

$$\textcircled{a} \text{ f) } f(t) = (at+b)(ct^2-d)$$

$$f'(t) = \overbrace{(a)}^{\text{red}}(ct^2-d) + \overbrace{(at+b)}^{\text{red}}(\overbrace{2ct}^{\text{red}})$$

$$f'(t) = \underline{act^2} - ad + \underline{2act} + \underline{2bct}$$

$$f'(t) = 3act^2 + 2bct - ad$$

$$\textcircled{a) } y = (1-2x)(3x-4), x=2$$

① Find y' :

$$y' = -2(3x-4) + (1-2x)(3)$$

$$y' = -6x + 8 + 3 - 6x$$

$$y' = -12x + 11$$

② sub in $x=2$

$$y' = -12x + 11$$

$$y' = -12(2) + 11$$

$$y' = -13$$

$$m = -13$$

$$\textcircled{a} y = (2-\sqrt{x})(1+\sqrt{x}+3x) \text{ @ } (1,5)$$

$$y = (2-x^{1/2})(1+x^{1/2}+3x)$$

① Find y' :

$$y' = \left(-\frac{1}{2}x^{-1/2}\right)(1+x^{1/2}+3x) + (2-x^{1/2})\left(\frac{1}{2}x^{-1/2}+3\right)$$

$$y' = \left(-\frac{1}{2\sqrt{x}}\right)(1+\sqrt{x}+3x) + (2-\sqrt{x})\left(\frac{1}{2\sqrt{x}}+3\right)$$

② sub in $x=1$

$$y' = \left(-\frac{1}{2\sqrt{1}}\right)(1+\sqrt{1}+3(1)) + (2-\sqrt{1})\left(\frac{1}{2\sqrt{1}}+3\right)$$

$$y' = \left(-\frac{1}{2}\right)(5) + (1)\left(\frac{1}{2}\right)$$

$$y' = \frac{-5}{2} + \frac{1}{2} = \frac{-4}{2} = -2$$

$$m = -2$$

$$\textcircled{a} y - y_1 = m(x - x_1)$$

$$y - 5 = 1(x - 1) \dots$$

$$y - 5 = x - 1$$

$$y = x + 4$$

$$\text{or } x - y + 4 = 0$$

$$\textcircled{a} \text{ h) } g(v) = (v - \sqrt{v})(v^2 + \sqrt{v})$$

$$g(v) = \underbrace{(v - v^{1/2})}_{f(x)} \underbrace{(v^2 + v^{1/2})}_{g(x)}$$

$$1 + \left(-\frac{1}{2}\right) \\ \frac{2}{2} + \left(-\frac{1}{2}\right)$$

$$\frac{-1}{2} + \frac{2}{2} \\ \frac{-1}{2} + \frac{4}{2}$$

$$g'(v) = \left(1 - \frac{1}{2}v^{-1/2}\right) \left(2v + v^{-1/2}\right) + (v - v^{1/2}) \left(2v + \frac{1}{2}v^{-1/2}\right)$$

$$g'(v) = \underline{v^2} + \underline{v^{1/2}} - \underline{\frac{1}{2}v^{3/2}} - \underline{\frac{1}{2}v^0} + \underline{2v^2} + \underline{\frac{1}{2}v^{1/2}} - \underline{2v^{3/2}} - \underline{\frac{1}{2}v^0}$$

$$g'(v) = \frac{3v^2}{1} - \frac{5}{2}v^{3/2} + \frac{3v^{1/2}}{2} - \frac{1}{1}$$

$$g'(v) = \frac{6v^2}{2} - \frac{5v^{3/2}}{2} + \frac{3v^{1/2}}{2} - \frac{2}{2}$$

$$g'(v) = \frac{6v^2 - 5v^{3/2} + 3v^{1/2} - 2}{2}$$

$$\textcircled{5} \quad y = (2 - \sqrt{x})(1 + \sqrt{x} + 3x) \text{ @ } (1, 5)$$

\uparrow \uparrow
 x_1 y_1

① Find derivative

$$y = \underbrace{(2 - x^{1/2})}_{f(x)} \underbrace{(1 + x^{1/2} + 3x)}_{g(x)}$$

$$y' = \underbrace{\left(-\frac{1}{2}x^{-1/2}\right)}_{f'(x)} \underbrace{(1 + \sqrt{x} + 3x)}_{g(x)} + \underbrace{(2 - \sqrt{x})}_{f(x)} \underbrace{\left(\frac{1}{2}x^{-1/2} + 3\right)}_{g'(x)}$$

$$y' = \left(-\frac{1}{2\sqrt{x}}\right)(1 + \sqrt{x} + 3x) + (2 - \sqrt{x})\left(\frac{1}{2\sqrt{x}} + 3\right)$$

② Solve for slope of tangent (m)

$$m = y'(1) = \left(-\frac{1}{2}\right)(5) + (1)\left(\frac{1}{2} + 3\right)$$

$$m = y'(1) = -\frac{5}{2} + 1\left(\frac{1}{2} + \frac{6}{2}\right)$$

$$m = y'(1) = -\frac{5}{2} + \frac{7}{2} = \frac{2}{2} = \underline{\underline{1}}$$

$m=1$ \swarrow

③ Find the equation

$$y - y_1 = m(x - x_1)$$

$$y - 5 = 1(x - 1)$$

$$y - 5 = x - 1$$

$$y = x + 4 \quad \checkmark$$

$$\boxed{0 = x - y + 4} \quad \checkmark$$

⑥ If:

$$\begin{aligned}f(a) &= \underline{3} \\f'(a) &= \underline{5} \\g(a) &= \underline{-1} \\g'(a) &= \underline{-4}\end{aligned}$$

Find $(fg)'(a)$

multiply (Product)

$$\begin{aligned}(fg)'(x) &= f'(x)g(x) + f(x)g'(x) \\(fg)'(a) &= \underline{f'(a)}\underline{g(a)} + \underline{f(a)}\underline{g'(a)} \\&= (5)(-1) + (3)(-4) \\&= -5 + -12 \\&= -17\end{aligned}$$

Differentiation Rules

Product Rule:

The Product Rule If f and g are both differentiable, then

$$\frac{d}{dx} [f(x)g(x)] = f(x) \frac{d}{dx} [g(x)] + g(x) \frac{d}{dx} [f(x)]$$

Express the product rule verbally if you are considering a function of the form...

$$f(x) = (\text{First}) \times (\text{Second})$$

In words, *the Product Rule* says that the *derivative of a product of two functions is: the derivative of the first function times the second function, plus the first function times the derivative of the second function*

Get in the habit of verbalizing the rule as you differentiate...it will help when the functions get more complicated.

$$(fg)'_x = f'(x)g(x) + f(x)g'(x)$$

Differentiate the following function and simplify your answer:

$$h(t) = \overset{f(x)}{(t^3 - 5t)} \overset{g(x)}{(6\sqrt{t} - t^{-5})}$$

$$h'(t) = (3t^2 - 5)(6t^{1/2} - t^{-5}) + (t^3 - 5t)(3t^{-1/2} + 5t^{-6})$$

$$h'(t) = \underline{18t^{5/2}} - \underline{3t^{-3}} - \underline{30t^{1/2}} + \underline{5t^{-5}} + \underline{3t^{5/2}} + \underline{5t^{-3}} - \underline{15t^{1/2}} - \underline{25t^{-5}}$$

$$h'(t) = \underline{21t^{5/2}} - \underline{45t^{1/2}} + \underline{2t^{-3}} - \underline{20t^{-5}}$$

$$h'(t) = 21t^{5/2} - 45t^{1/2} + \frac{2}{t^3} - \frac{20}{t^5}$$

Quotient Rule:

The Quotient Rule If f and g are differentiable, then

$$\frac{d}{dx} \left[\frac{f(x)}{g(x)} \right] = \frac{g(x) \frac{d}{dx} [f(x)] - f(x) \frac{d}{dx} [g(x)]}{[g(x)]^2}$$

Express the quotient rule verbally if you are considering a function of the form...

$$f(x) = \frac{\text{(First)}}{\text{(Second)}}$$

In words, *the Quotient Rule* says that the *derivative of a quotient is: the denominator times the derivative of the numerator, minus the numerator times the derivative of the denominator, all divided by the square of the denominator.*

$$\left(\frac{f}{g} \right)' (x) = \frac{f'(x)g(x) - f(x)g'(x)}{g(x)^2}$$

Examples:

$$\left(\frac{f}{g}\right)'(x) = \frac{f'(x)g(x) - f(x)g'(x)}{g(x)^2}$$

Differentiate the following functions and simplify your answers:

$$F(x) = \frac{x^2 + 2x - 3}{x^3 + 1} \quad \begin{array}{l} f(x) \\ g(x) \end{array}$$

$$F'(x) = \frac{(2x+2)(x^3+1) - 3x^2(x^2+2x-3)}{(x^3+1)^2}$$

$$F'(x) = \frac{2x^4 + 2x + 2x^3 + 2 - 3x^4 - 6x^3 + 9x^2}{(x^3+1)^2}$$

$$F'(x) = \frac{-x^4 - 4x^3 + 9x^2 + 2x + 2}{(x^3+1)^2}$$

$$F(x) = \frac{\sqrt{x}}{1+2x} \quad \begin{array}{l} f(x) \\ g(x) \end{array} = \frac{x^{1/2}}{1+2x}$$

$$F'(x) = \frac{\frac{1}{2}x^{-1/2}(1+2x) - 2\sqrt{x}}{(1+2x)^2}$$

$$F'(x) = \frac{\frac{1}{2\sqrt{x}}(1+2x) - 2\sqrt{x}}{(1+2x)^2}$$

$$F'(x) = \frac{\frac{1+2x}{2\sqrt{x}} - 2\sqrt{x} \cdot 2\sqrt{x}}{2\sqrt{x}(1+2x)^2} \quad \text{CD: } 2\sqrt{x}$$

$$F'(x) = \frac{1+2x-4x}{2\sqrt{x}(1+2x)^2} = \frac{1-2x}{2\sqrt{x}(1+2x)^2}$$

Differentiate the following functions, do not simplify your answers:

$$f(x) = \frac{8 - 9x^7}{3x - 7}$$

$$f'(x) = \frac{-63x^6(3x-7) - 3(8-9x^7)}{(3x-7)^2}$$

$$f(x) = \frac{x^3 - 7x^2 + 2}{x^8 - 4x^5}$$

$$f'(x) = \frac{(3x^2 - 14x)(x^8 - 4x^5) - (x^3 - 7x^2 + 2)(8x^7 - 20x^4)}{(x^8 - 4x^5)^2}$$

Homework

$$y = \frac{x}{x-2} \quad (\underline{4}, 2)$$

$$\textcircled{1} \quad y' = \frac{1(x-2) - x}{(x-2)^2} = \frac{x-2-x}{(x-2)^2} = \frac{-2}{(x-2)^2}$$

$$\textcircled{2} \quad y' = \frac{-2}{(x-2)^2} = \frac{-2}{(4-2)^2} = \frac{-2}{(2)^2} = \frac{-2}{4} = \left(\frac{-1}{2}\right)$$

$$\textcircled{3} \quad y - y_1 = m(x - x_1)$$

$$y - 2 = -\frac{1}{2}(x - 4)$$

$$y - 2 = -\frac{1}{2}x + 2$$

$$y = -\frac{1}{2}x + 4$$

Exercise 2.5

$$\textcircled{1} d) \quad g(x) = \frac{x^3 - 1}{x^2 + x + 1} \quad f(x)$$

$$\left(\frac{f}{g}\right)'(x) = \frac{f'(x)g(x) - f(x)g'(x)}{[g(x)]^2}$$

$$g'(x) = \frac{3x^2(x^2 + x + 1) - (x^3 - 1)(2x + 1)}{(x^2 + x + 1)^2}$$

$$g'(x) = \frac{3x^4 + 3x^3 + 3x^2 - (2x^4 + x^3 - 2x - 1)}{(x^2 + x + 1)^2}$$

$$g'(x) = \frac{x^4 + 2x^3 + 3x^2 + 2x + 1}{(x^2 + x + 1)^2}$$

$$g'(x) = \frac{x^4 + 2x^3 + 3x^2 + 2x + 1}{(x^2 + x + 1)(x^2 + x + 1)}$$

$$g'(x) = \frac{x^4 + 2x^3 + 3x^2 + 2x + 1}{x^4 + x^3 + x^2 + x^3 + x^2 + x + x^2 + x + 1}$$

$$g'(x) = \frac{x^4 + 2x^3 + 3x^2 + 2x + 1}{x^4 + 2x^3 + 3x^2 + 2x + 1}$$

$$g'(x) = 1$$

$$\textcircled{3} \text{ d), } y = \frac{x^3 - 1}{1 + 2x^2}, \quad (1, 0)$$

x_1, y_1

(i) Find derivative

$$y' = \frac{3x^2(1+2x^2) - 4x(x^3-1)}{(1+2x^2)^2}$$

$$y' = \frac{3x^2 + 6x^4 - 4x^4 + 4x}{(1+2x^2)^2}$$

$$y' = \frac{2x^4 + 3x^2 + 4x}{(1+2x^2)^2}$$

(ii) Find m (sub in $x=1$)

$$m = y'(1) = \frac{2(1)^4 + 3(1)^2 + 4(1)}{[1+2(1)^2]^2}$$

$$m = y'(1) = \frac{2+3+4}{9} = \frac{9}{9} = 1$$

↑
m

$$\textcircled{iii} \quad y - y_1 = m(x - x_1)$$

$$y - 0 = 1(x - 1)$$

$$\boxed{y = x - 1}$$

$$\boxed{x - y - 1 = 0}$$

Exercise 2.5

③ Find an equation of the tangent line to the curve at the given point.

$$c) y = \frac{1}{x^2+1}, \quad (-2, \frac{1}{5}) \quad \begin{matrix} x_1 = -2 \\ y_1 = \frac{1}{5} \end{matrix}$$

(i) Find the derivative:

$$y = \frac{1}{x^2+1} \quad \begin{matrix} f(x) \\ \swarrow \text{division} \\ g(x) \end{matrix}$$

$$y' = \frac{(0)(x^2+1) - (1)(2x)}{(x^2+1)^2}$$

$$y' = \frac{-2x}{(x^2+1)^2}$$

(ii) Sub in x-value to find m

$$y'(-2) = \frac{-2(-2)}{((-2)^2+1)^2} = \boxed{\frac{4}{25}} m$$

(iii) Find the equation of the tangent line

$$y - y_1 = m(x - x_1)$$

$$y - \frac{1}{5} = \frac{4}{25}(x + 2)$$

$$y - \frac{1}{5} = \frac{4}{25}x + \frac{8}{25}$$

$$y = \frac{4}{25}x + \frac{8}{25} + \frac{1}{5}$$

$$y = \frac{4}{25}x + \frac{8}{25} + \frac{5}{25}$$

$$\boxed{y = \frac{4}{25}x + \frac{13}{25}} \quad \checkmark$$

$$25y = 4x + 13$$

$$\boxed{0 = 4x - 25y + 13} \quad \checkmark$$

Exercise 2.5

#6 slope of the tangent equals 0 (horizontal)

① Find derivative:

$$y = \frac{x^2}{2x+5}$$

$f(x)$
 $g(x)$

$$y' = \frac{2x(2x+5) - x^2(2)}{(2x+5)^2}$$

$$y' = \frac{4x^2 + 10x - 2x^2}{(2x+5)^2}$$

$$y' = \frac{2x^2 + 10x}{(2x+5)^2}$$

② Solve for x:

$$y' = \frac{2x^2 + 10x}{(2x+5)^2}$$

$$0 = \frac{2x^2 + 10x}{(2x+5)^2}$$

$$0 = 2x^2 + 10x$$

$$0 = 2x(x+5)$$

$$\begin{array}{l|l} 2x=0 & x+5=0 \\ x=0 & x=-5 \end{array}$$

③ Solve for y:

if $x=0$

$$y = \frac{x^2}{2x+5}$$

$$y = \frac{(0)^2}{2(0)+5} = \frac{0}{5} = 0$$

$$(0, 0)$$

if $x=-5$

$$y = \frac{x^2}{2x+5}$$

$$y = \frac{(-5)^2}{2(-5)+5} = \frac{25}{-5} = -5$$

$$(-5, -5)$$

Review to Date:

$$\textcircled{a} \text{ e) } f(x) = 6x\sqrt{x^3} - 2(\sqrt[3]{x})$$

$$6 \cdot \frac{5}{2} = \frac{30}{2} = 15$$

$$f(x) = 6x'(x^{3/2}) - 2x^{1/3}$$

$$-2 \cdot \frac{1}{3} = -\frac{2}{3}$$

$$f(x) = 6x^{5/2} - 2x^{1/3}$$

$$f'(x) = 15x^{3/2} - \frac{2}{3}x^{-2/3}$$

$$f'(x) = 15x^{3/2} - \frac{2}{3x^{2/3}}$$

$$\textcircled{c} \text{ b) } f(x) = \frac{x+3}{4-x^2} \quad \begin{matrix} f(x) \\ g(x) \end{matrix}$$

$$f'(x) = \frac{1(4-x^2) + 2x(x+3)}{(4-x^2)^2}$$

$$f'(x) = \frac{4-x^2 + 2x^2 + 6x}{(4-x^2)^2} = \frac{x^2 + 6x + 4}{(4-x^2)^2}$$

Review to Date:

$$\textcircled{4} \text{ c) } f(x) = \sqrt[3]{x^2} (x^4 - 5\sqrt[3]{x^5})$$

$$f(x) = x^{2/3} (x^4 - 5x^{5/3})$$

$$f'(x) = \frac{2}{3}x^{-1/3}(x^4 - 5x^{5/3}) + x^{2/3}(4x^3 - \frac{25}{3}x^{2/3})$$

$$f'(x) = \frac{2}{3}x^{11/3} - \frac{10}{3}x^{4/3} + 4x^{11/3} - \frac{25}{3}x^{4/3}$$

$$f'(x) = \frac{14}{3}x^{11/3} - \frac{35}{3}x^{4/3}$$

$$-\frac{1}{2}x^{-1/2}$$

$$3(2) = 6$$

$$2(3) = 6$$

$$\textcircled{5} \text{ c) } g(x) = \frac{7\sqrt{x} - 10}{1 - \sqrt{x}} = \frac{7x^{1/2} - 10}{1 - x^{1/2}}$$

$$g'(x) = \frac{7x^{-1/2}(1 - x^{1/2}) + \frac{1}{2}x^{-1/2}(7x^{1/2} - 10)}{(1 - \sqrt{x})^2}$$

$$g'(x) = \frac{7(1 - x^{1/2}) + \frac{1}{2}(7x^{1/2} - 10)}{2x^{1/2}(1 - \sqrt{x})^2}$$

$$g'(x) = \frac{7(1 - x^{1/2}) + 1(7x^{1/2} - 10)}{2x^{1/2}(1 - \sqrt{x})^2}$$

$$g'(x) = \frac{7 - 7x^{1/2} + 7x^{1/2} - 10}{2\sqrt{x}(1 - \sqrt{x})^2} = \frac{-3}{2\sqrt{x}(1 - \sqrt{x})^2}$$

Equation of tangent

$$\textcircled{3} \quad y = x^3 + 3x \quad x = \underline{1}$$

(i) Find y

$$y = (1)^3 + 3(1)$$

$$y = \underline{4}$$

(ii) Find the derivative

$$y = x^3 + 3x$$

$$y' = 3x^2 + 3$$

(iii) Find m .

$$y'(1) = 3(1)^2 + 3$$

$$y'(1) = 6$$

$$m = 6$$

$$(iv) \quad y - y_1 = m(x - x_1)$$

$$y - 4 = 6(x - 1)$$

$$y - 4 = 6x - 6$$

$$\boxed{y = 6x - 2}$$

$$\boxed{0 = 6x - y - 2}$$

$$\textcircled{2} \text{ e) } f(x) = 6x\sqrt{x^3} - 2\sqrt[3]{x}$$

$$f(x) = 6x(x)^{3/2} - 2x^{1/3}$$

$$f(x) = 6x^{5/2} - 2x^{1/3}$$

$$f'(x) = 15x^{3/2} - \frac{2}{3}x^{-2/3}$$

$$6x - y = 4$$

$$6x - 4 = y$$

$$y = 6x - 4$$

$$m = 6$$

$$y = x\sqrt{x} = x(x^{1/2}) = x^{3/2}$$

$$y' = \frac{3}{2}x^{1/2}$$

$$6 = \frac{3}{2}x^{1/2}$$

$$12 = 3x^{1/2}$$

$$4 = x^{1/2}$$

$$16 = x$$

$$\textcircled{1} \text{ a) } f(x) = \underline{2x^3 + 3x} \quad \textcircled{1} f(x+h) = \underline{2(x+h)^3 + 3(x+h)}$$

$$f(x+h) = 2(\underline{x^3 + 3x^2h + 3xh^2 + h^3}) + 3(\underline{x+h})$$

$$f(x+h) = \underline{2x^3 + 6x^2h + 6xh^2 + 2h^3 + 3x + 3h}$$

$$\textcircled{2} f'(x) = \lim_{h \rightarrow 0} \frac{\cancel{2x^3} + 6x^2h + 6xh^2 + \cancel{2h^3} + 3x + 3h - (\cancel{2x^3} + \cancel{3x})}{h}$$

$$f'(x) = \lim_{h \rightarrow 0} \frac{6x^2h + 6xh^2 + 2h^3 + 3h}{h}$$

$$f'(x) = \lim_{h \rightarrow 0} \frac{\cancel{h}(6x^2 + 6x\cancel{h} + 2\cancel{h^2} + 3)}{\cancel{h}} = 6x^2 + 3$$

$$\textcircled{1} \quad f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h} \quad \left| \quad \begin{array}{l} (x+h)^3 \\ x^3 + 3x^2h + 3xh^2 + h^3 \end{array} \right.$$
$$\textcircled{5} \quad y - y_1 = m(x - x_1)$$