Questions from Homework

(17) j) $f(x)=\frac{6x^3-30x^2-84x}{2x^3+3x^2+x} = \frac{6x(x^2-5x-14)}{x(2x^2+3x+1)} = \frac{6x(x-7)(x+2)}{x(x+\frac{1}{2})(x+\frac{2}{2})}$

$$= \frac{6\cancel{x}(x-7)(x+2)}{\cancel{x}(2x+1)(x+1)}$$

(1) HA:

$$\lim_{x\to\infty} \frac{6x^3-30x^2-84x}{2x^3+3x^2+x} = \frac{6}{2} = 3$$

$\boxed{y=3}$

(2) VA:

$(2x+1)(x+1)=0$

$2x+1=0$

$2x=-1$

$\boxed{x=-\frac{1}{2}}$

$x+1=0$

$\boxed{x=-1}$

(3) Point of Discontinuity (Hole)

$\boxed{x=0}$

$\frac{6(-7)(2)}{(1)(1)}=-84$

$(0,-84)$

Questions from Homework

k) $f(x) = \dfrac{x^2+5x+4}{x^2+6x+8} = \dfrac{(x+1)\cancel{(x+4)}}{(x+2)\cancel{(x+4)}} = \dfrac{x+1}{x+2}$

HA: $\lim\limits_{x\to\infty} \dfrac{x^2+5x+4}{x^2+6x+8} = \dfrac{1}{1}$

$\boxed{y=1}$

VA: $x+2=0$

$\boxed{x=-2}$

$\lim\limits_{x\to -2^-} \dfrac{(-)}{(-)} = +\infty$

$(x=-2.1)$

$\lim\limits_{x\to -2^+} \dfrac{(-)}{(+)} = -\infty$

$(x=-1.9)$

Hole: $x+4=0$

$\boxed{x=-4}$

sub in $x=-4$

$f(-4) = \dfrac{-4+1}{-4+2}$

$= \dfrac{-3}{-2} = 1.5$

$\boxed{(-4, 1.5)}$

Questions from Homework

17 b) $f(x) = \frac{x^2+2x-3}{x^2+3x-4} = \frac{(\cancel{x-1})(x+3)}{(\cancel{x-1})(x+4)} = \frac{x+3}{x+4}$

HA: $\lim_{x\to\infty} \frac{x^2+2x-3}{x^2+3x-4} = \frac{1}{1}$

$\boxed{y=1}$

VA: $x+4=0$

$\boxed{x=-4}$

$\lim_{x\to -4^-} \frac{(-)}{(-)} = +\infty$

$(x=-4.1)$

$\lim_{x\to -4^+} \frac{(-)}{(+)} = -\infty$

$(x=-3.9)$

Hole: $x-1=0$

$\boxed{x=1}$

sub in $x=1$

$f(1) = \frac{1+3}{1+4} = \frac{4}{5} = 0.8$

$\boxed{(1, 0.8)}$

Curve Sketching

In this chapter we look at further aspects of curves such as vertical and horizontal asymptotes, concavity, and inflections points. Then we use them, together with intervals of increase and decrease and maximum and minimum values, to develop a procedure for curve sketching.

Slant Asymptotes

For rational functions, slant asymptotes occur when the degree of the numerator is one more than the degree of the denominator and can be found by division. (ignore the remainder)

Example

Find the slant asymptote of the curve $y = \dfrac{2x^3 - 3x^2 + x - 3}{x^2 + 1}$

SA:

$$\begin{array}{r} 2x - 3 \\ x^2+1 \overline{\smash{)}\,2x^3 - 3x^2 + x - 3} \\ -(2x^3 \quad\quad + 2x) \\ \hline -3x^2 - x - 3 \\ -(-3x^2 \quad - 3) \\ \hline -x \; R \end{array}$$

$2x^3-3x^2+x-3-(2x^3+2x)$

$\cancel{2x^3}-3x^2+x-3-\cancel{2x^3}-2x$

$-3x^2-x-3$

$-3x^2-x-3-(-3x^2-3)$

$\cancel{-3x^2}-x-\cancel{3}+\cancel{3x^2}+\cancel{3}$

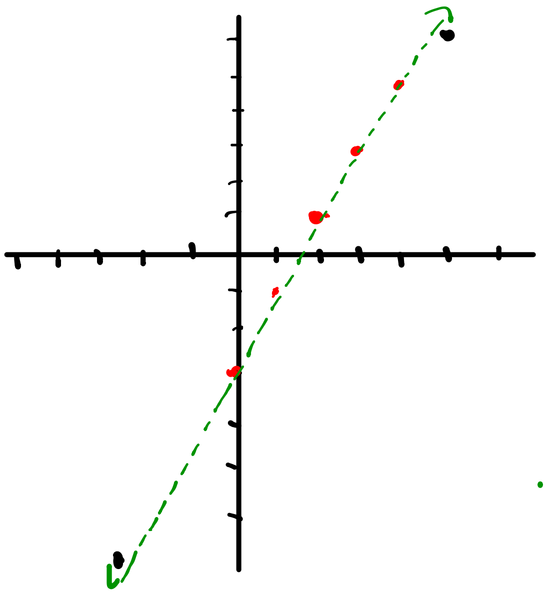
$-x$

SA: $\boxed{y = 2x - 3}$

$y = mx + b$

$m = \dfrac{2}{1}$ $\dfrac{\text{rise}}{\text{run}}$

$b = -3$ (y-int)



Example

Find the slant asymptote of the curve $y = \dfrac{1+x-x^2}{x-1}$

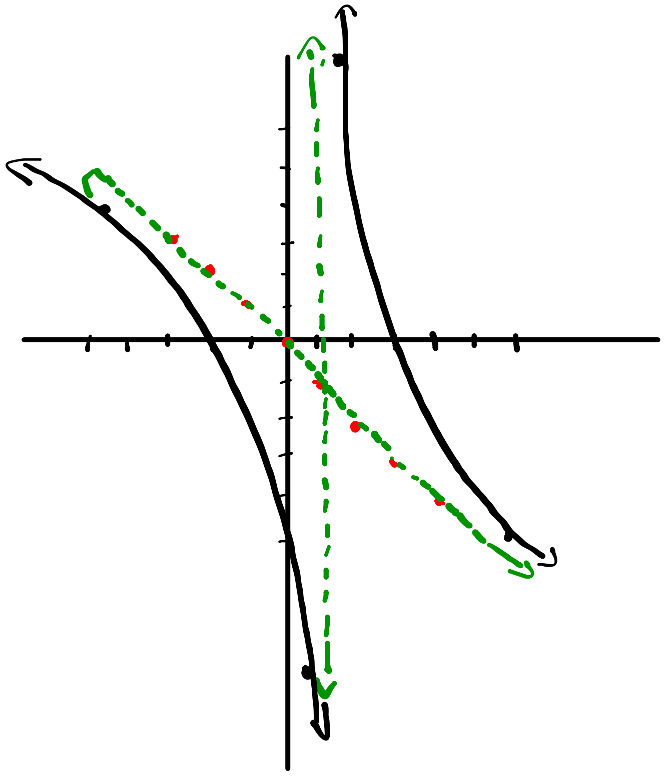
① SA: $x-1 \overline{)\,-x^2+x+1}$ quotient $-x$

$-(-x^2+x)$

1 R

$\boxed{y=-x}$ $m = \frac{-1}{1}$ $\frac{\text{rise}}{\text{run}}$

$b = 0$



② VA: $x-1=0$

$\boxed{x=1}$

$\lim\limits_{x\to 1^-} \frac{(+)}{(-)} = -\infty$

$(x=0.9)$

$\lim\limits_{x\to 1^+} \frac{(+)}{(+)} = +\infty$

$(x=1.1)$

Homework