

Questions from Homework

$$\textcircled{1} \text{ g) } y = \frac{x+1}{\sqrt{x}} = \frac{x+1}{x^{1/2}} = x^{-1/2}(x+1) = x^{1/2} + x^{-1/2}$$

$$y' = \frac{1}{2}x^{-1/2} - \frac{1}{2}x^{-3/2} = \frac{1}{2x^{1/2}} - \frac{1}{2x^{3/2}}$$

$$\text{k) } u(t) = a + \frac{b}{t} + \frac{c}{t^2} = a + bt^{-1} + ct^{-2}$$

$$u'(t) = -bt^{-2} - 2ct^{-3} = -\frac{b}{t^2} - \frac{2c}{t^3}$$

$$\text{l) } v(r) = \sqrt{r}(2+3r) = r^{1/2}(2+3r) = 2r^{1/2} + 3r^{3/2}$$

$$v'(r) = r^{-1/2} + \frac{9}{2}r^{1/2} = \frac{1}{\sqrt{r}} + \frac{9\sqrt{r}}{2}$$

Questions from Homework

$$\textcircled{3} \text{ c) } y = x + \frac{6}{x} ; (\underline{2}, \underline{5})$$

$$x_1 = 2 \quad m = -\frac{1}{2}$$

$$y_1 = 5$$

$$\text{(i) } y = x + 6x^{-1}$$

$$y' = 1 - 6x^{-2}$$

$$y' = 1 - \frac{6}{x^2}$$

$$\text{(ii) } y' = 1 - \frac{6}{(2)^2}$$

$$y' = 1 - \frac{3}{2}$$

$$y' = \frac{2}{2} - \frac{3}{2}$$

$$y' = -\frac{1}{2}$$

$$m = -\frac{1}{2}$$

$$\text{(iii) } y - y_1 = m(x - x_1)$$

$$y - 5 = -\frac{1}{2}(x - 2)$$

$$y - 5 = -\frac{1}{2}x + 1$$

$$\boxed{y = -\frac{1}{2}x + 6}$$

$$2y = -x + 12$$

$$x + 2y - 12 = 0$$

Questions from Homework

④ $y = 2\sqrt{x}$ $(9, 6)$
 $x_1 = 9$ $y_1 = 6$

(i) Differentiate

$$y = 2\sqrt{x}$$

$$y = 2x^{1/2}$$

$$y' = 1x^{-1/2}$$

$$y' = \frac{1}{x^{1/2}}$$

$$y' = \frac{1}{\sqrt{x}}$$

(ii) Sub in x-value

$$y' = \frac{1}{\sqrt{9}}$$

$$y' = \frac{1}{3}$$

$$m = \frac{1}{3}$$

(iii) $y - y_1 = m(x - x_1)$

$$y - 6 = \frac{1}{3}(x - 9)$$

$$y - 6 = \frac{1}{3}x - 3$$

$$y = \frac{1}{3}x - 3 + 6$$

$$y = \frac{1}{3}x + 3$$

$$0 = \frac{1}{3}x - y + 3$$

or $0 = x - 3y + 9$

Questions from Homework

① At what point on the parabola $y = 3x^2$ is the slope of the tangent line equal to 24 \uparrow m

① $y = 3x^2$
 $y' = 6x$

② $y' = 6x$
 $24 = 6x$
 $4 = x$

③ $y = 3x^2$
 $y = 3(4)^2$
 $y = 3(16)$
 $y = 48$

④ $(4, 48)$

Questions from Homework

(x, y)

- ⑧ Find the point on the curve $y = x\sqrt{x}$ where the tangent line is parallel to $6x - y = 4$
same slope.

(i) Find slope of

$$6x - y = 4$$

$$-y = \frac{-6x + 4}{-1}$$

$$y = 6x - 4$$

$$(y = mx + b)$$

$$m = 6$$

(iv) Solve for y

$$y = x\sqrt{x}$$

$$y = (16)\sqrt{(16)}$$

$$y = 16(4)$$

$$y = 64$$

(ii) Find y' (slope of tangent)

$$y = x\sqrt{x} = x \cdot x^{1/2} = x^{3/2}$$

$$y' = \frac{3}{2}x^{1/2} = \frac{3\sqrt{x}}{2} \leftarrow m$$

(iii) set slopes equal

$$\frac{3\sqrt{x}}{2} = 6$$

$$\frac{3\sqrt{x}}{3} = \frac{12}{3}$$

$$\sqrt{x} = 4$$

$$(x^{1/2})^2 = (4)^2$$

$$x = 16$$

(16, 64) is the point

Warm Up

Differentiate the following:

$$f(x) = -4x^2 - 5x(x^3 + 7)^2 + 2\sqrt[5]{x^9} - \frac{5}{x^{10}} + \frac{7x^2}{\sqrt{x}} - e^6$$

$$f(x) = -4x^2 - 5x(x^6 + 14x^3 + 49) + 2x^{9/5} - 5x^{-10} + 7x^2(x^{-1/2})$$

$$f(x) = -4x^2 - 5x^7 - 70x^4 - 245x + 2x^{9/5} - 5x^{-10} + 7x^{3/2}$$

$$f'(x) = -8x - 35x^6 - 280x^3 - 245 + \frac{18}{5}x^{4/5} + 50x^{-11} - \frac{21}{2}x^{1/2}$$

Differentiation Rules

Product Rule:

The Product Rule If f and g are both differentiable, then

$$\frac{d}{dx} [f(x)g(x)] = f(x) \frac{d}{dx} [g(x)] + g(x) \frac{d}{dx} [f(x)]$$

Express the product rule verbally if you are considering a function of the form...

$$f(x) = (\text{First}) \times (\text{Second})$$

In words, *the Product Rule* says that the *derivative of a product of two functions is: the derivative of the first function times the second function, plus the first function times the derivative of the second function*

Get in the habit of verbalizing the rule as you differentiate...it will help when the functions get more complicated.

$$(fg)'(x) = f'(x)g(x) + f(x)g'(x)$$

Examples:

$$y = f(x)g(x)$$

$$y' = f'(x)g(x) + f(x)g'(x)$$

$$y = (2x^3 + 5)(3x^2 - x)$$

$$y' = \overset{\text{first}}{6x^2}(3x^2 - x) + \overset{\text{second}}{(2x^3 + 5)}(6x - 1)$$

$$y' = \underline{18x^4} - \underline{6x^3} + \underline{12x^4} - \underline{2x^3} + 30x - 5$$

$$y' = 30x^4 - 8x^3 + 30x - 5$$

$$f(x) = \sqrt{x}(2 - 3x) = x^{1/2}(2 - 3x)$$

$$f'(x) = \frac{1}{2}x^{-1/2}(2 - 3x) + x^{1/2}(-3)$$

$$f'(x) = \frac{1}{2x^{1/2}}(2 - 3x) - 3x^{1/2}$$

$$f'(x) = \frac{2 - 3x}{2x^{1/2}} - \frac{3x^{1/2}}{1} \quad \text{co: } 2x^{1/2}$$

$$f'(x) = \frac{2 - 3x}{2x^{1/2}} - \frac{6x}{2x^{1/2}} = \boxed{\frac{2 - 9x}{2\sqrt{x}}}$$

Examples:

$$f(x) = (7x^3 - x^2 + 5)(x^9 + 3x - 5)$$

$$f'(x) = (21x^2 - 2x)(x^9 + 3x - 5) + (7x^3 - x^2 + 5)(9x^8 + 3)$$

$$h(t) = (t^3 - 5t)(6\sqrt{t} - t^{-5})$$

$$h'(t) = (3t^2 - 5)(6\sqrt{t} - t^{-5}) + (t^3 - 5t)(3t^{-1/2} + 5t^{-6})$$

Homework

$$(f \cdot g)'(x) = f'(x)g(x) + f(x)g'(x)$$

$$\textcircled{1} \text{ a) } y = (2x-1)(x^2+1)$$

$$y' = 2(x^2+1) + (2x-1)(2x)$$

$$\text{b) } f(t) = \sqrt[3]{t}(1-t) = t^{1/3}(1-t)$$

$$f'(t) = \frac{1}{3}t^{-2/3}(1-t) + t^{1/3}(-1)$$

$$\text{c) } F(y) = \sqrt{y}(y-2\sqrt{y}+2) = \underbrace{y^{1/2}}_{f(x)} \underbrace{(y-2y^{1/2}+2)}_{g(x)}$$

$$F'(y) = \frac{1}{2}y^{-1/2}(y-2y^{1/2}+2) + y^{1/2}(1-y^{-1/2})$$

$$= \frac{1}{2\sqrt{y}}(y-2\sqrt{y}+2) + \sqrt{y}\left(1-\frac{1}{\sqrt{y}}\right)$$

$$(f \cdot g)'(x) = f'(x)g(x) + f(x)g'(x)$$

$$\textcircled{1} \text{ d) } y = \underbrace{(x^3+x^2+1)}_{f(x)} \underbrace{(x^2+2)}_{g(x)}$$

$$y' = (3x^2+2x)(x^2+2) + (x^3+x^2+1)(2x)$$

$$\text{e) } f(t) = \sqrt[3]{t}(1-t) = \underbrace{t^{1/3}}_{f(x)} \underbrace{(1-t)}_{g(x)}$$

$$f'(t) = \frac{1}{3}t^{-2/3}(1-t) + t^{1/3}(-1)$$

$$f'(t) = \frac{1}{3t^{2/3}}(1-t) - t^{1/3}$$

$$f'(t) = \frac{1-t}{3t^{2/3}} - \frac{t^{1/3}}{1} \quad \text{CD: } 3t^{2/3}$$

$$f'(t) = \frac{1-t}{3t^{2/3}} - \frac{3t}{3t^{2/3}} = \frac{1-t-3t}{3t^{2/3}} = \frac{1-4t}{3\sqrt[3]{t^2}}$$

$$\textcircled{a} \text{ b) } y = x^{-2}(x^3 - 3x^2 + 6)$$

$$y' = -2x^{-3}(x^3 - 3x^2 + 6) + x^{-2}(3x^2 - 6x)$$

$$y' = -2 + \underline{6x^{-1}} - 12x^{-3} + \underline{3} - \underline{6x^{-1}}$$

$$y' = 1 - 12x^{-3}$$

$$\textcircled{a} \text{ e) } f(t) = (6+t^{-2})(8t^{10} - 5t^3)$$

$$f'(t) = (-2t^{-3})(8t^{10} - 5t^3) + (6+t^{-2})(80t^9 - 15t^2)$$

$$f'(t) = -16t^7 + 10t^0 + 480t^9 - 90t^2 + 80t^7 - 15t^0$$

$$f'(t) = 480t^9 + 64t^7 - 90t^2 - 5t^0 \quad t^0 = 1$$

$$f'(t) = 480t^9 + 64t^7 - 90t^2 - 5$$

$$\text{f) } f(t) = (at + b)(ct^2 - d)$$

$$f'(t) = a(ct^2 - d) + (at + b)(2ct)$$

$$f'(t) = act^2 - ad + 2act^2 + 2bct$$

$$f'(t) = 3act^2 + 2bct - ad$$

$$\textcircled{5} \quad y = (2 - \sqrt{x})(1 + \sqrt{x} + 3x) \quad @ \quad (1, 5)$$

$$\textcircled{i} \quad y' = \left(-\frac{1}{2}x^{-\frac{1}{2}}\right)(1 + \sqrt{x} + 3x) + (2 - \sqrt{x})\left(\frac{1}{2}x^{-\frac{1}{2}} + 3\right)$$

$$y' = \left(-\frac{1}{2\sqrt{x}}\right)(1 + \sqrt{x} + 3x) + (2 - \sqrt{x})\left(\frac{1}{2\sqrt{x}} + 3\right)$$

$$\textcircled{ii} \quad y' = \left(-\frac{1}{2\sqrt{1}}\right)(1 + \sqrt{1} + 3(1)) + (2 - \sqrt{1})\left(\frac{1}{2\sqrt{1}} + 3\right)$$

$$y' = \left(-\frac{1}{2}\right)(5) + 1\left(\frac{1}{2} + 3\right)$$

$$y' = -\frac{5}{2} + 1\left(\frac{7}{2}\right)$$

$$y' = -\frac{5}{2} + \frac{7}{2} = \frac{2}{2} = \underline{1} \quad \leftarrow m$$

$$\textcircled{iii} \quad y - y_1 = m(x - x_1)$$

$$y - 5 = 1(x - 1)$$

$$y - 5 = x - 1$$

$$y = x + 4$$

$$\text{or } \boxed{x - y + 4 = 0}$$