Example 4

Model the Speed of Sound

Justin's physics textbook states that the speed, s, in metres per second, of sound in dry air is related to the air temperature, T, in degrees Celsius, by the function $s = 331.3\sqrt{1 + \frac{T}{273.15}}$.

a) Determine the domain and range in this context.
b) On the Internet, Justin finds another formula for the speed of sound, $s = 20\sqrt{T + 273}$. Use algebra to show that the two functions are approximately equivalent.
c) How is the graph of this function related to the graph of the base square root function? Which transformation do you predict will be the most noticeable on a graph?
d) Graph the function $s = 331.3\sqrt{1 + \frac{T}{273.15}}$ using technology.
e) Determine the speed of sound, to the nearest metre per second, at each of the following temperatures.
  i) 20 °C (normal room temperature)
  ii) 0 °C (freezing point of water)
  iii) −63 °C (coldest temperature ever recorded in Canada)
  iv) −89 °C (coldest temperature ever recorded on Earth)

a) Domain: $1+\frac{T}{273.15} \geq 0$ $\{T \mid T \geq -273.15, T \in R\}$

$\frac{T}{273.15} \geq -1$

$T \geq -273.15°$

Range: $\{s \mid s \geq 0, s \in R\}$

b) $s = 331.3\sqrt{1+\frac{T}{273.15}}$

$s = 331.3\sqrt{\frac{273.15}{273.15}+\frac{T}{273.15}}$

$s = 331.3\sqrt{\frac{273.15+T}{273.15}}$

$s = 331.3\frac{\sqrt{273.15+T}}{\sqrt{273.15}}$

$s = 331.3\frac{\sqrt{273.15+T}}{16.527}$

$s = 20.04\sqrt{273.15+T}$

$s \approx 20\sqrt{T+273}$

The graph of $s = \sqrt{T}$ is stretched vertically by a factor of about 20 and then translated about 273 units to the left. Translating 273 units to the left will be most noticeable on the graph of the function.

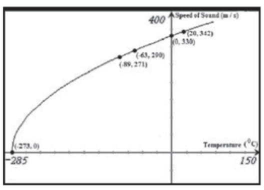
Are these transformations consistent with the domain and range?

c) $s = 20\sqrt{T+273}$

$a = 20 \rightarrow$ vertical stretch by a factor of 20

$h = -273 \rightarrow$ translation of 273 units to the left

d)



Are your answers to part c) confirmed by the graph?

e)

	Temperature (°C)	Approximate Speed of Sound (m/s)
i)	20	343
ii)	0	331
iii)	−63	291
iv)	−89	272

Solving Radical Equations Graphically + Algebraically

Focus on...

- relating the roots of radical equations and the x-intercepts of the graphs of radical functions
- determining approximate solutions of radical equations graphically

Example 1

Relate Roots and x-Intercepts

a) Determine the root(s) of $\sqrt{x+5} - 3 = 0$ algebraically. (Solve for x)

b) Using a graph, determine the x-intercept(s) of the graph of $y = \sqrt{x+5} - 3$. (Let y = 0)

c) Describe the connection between the root(s) of the equation and the x-intercept(s) of the graph of the function.

a) $\sqrt{x+5} - 3 = 0$ (Isolate the radical)

$\left(\sqrt{x+5}\right)^2 = (3)^2$ (Square both sides)

$x + 5 = 9$ (Solve for x)

$\boxed{x = 4}$

b) $y = \sqrt{x+5} - 3$ a = 1 b = 1 h = -5 k = -3

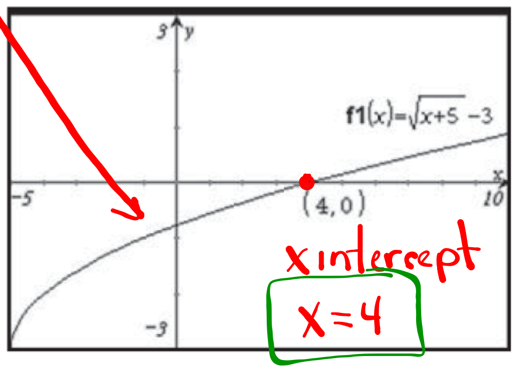
$(x, y) \rightarrow (x - 5, y - 3)$

$y = \sqrt{x}$

x	y
0	0
1	1
4	2
9	3
16	4

$y = \sqrt{x+5} - 3$

x	y
-5	-3
-4	-2
-1	-1
4	0
11	1



c) The roots and the x-intercepts are equal

Example 2

Solve a Radical Equation Involving an Extraneous Solution

Solve the equation $\sqrt{x+5} = x + 3$ algebraically and graphically.

① Isolate the radical
② Square both sides
③ Solve for x

$(\sqrt{x+5})^2 = (x+3)^2$ (Square both sides)

$x+5 = (x+3)(x+3)$

$x+5 = x^2+3x+3x+9$

$x+5 = x^2+6x+9$ (Bring everything to one side)

$0 = x^2+6x+9-x-5$

$0 = x^2+5x+4$ (Factor) $1+4=5$ $1 \times 4 = 4$ 4 1x4 2x2

$0 = (x+1)(x+4)$

$x+1=0$ | $x+4=0$

$x=-1$ | $x=-4$

Test $x=-1$

$\sqrt{x+5} = x+3$

$\sqrt{-1+5} = -1+3$

$\sqrt{4} = 2$

$2 = 2$ ✓

x=-1 is a solution

Test $x=-4$

$\sqrt{x+5} = x+3$

$\sqrt{-4+5} = -4+3$

$\sqrt{1} = -1$

$1 = -1$ ✗

x=-4 is not a solution (extraneous)

Example 3 → Decimals

Approximate Solutions to Radical Equations

a) Solve the equation $\sqrt{3x^2 - 5} = x + 4$ graphically. Express your answer to the nearest tenth.

b) Verify your solution algebraically.

$$\left(\sqrt{3x^2-5}\right)^2 = (x+4)^2$$

$$3x^2-5 = (x+4)(x+4)$$

$$3x^2-5 = x^2+4x+4x+16$$

$$3x^2-5 = x^2+8x+16$$

$$3x^2-5-x^2-8x-16 = 0$$

$$2x^2-8x-21 = 0$$

$a=2 \quad b=-8 \quad c=-21$

$?\_ + ?\_ = -8$

$?\_ \times ?\_ = -42$

42
1×42
2×21
3×14
6×7

$$x = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$$

$$x = \frac{-(-8) \pm \sqrt{(-8)^2-4(2)(-21)}}{2(2)}$$

$$x = \frac{8 \pm \sqrt{64+168}}{4}$$

$$x = \frac{8 \pm \sqrt{232}}{4}$$

$$x = \frac{8 \pm 15.23}{4}$$

$$x = \frac{8+15.23}{4} \quad \text{or} \quad x = \frac{8-15.23}{4}$$

$$x = \frac{23.23}{4} \qquad x = \frac{-7.23}{4}$$

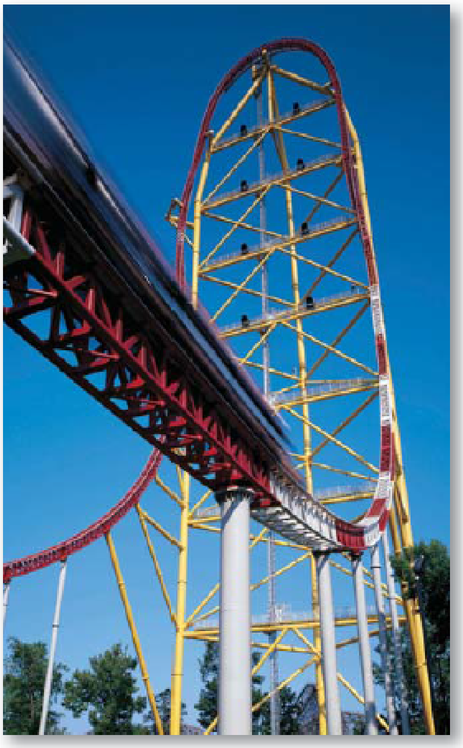
$$x = 5.8 \qquad x = -1.8$$

Test your answers.

Example 4

Solve a Problem Involving a Radical Equation

An engineer designs a roller coaster that involves a vertical drop section just below the top of the ride. She uses the equation $v = \sqrt{(v_0)^2 + 2ad}$ to model the velocity, v, in feet per second, of the ride's cars after dropping a distance, d, in feet, with an initial velocity, v_0, in feet per second, at the top of the drop, and constant acceleration, a, in feet per second squared. The design specifies that the speed of the ride's cars be 120 ft/s at the bottom of the vertical drop section. If the initial velocity of the coaster at the top of the drop is 10 ft/s and the only acceleration is due to gravity, 32 ft/s², what vertical drop distance should be used, to the nearest foot?



Key Ideas

- You can solve radical equations algebraically and graphically.
- The solutions or roots of a radical equation are equivalent to the x-intercepts of the graph of the corresponding radical function. You can use either of the following methods to solve radical equations graphically:
  - Graph the corresponding function and identify the value(s) of the x-intercept(s).
  - Graph the system of functions that corresponds to the expression on each side of the equal sign, and then identify the value(s) of x at the point(s) of intersection.

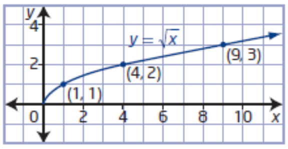
Homework

Finish Chapter Review

Page 99

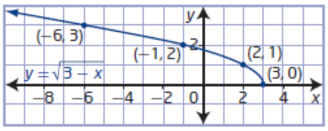
Chapter 2 Review, pages 99 to 101

1. a)



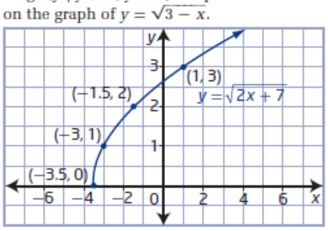
domain $\{x \mid x \geq 0, x \in \mathrm{R}\}$
range $\{y \mid y \geq 0, y \in \mathrm{R}\}$ All values in the table lie on the smooth curve graph of $y = \sqrt{x}$.

b)



domain $\{x \mid x \leq 3, x \in \mathrm{R}\}$
range $\{y \mid y \geq 0, y \in \mathrm{R}\}$ All points in the table lie on the graph of $y = \sqrt{3 - x}$.

c)



domain $\{x \mid x \geq -3.5, x \in \mathrm{R}\}$
range $\{y \mid y \geq 0, y \in \mathrm{R}\}$ All points in the table lie on the graph of $y = \sqrt{2x + 7}$.

2. Use $y = a\sqrt{b(x - h)} + k$ to describe transformations.

a) $a = 5 \rightarrow$ vertical stretch factor of 5
$h = -20 \rightarrow$ horizontal translation left 20 units;
domain $\{x \mid x \geq -20, x \in \mathrm{R}\}$; range $\{y \mid y \geq 0, y \in \mathrm{R}\}$

b) $b = -2 \rightarrow$ horizontal stretch factor of $\frac{1}{2}$, then reflected on y-axis: $k = -8 \rightarrow$ vertical translation of 8 units down.
domain $\{x \mid x \leq 0, x \in \mathrm{R}\}$; range $\{y \mid y \geq -8, y \in \mathrm{R}\}$

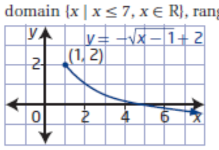
c) $a = -1 \rightarrow$ reflect in x-axis
$b = \frac{1}{6} \rightarrow$ horizontal stretch factor of 6
$h = 11 \rightarrow$ horizontal translation right 11 units;
domain $\{x \mid x \geq 11, x \in \mathrm{R}\}$, range $\{y \mid y \leq 0, y \in \mathrm{R}\}$.

3. a) $y = \sqrt{\frac{1}{10}x} + 12$, domain $\{x \mid x \geq 0, x \in \mathrm{R}\}$,
range $\{y \mid y \geq 12, y \in \mathrm{R}\}$

b) $y = -2.5\sqrt{x + 9}$
domain $\{x \mid x \geq -9, x \in \mathrm{R}\}$, range $\{y \mid y \leq 0, y \in \mathrm{R}\}$

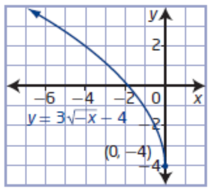
c) $y = \frac{1}{20}\sqrt{-\frac{2}{5}(x - 7)} - 3$,
domain $\{x \mid x \leq 7, x \in \mathrm{R}\}$, range $\{y \mid y \geq -3, y \in \mathrm{R}\}$

4. a)



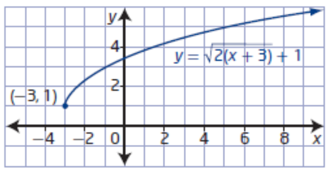
domain
$\{x \mid x \geq 1, x \in \mathrm{R}\}$,
range
$\{y \mid y \leq 2, y \in \mathrm{R}\}$

b)



domain
$\{x \mid x \leq 0, x \in \mathrm{R}\}$,
range
$\{y \mid y \geq -4, y \in \mathrm{R}\}$

c)

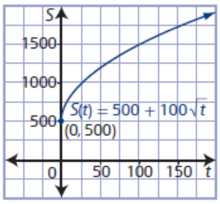


domain $\{x \mid x \geq -3, x \in \mathrm{R}\}$, range $\{y \mid y \geq 1, y \in \mathrm{R}\}$

5. The domain is affected by a horizontal translation of 4 units right and by no reflection on the y-axis. The domain will have values of x greater than or equal to 4, due to a translation of the graph 4 units right. The range is affected by vertical translation of 9 units up and a reflection on the x-axis. The range will be less than or equal to 9, because the graph has been moved up 9 units and reflected on the x-axis, therefore the range is less than or equal to 9, instead of greater than or equal to 9.

6. a) Given the general equation $y = a\sqrt{b(x - h)} + k$ to describe transformations, $a = 100$ indicates a vertical stretch by a factor of 100, $k = 500$ indicates a vertical translation up 500 units.

b)



Since the minimum value of the graph is 500, the minimum estimated sales will be 500 units.

c) domain $\{t \mid t \geq 0, t \in R\}$ The domain means that time is positive in this situation.
range $\{S(t) \mid S(t) \geq 500, S(t) \in W\}$. The range means that the minimum sales are 500 units.

d) about 1274 units

7. a) $y = \sqrt{\frac{1}{4}(x + 3)} + 2$ **b)** $y = -2\sqrt{x + 4} + 3$

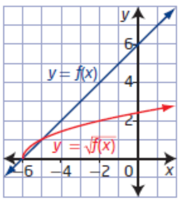
c) $y = 4\sqrt{-(x - 6)} - 4$

8. a) For $y = x - 2$, domain $\{x \mid x \in R\}$, range $\{y \mid y \in R\}$; for $y = \sqrt{x - 2}$, domain $\{x \mid x \geq 2, x \in R\}$, range $\{y \mid y \geq 0, y \in R\}$. The domain changes because the square root function has restrictions. The range changes because the function only exists on or above the x-axis.

b) For $y = 10 - x$, domain $\{x \mid x \in R\}$, range $\{y \mid y \in R\}$; for $y = \sqrt{10 - x}$, domain $\{x \mid x \leq 10, x \in R\}$, range $\{y \mid y \geq 0, y \in R\}$ The domain changes because the square root function has restrictions. The range changes because the function only exists on or above the x-axis.

c) For $y = 4x + 11$, domain $\{x \mid x \in R\}$, range $\{y \mid y \in R\}$; for $y = \sqrt{4x + 11}$, domain $\left\{x \mid x \geq -\frac{11}{4}, x \in R\right\}$, range $\{y \mid y \geq 0, y \in R\}$. The domain changes because the square root function has restrictions. The range changes because the function only exists on or above the x-axis.

9. a)



Plot invariant points at the intersection of the graph and lines $y = 0$ and $y = 1$. Plot any points $(x, \sqrt{y})$ where the value of y is a perfect square. Sketch a smooth curve through the invariant points and points satisfying $(x, \sqrt{y})$.

b) $y = \sqrt{f(x)}$ is positive when $f(x) > 0$,
$y = \sqrt{f(x)}$ does not exist when $f(x) < 0$.
$\sqrt{f(x)} > f(x)$ when $0 < f(x) < 1$ and
$f(x) > \sqrt{f(x)}$ when $f(x) > 1$

c) For $f(x)$: domain $\{x \mid x \in R\}$, range $\{y \mid y \in R\}$; for $\sqrt{f(x)}$, domain $\{x \mid x \geq -6, x \in R\}$, range $\{y \mid y \geq 0, y \in R\}$, since $\sqrt{f(x)}$ is undefined when $f(x) < 0$.

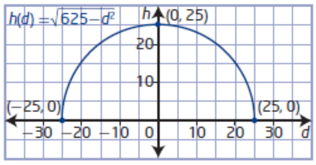
10. a) $y = 4 - x^2 \rightarrow$ domain $\{x \mid x \in R\}$, range $\{y \mid y \leq 4, y \in R\}$ for $y = \sqrt{4 - x^2} \rightarrow$ domain $\{x \mid -2 \leq x \leq 2, x \in R\}$, range $\{y \mid 0 \leq y \leq 2, y \in R\}$, since $4 - x^2 > 0$ only between -2 and 2 then the domain of $y = \sqrt{4 - x^2}$ is $-2 \leq x \leq 2$. In the domain of $-2 \leq x \leq 2$ the maximum value of $y = 4 - x^2$ is 4, so the maximum value of $y = \sqrt{4 - x^2}$ is $\sqrt{4} = 2$ then the range of the function $y = \sqrt{4 - x^2}$ will be $0 \leq y \leq 2$.

b) $y = 2x^2 + 24 \rightarrow$ domain $\{x \mid x \in R\}$, range $\{y \mid y \geq 24, y \in R\}$ for $y = \sqrt{2x^2 + 24} \rightarrow$ domain $\{x \mid x \in R\}$, range $\{y \mid y \geq \sqrt{24}, y \in R\}$. The domain does not change since the entire graph of $y = 2x^2 + 24$ is above the x-axis. The range changes since the entire graph moves up 24 units and the graph itself opens up, so the range becomes $y \geq \sqrt{24}$.

c) $y = x^2 - 6x \rightarrow$ domain $\{x \mid x \in R\}$, range $\{y \mid y \geq -9, y \in R\}$ for $y = \sqrt{x^2 - 6x} \rightarrow$ domain $\{x \mid x \leq 0 \text{ or } x \geq 6, x \in R\}$, range $\{y \mid y \geq 0, y \in R\}$, since $x^2 - 6x < 0$ between 0 and 6, then the domain is undefined in the interval $(0, 6)$ and exists when $x \leq 0$ or $x \geq 6$. The range changes because the function only exists above the x-axis.

11. a) $h(d) = \sqrt{625 - d^2}$

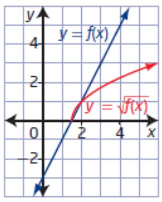
b)



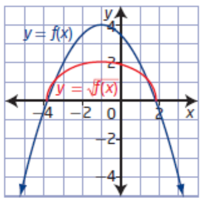
domain $\{d \mid -25 \leq d \leq 25, d \in R\}$
range $\{h \mid 0 \leq h \leq 25, h \in R\}$

c) In this situation, the values of h and d must be positive to express a positive distance. Therefore the domain changes to $\{d \mid 0 \leq d \leq 25, d \in R\}$. Since the range of the original function $h(d) = \sqrt{625 - d^2}$ is always positive then the range does not change.

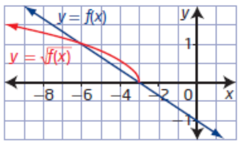
12. a)



b)

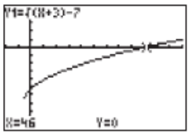


c)



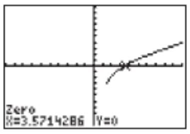
13. a) $x = 46$

b)

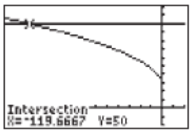


c) The root of the equation and the x value of the x-intercept are the same.

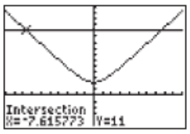
14. a) $x \approx 3.571$

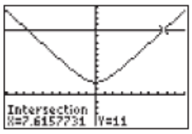


b) $x \approx -119.667$



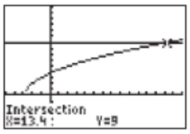
c) $x \approx -7.616$ and $x \approx 7.616$



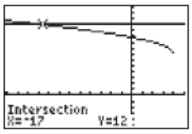


15. 4.13 m

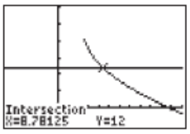
16. a) $x = 13.4$



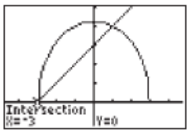
b) $x = -17$

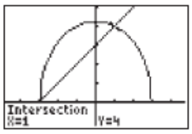


c) $x \approx 8.781$



d) $x = -3$ and 1





17. a) Jaime found two possible answers which are determined by solving a quadratic equation.

b) Carly found only one intersection at (5, 5) or x-intercept (5, 0) determined by possibly graphing.

c) Atid found an extraneous root of $x = 2$.

18. a) 130 m^2 **b)** 6 m