a trigonometric equation
- identifying exact and approximate solutions of a trigonometric equation in a restricted domain
- determining the general solution of a trigonometric equation

### Did You Know?

In equations, mathematicians often use the notation  $\cos^2 \theta$ . This means the same as  $(\cos \theta)^2$ .

Let's start with basic LINEAR trigonometric equations...

Solve:  $\sin \theta = 0.9659$ ,  $-360^\circ < \theta < 720^\circ$

...Pre-Calculus 110

- Reference angle?
- Which quadrants?
- Any co-terminal angles acceptable?

Domain in degrees

- If the domain is in degrees, give solutions in degrees.
- If the domain is in radians, give solutions in radians.

$$\sin \theta = 0.9659$$

① Find  $\bar{\theta}$

② where is  $\sin \theta > 0$

S	A
T	C

$$\bar{\theta} = \sin^{-1}(0.9659)$$

③ Find  $\theta$

$$\bar{\theta} = 75^\circ$$

Q1	Q2
$\theta = \bar{\theta}$	$\theta = 180^\circ - \bar{\theta}$
$\theta = 75^\circ$	$\theta = 180^\circ - 75^\circ = 105^\circ$
$\theta = 75^\circ - 360^\circ = -285^\circ$	$\theta = 105^\circ - 360^\circ = -255^\circ$
$\theta = 75^\circ + 360^\circ = 435^\circ$	$\theta = 105^\circ + 360^\circ = 465^\circ$

Solve:  $\sec \theta = -1.3054$ ,  $-2\pi \leq \theta \leq 2\pi$  (Radians)  $-6.28 \leq \theta \leq 6.28$   
 (Approx.)

$$\cos \theta = \frac{1}{-1.3054}$$

$$\cos \theta = -0.7660$$

① Find  $\bar{\theta}$

$$\bar{\theta} = \cos^{-1}(0.7660)$$

$$\bar{\theta} = 0.7 \text{ rads}$$

② where is  $\cos \theta < 0$   $\frac{S}{A}$   
 $\frac{+}{-}$

③ Find  $\theta$

Q2	Q3
$\theta = \pi - \bar{\theta}$	$\theta = \pi + \bar{\theta}$
$\theta = 3.14 - 0.7 = 2.44$	$\theta = 3.14 + 0.7 = 3.84$
$\theta = 2.44 - 6.28 = -3.84$	$\theta = 3.84 - 6.28 = -2.44$

## Warm-up

(Approx.)

Ex:  $\tan \theta = -0.8524$ ,  $-360^\circ \leq \theta \leq 360^\circ$  (Degrees)

① Find  $\bar{\theta}$ 

$$\bar{\theta} = \tan^{-1}(0.8524) \leftarrow \text{ignore negative}$$

$$\bar{\theta} = 40.4^\circ$$

② Where is  $\tan \theta < 0$ 

S	A
T	C

③ Find  $\theta$ 

Q2	Q4
$\theta = 180^\circ - \bar{\theta}$	$\theta = 360^\circ - \bar{\theta}$
$\theta = 180^\circ - 40.4^\circ = 139.6^\circ$	$\theta = 360^\circ - 40.4^\circ = 319.6^\circ$
$\theta = 139.6^\circ - 360^\circ = -220.4^\circ$	$\theta = 319.6^\circ - 360^\circ = -40.4^\circ$

(Exact)

$$\frac{-1}{\sqrt{2}} = -0.7071$$

Ex.  $\sqrt{2} \cos \theta + 1 = 0, -360^\circ \leq \theta \leq 720^\circ$  (Degrees)

$$\frac{\sqrt{2} \cos \theta}{\sqrt{2}} = \frac{-1}{\sqrt{2}}$$

$$\cos \theta = \frac{-1}{\sqrt{2}}$$

① Find  $\bar{\theta}$ 

$$\bar{\theta} = \cos^{-1}\left(\frac{1}{\sqrt{2}}\right) \quad (\text{Triangle \#1}) \quad \cos \theta = \frac{a}{r} = \frac{x}{r}$$

$$\bar{\theta} = 45^\circ$$

② Where is  $\cos \theta < 0$ 

$$\begin{array}{c|c} S & A \\ \hline \checkmark & c \end{array}$$

③ Find  $\theta$ 

Q2	Q3
$\theta = 180^\circ - \bar{\theta}$	$\theta = 180^\circ + \bar{\theta}$
$\theta = 180^\circ - 45^\circ = 135^\circ$	$\theta = 180^\circ + 45^\circ = 225^\circ$
$\theta = 135^\circ - 360^\circ = -225^\circ$	$\theta = 225^\circ - 360^\circ = -135^\circ$
$\theta = 135^\circ + 360^\circ = 495^\circ$	$\theta = 225^\circ + 360^\circ = 585^\circ$



(Exact)

Ex.  $\sin x + 1 = 0$ ,  $-2\pi \leq x \leq 4\pi$  (Radians)  $-\frac{4\pi}{2} \leq x \leq \frac{8\pi}{2}$

$$\sin x = -1$$

① Find  $x$  (Unit Circle)

$$x = \frac{3\pi}{2}$$

$$x = \frac{3\pi}{2} - \frac{2\pi}{1} = \frac{3\pi}{2} - \frac{4\pi}{2} = -\frac{\pi}{2}$$

$$x = \frac{3\pi}{2} + \frac{2\pi}{1} = \frac{3\pi}{2} + \frac{4\pi}{2} = \frac{7\pi}{2}$$

**Your Turn**

Solve each trigonometric equation in the specified domain.

a)  $3 \cos \theta - 1 = \cos \theta + 1, -2\pi \leq \theta \leq 2\pi$

b)  $4 \sec x + 8 = 0, 0^\circ \leq x < 360^\circ$

(Exact)

$$\text{a) } 3 \cos \theta - 1 = \cos \theta + 1, \quad -2\pi \leq \theta \leq 2\pi \quad (\text{Radians})$$

$$3 \cos \theta - \cos \theta = 1 + 1$$

$$\frac{2 \cos \theta}{2} = \frac{2}{2}$$

$$\cos \theta = 1$$

① Find  $\theta$  (unit circle)

$$\theta = 0, 2\pi$$

$$\theta = 0 - 2\pi = -2\pi$$

$$\text{b) } 4 \sec x + 8 = 0, \quad 0^\circ \leq x < 360^\circ$$

$$4 \sec x = -8$$

$$\sec x = -2$$

$$\cos x = -\frac{1}{2}$$

① Find  $\bar{x}$

$$\bar{x} = \cos^{-1}\left(\frac{1}{2}\right) \quad (\text{Triangle } \theta)$$

$$\bar{x} = 60^\circ$$

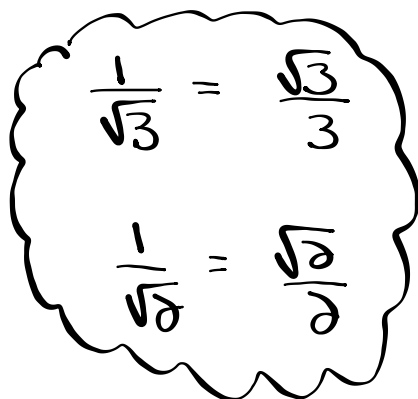
② where is  $\cos x < 0$   $\frac{S}{T} \mid \frac{A}{C}$

③ Find  $x$

Q2	Q3
$x = 180^\circ - \bar{x}$	$x = 180^\circ + \bar{x}$
$x = 180^\circ - 60^\circ = 120^\circ$	$x = 180^\circ + 60^\circ = 240^\circ$

## Homework

Finish worksheet and Page 211 #1-5



Hand-drawn cloud containing two trigonometric identities:

$$\frac{1}{\sqrt{3}} = \frac{\sqrt{3}}{3}$$
$$\frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2}$$

Worksheet Solutions

①  $\sin \theta = -\frac{\sqrt{3}}{2}$  where is  $\sin \theta < 0$   $\frac{S}{A}$   
 $\bar{\theta} = \sin^{-1}\left(\frac{\sqrt{3}}{2}\right)$  *use positive*  $\frac{Q3}{Q4}$   
 $\bar{\theta} = 60^\circ$   
 $\theta = 180^\circ + 60^\circ = 240^\circ$   $\theta = 360^\circ - 60^\circ = 300^\circ$   
 $240^\circ \pm 360^\circ n, n \in \mathbb{N}$   $300^\circ \pm 360^\circ n, n \in \mathbb{N}$

②  $\cos \theta = -\frac{1}{2}$  where is  $\cos \theta < 0$   $\frac{S}{A}$   
 $\bar{\theta} = \cos^{-1}\left(\frac{1}{2}\right)$   $\frac{Q2}{Q3}$   
 $\bar{\theta} = 60^\circ$   
 $\theta = 180^\circ - 60^\circ = 120^\circ$   $\theta = 180^\circ + 60^\circ = 240^\circ$   
 $120^\circ \pm 360^\circ n, n \in \mathbb{N}$   $240^\circ \pm 360^\circ n, n \in \mathbb{N}$

③  $\tan \theta = -\frac{\sqrt{3}}{3}$  where is  $\tan \theta < 0$   $\frac{S}{A}$   
 $\tan \theta = -\frac{1}{\sqrt{3}}$   $\frac{Q2}{Q4}$   
 $\bar{\theta} = \tan^{-1}\left(\frac{1}{\sqrt{3}}\right)$   
 $\bar{\theta} = 30^\circ$   
 $\theta = 180^\circ - 30^\circ = 150^\circ$   $\theta = 360^\circ - 30^\circ = 330^\circ$   
 $150^\circ \pm 360^\circ n, n \in \mathbb{N}$   $330^\circ \pm 360^\circ n, n \in \mathbb{N}$

④  $\tan \theta = \frac{\sqrt{3}}{1}$  where is  $\tan \theta > 0$   $\frac{S}{A}$   
 $\bar{\theta} = \tan^{-1}\left(\frac{\sqrt{3}}{1}\right)$   $\frac{Q1}{Q3}$   
 $\bar{\theta} = 60^\circ$   
 $\theta = 60^\circ$   $\theta = 360^\circ - 60^\circ = 300^\circ$   
 $60^\circ \pm 360^\circ n, n \in \mathbb{N}$   $300^\circ \pm 360^\circ n, n \in \mathbb{N}$

⑤  $\sin \theta = -1$   
 \* Unit Circle  
 $\theta = 270^\circ \rightarrow 270^\circ \pm 360^\circ n, n \in \mathbb{N}$

⑥  $5 \sin \theta - 4 = 0$  where is  $\sin \theta > 0$   $\frac{S}{A}$   
 $\sin \theta = \frac{4}{5}$   $\frac{TK}{TK}$   
 $\sin \theta = 0.8$  (approx. value)  $\frac{Q1}{Q2}$   
 $\bar{\theta} = \sin^{-1}(0.8)$   
 $\bar{\theta} = 53.1^\circ$   
 $\theta = 53.1^\circ$   $\theta = 180^\circ - 53.1^\circ = 126.9^\circ$   
 $53.1^\circ \pm 360^\circ n, n \in \mathbb{N}$   $126.9^\circ \pm 360^\circ n, n \in \mathbb{N}$

Check-Up:

Solve:

$$\textcircled{1} \cot \theta = 0.7834, \quad -\frac{\pi}{2} < \theta < -\pi$$

$$\textcircled{2} 3\cos x + 5 = 6, \quad -360^\circ \leq x \leq 720^\circ$$

$$\textcircled{3} 2\csc x (1 - \csc x) = 0, \quad -4\pi < x < 4\pi$$

$$\textcircled{1} \cot \theta = 0.7834, \quad -\frac{\pi}{2} < \theta < -\pi$$

$$\textcircled{2} 3\cos x + 5 = 6, \quad -360^\circ \leq x \leq 720^\circ$$

$$\textcircled{3} 2\csc x (1 - \csc x) = 0, \quad -4\pi < x < 4\pi$$



Let's move onto QUADRATIC trigonometric equations...

*...Pre-Calculus 110*

- What strategies can we use to solve quadratic equations?
- Quadratic trigonometric equations will ultimately become TWO linear trigonometric equations.

Solve:  $2x^2 + x = 1$

Solve:  $2\sin^2 x + \sin x = 1, 0 \leq x \leq 4\pi$

$$\text{Ex. } \cos^2 \theta - \frac{1}{2} \cos \theta = 0, -2\pi \leq \theta \leq 4\pi$$

$$\text{Ex. } 6 \sin^2 x - \sin x = 2, -2\pi \leq \theta \leq 4\pi$$

**Your Turn**

Solve for  $\theta$ .

$$\cos^2 \theta - \cos \theta - 2 = 0, 0^\circ \leq \theta < 360^\circ$$

Give solutions as exact values where possible. Otherwise, give approximate measures to the nearest thousandth of a degree.

## General Solution of a Trigonometric Equation

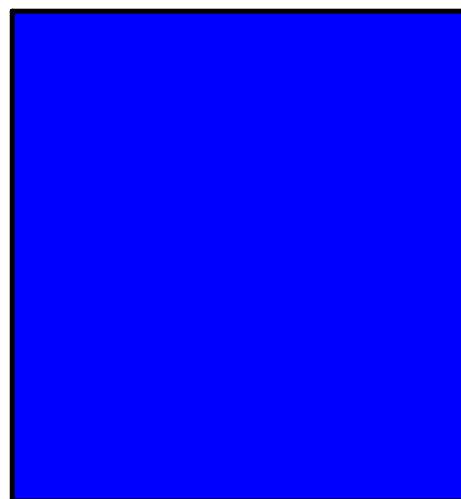
*Solve:*  $3\cos^2 \theta - \cos \theta = 2; \theta \in \mathbb{R}$

Determine the general solution for  $\sin^2 x - 1 = 0$  over the real numbers if  $x$  is measured in radians.

**Did You Know?**

$2n$ , where  $n \in \mathbb{I}$ ,  
represents all even  
integers.

$2n + 1$ , where  $n \in \mathbb{I}$ ,  
is an expression for all  
odd integers.



Determine the general solution for  $\cos^2 x - 1 = 0$ , where the domain is real numbers measured in degrees.

Practice Problems:

Pages 212 - 214

#11 - 23



Check-Up problem...

Solve:

$$\sin x \sec x + 2 \sin x = 0, \quad x \in R \quad (x \text{ is measured in radians})$$

Unit Review...

What topics have we covered??

## Review...

- C4 a)** Determine all solutions for the equation  $2 \sin^2 \theta = 1 - \sin \theta$  in the domain  $0^\circ \leq \theta < 360^\circ$ .
- b)** Are your solutions exact or approximate? Why?
- c)** Show how you can check one of your solutions to verify its correctness.

A grandfather clock shows a time of 7 o'clock. What is the exact radian measure of the angle between the hour hand and the minute hand?

Determine the angular velocity of the minute hand on a clock.

Solve:  $6 \sin^2 \theta - 3 \sin \theta = 0$ ,  $0 \leq \theta \leq 360^\circ$

[A]  $0^\circ, 30^\circ, 180^\circ, 330^\circ, 360^\circ$

[B]  $0^\circ, 30^\circ, 180^\circ, 150^\circ, 360^\circ$

[C]  $30^\circ, 90^\circ, 120^\circ, 270^\circ$

[D]  $0^\circ, 180^\circ, 210^\circ, 330^\circ, 360^\circ$

If  $\csc \theta < 0$  and  $\tan \theta > 0$ , then which of the following could be a possible measure of angle  $\theta$ ?

[A]  $\frac{11\pi}{6}$

[B]  $\frac{4\pi}{3}$

[C]  $\frac{3\pi}{4}$

[D]  $\frac{\pi}{2}$

What is the principal angle of  $-\frac{25\pi}{4}$ ?

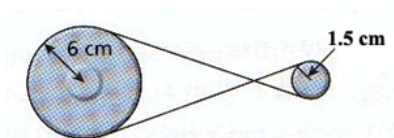
[A]  $\frac{3\pi}{4}$

[B]  $\frac{\pi}{4}$

[C]  $-\frac{\pi}{4}$

[D]  $\frac{7\pi}{4}$

If the belt in the pulley system below travels 30 cm, what is the angle of rotation of the smaller pulley?



[A]  $\frac{\pi}{9}$  radians

[B]  $20^\circ$

[C] 20 radians

[D]  $5^\circ$

Nibbles the hamster is running at 0.02 m/s on an exercise wheel of radius 8 cm. What is the angular velocity of this wheel?

[A] 0.15 rad/minute

[B] 240 rad/minute

[C] 0.25 rad/minute

[D] 15 radians/minute

Solve:  $2(1 - \sin \theta)^2 + \sin \theta = 2(3 - 4 \sin^2 \theta)$ ,  $-360^\circ \leq \theta \leq 720^\circ$

Little Johnny has a rock tied to the end of a piece of rope 1.5 m long and he is swinging it around his head in a circular pattern. Mrs. Centripetal, his physics teacher, is watching Johnny out the window of her physics lab and notes that the rock is making 12 revolutions every 48 seconds.

- (a) Determine the angular velocity with which little Johnny is twirling the rope above his head. [2]

- (b) The rock comes flying from the rope 3 minutes after Mrs. Centripetal started to time little Johnny. How far did the rock travel during the 3 minutes? [2]

## Attachments

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Worksheet - Sketching Angles in Radians.doc