Correct Homework Sheet

① a) $f(x) = 3x^{-2} + 3x^3 + 1$

$f'(x) = -6x^{-3} + 9x^2$

b) $y = \dfrac{4x+1}{6-2x^5}$

$f' \quad g \quad g' \quad f$

$y' = \dfrac{4(6-2x^5) - (-10x^4)(4x+1)}{(6-2x^5)^2}$

c) $f(x) = 3(2x^5 + x - 5)^{10}$

$f'(x) = 30(2x^5 + x - 5)^9 (10x^4 + 1)$

d) $h(x) = (x^2 - x)\sqrt{4-9x} = (x^2-x)(4-9x)^{1/2}$

$f' \quad g \quad f \quad g'$

$h'(x) = (2x-1)(4-9x)^{1/2} + (x^2-x)\left(\frac{1}{2}\right)(4-9x)^{-1/2}(-9)$

② a) $y = \dfrac{2}{x} + \dfrac{3}{5x^3} - 6\sqrt{x} + \sqrt[3]{9x^8} - 8\pi$

$y = 2x^{-1} + \frac{3}{5}x^{-3} - 6x^{1/2} + (9x^8)^{1/3} - 8\pi$

$y' = -2x^{-2} - \frac{9}{5}x^{-4} - 3x^{-1/2} + \frac{1}{3}(9x^8)^{-2/3}(72x^7) - 0$

b) $y = \sqrt[3]{\dfrac{1-x^6}{2+(5x-1)^4}} = \left(\dfrac{1-x^6}{2+(5x-1)^4}\right)^{1/3}$

$y' = \frac{1}{3}\left(\dfrac{1-x^6}{2+(5x-1)^4}\right)^{-2/3}\left[\dfrac{-6x^5(2+(5x-1)^4) - (1-x^6)(4)(5x-1)^3(5)}{[2+(5x-1)^4]^2}\right]$

c) $g(x) = (x-5)^3(7x^5+2x)^9(4-2x^3)^5$

$g'(x) = 3(x-5)^2(1)(7x^5+2x)^9(4-2x^3)^5 + 9(7x^5+2x)^8(35x^4+2)(x-5)^3(4-2x^3)^5 + 5(4-2x^3)^4(-6x^2)(x-5)^3(7x^5+2x)^9$

② d) $f(x) = \sqrt{25+4(2x-1)^4} = (25+4(2x-1)^4)^{1/2}$

$f'(x) = \frac{1}{2}(25+4(2x-1)^4)^{-1/2}(16(2x-1)^3(2))$

③ a) $y = \sqrt{x^2 - 5x\sqrt{2x^3 + 3\sqrt{x}}} = \left[x^2 - 5x(2x^3+3x^{1/2})^{1/2}\right]^{1/2}$

$y' = \frac{1}{2}\left[x^2 - 5x(2x^3+3x^{1/2})^{1/2}\right]^{-1/2}\left(2x - \left[5(2x^3+3x^{1/2})^{1/2} + 5x\left(\frac{1}{2}\right)(2x^3+3x^{1/2})^{-1/2}\left(6x^2 + \frac{3}{2}x^{-1/2}\right)\right]\right)$

To be handed in today

$$f(x)=\sqrt[7]{\frac{9+16x^4}{\left[4x^5(3x^8+8x-2)\right]^5}} = \left[\frac{9+16x^4}{\left[4x^5(3x^8+8x-2)\right]^5}\right]^{1/7}$$

$$= \frac{1}{7}\left[\frac{9+16x^4}{\left[4x^5(3x^8+8x-2)\right]^5}\right]^{-6/7}\left[\frac{64x^3\left[4x^5(3x^8+8x-2)\right]^5-(9+16x^4)(5)\left[4x^5(3x^8+8x-2)\right]^4\left[(20x^4)(3x^8+8x-2)+4x^5(24x^7+8)\right]}{\left[4x^5(3x^8+8x-2)\right]^{10}}\right]$$

$f(x) = (x^2+5x+7)^4$

$f'(x) = 4(x^2+5x+7)^3(2x+5)$

$f(x) = (x)^4$

$f'(x) = 4(x)^3(1)$

$= 4x^3$

$f(x) = 5(x^2+5x+7)^4$

$f'(x) = 20(x^2+5x+7)^3(2x+5)$

$f(x) = 5x(x^2+5x+7)^4$

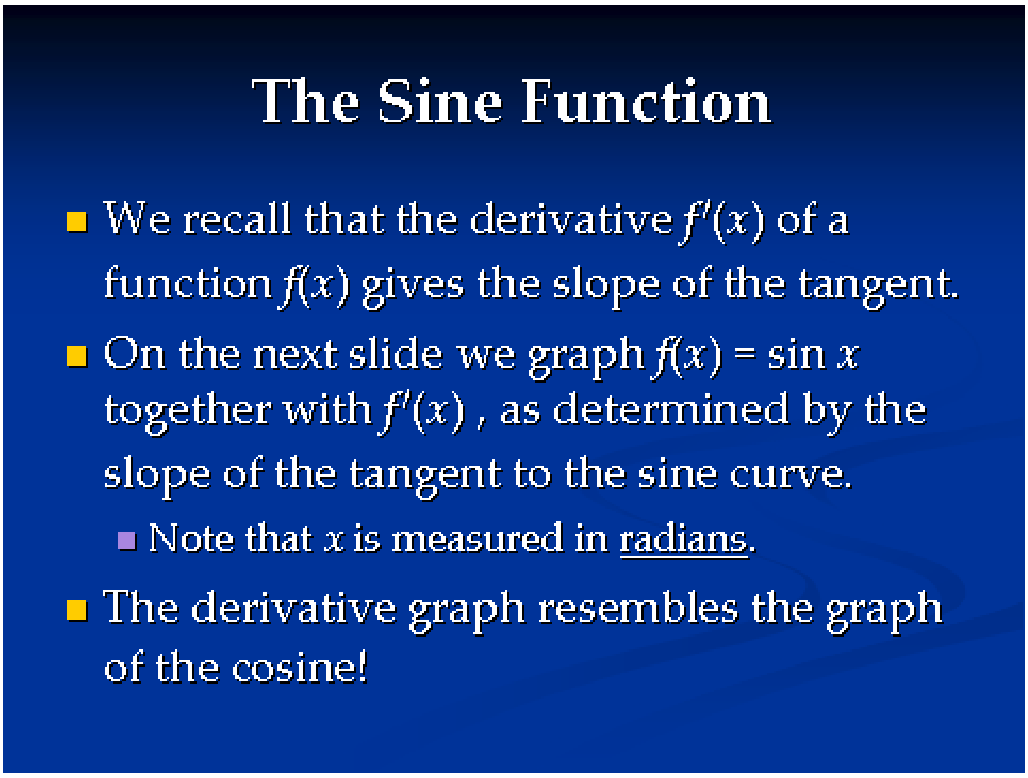
$f'(x) = \underset{f'}{5}\,\underset{g}{(x^2+5x+7)^4} + \underset{f}{5x}\,\underbrace{(4)(x^2+5x+7)^3(2x+5)}_{g'}$

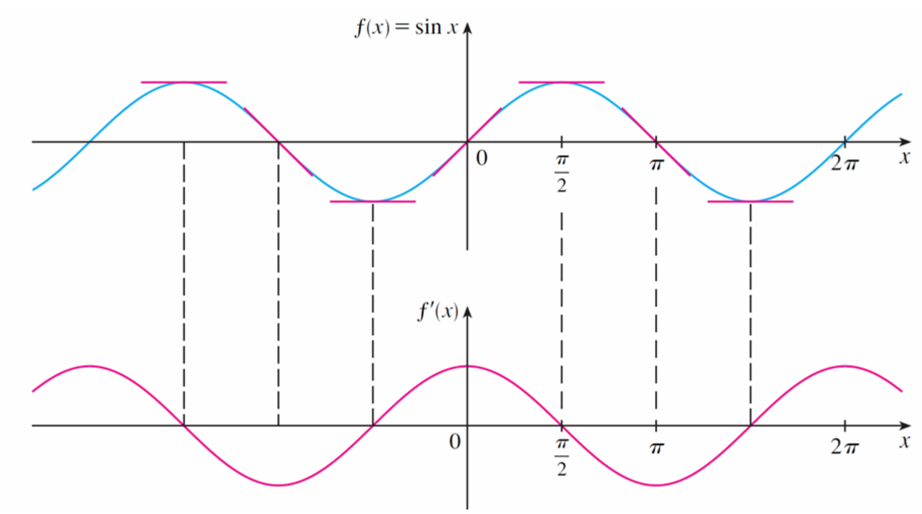
Quotient: $\dfrac{f'(x)g(x) - f(x)g'(x)}{[g(x)]^2}$

Product: $f'(x)g(x) + f(x)g'(x)$

Chain: $f'(g(x)) \cdot g'(x)$

Derivatives of Trigonometric Functions





Let's check this using the definition of a derivative...

$$f'(x) = \lim_{h\to 0} \frac{f(x+h) - f(x)}{h} = \lim_{h\to 0} \frac{\sin(x+h) - \sin x}{h}$$

$$= \lim_{h\to 0} \frac{\sin x \cos h + \cos x \sin h - \sin x}{h}$$

$$= \lim_{h\to 0} \left[\frac{\sin x \cos h - \sin x}{h} + \frac{\cos x \sin h}{h} \right]$$

$$= \lim_{h\to 0} \left[\frac{\sin x \cos h - \sin x}{h} + \frac{\cos x \sin h}{h} \right]$$

$$= \lim_{h\to 0} \left[\sin x \left(\frac{\cos h - 1}{h} \right) + \cos x \left(\frac{\sin h}{h} \right) \right]$$

$$= \lim_{h\to 0} \sin x \cdot \lim_{h\to 0} \frac{\cos h - 1}{h} + \lim_{h\to 0} \cos x \cdot \lim_{h\to 0} \frac{\sin h}{h}$$

- Our calculations have brought us to four limits, two of which are easy:
  - Since x is constant while $h \to 0$,

    $$\lim_{h\to 0} \sin x = \sin x \quad \text{and} \quad \lim_{h\to 0} \cos x = \cos x$$
- With some work we can also show that

$$\lim_{h\to 0} \frac{\sin h}{h} = 1 \quad \text{and} \quad \lim_{h\to 0} \frac{\cos h - 1}{h} = 0$$

- Thus our guess is confirmed:

$$f'(x) = \lim_{h\to 0} \sin x \cdot \lim_{h\to 0} \frac{\cos h - 1}{h} + \lim_{h\to 0} \cos x \cdot \lim_{h\to 0} \frac{\sin h}{h}$$

$$= (\sin x) \cdot 0 + (\cos x) \cdot 1 = \cos x$$

Rules to differentiate trigonometric functions:

Given that "u" represents some differentiable function...

$\frac{d}{du}(\sin u) = \cos u \bullet du$	$\frac{d}{du}(\csc u) = -\csc u \cot u \bullet du$
$\frac{d}{du}(\cos u) = -\sin u \bullet du$	$\frac{d}{du}(\sec u) = \sec u \tan u \bullet du$
$\frac{d}{du}(\tan u) = \sec^2 u \bullet du$	$\frac{d}{du}(\cot u) = -\csc^2 u \bullet du$

Let's Practice...

Differentiate the following:

$y = \sin 3x$ $y = \sin(x+2)$ $y = \sin(kx+d)$

Ex #2.

Differentiate:

a) $y = \sin(x^3)$ b) $y = \sin^3 x$ c) $y = \sin^3(x^2 - 1)$

Ex #3.

Differentiate:

$$y = x^2 \cos x$$

Homework

Worksheet on derivatives of trigonometric functions

Attachments

Derivatives Worksheet.doc