Questions from Homework

8. If seafood is not kept frozen (below 0 °C), it will spoil due to bacterial growth. The relative rate of spoilage increases with temperature according to the model $R = 100(2.7)^{\frac{T}{8}}$, where T is the temperature, in degrees Celsius, and R is the relative spoilage rate.

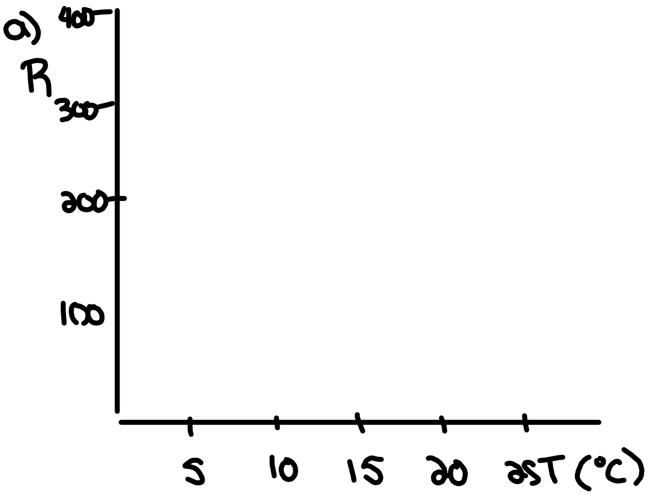
a) Sketch a graph of the relative spoilage rate R versus the temperature T from 0 °C to 25 °C.

b) Use your graph to predict the temperature at which the relative spoilage rate doubles to 200.

c) What is the relative spoilage rate at 15 °C?

d) If the maximum acceptable relative spoilage rate is 500, what is the maximum storage temperature?

a)



Assignment:

y-int ($x=0$)

$y = 4^{-2(x+5)} - 3$

$y = 4^{-2(0+5)} - 3$

$y = 4^{-2(5)} - 3$

$y = 4^{-10} - 3$

$y = \left(\frac{1}{4}\right)^{10} - 3$

$y = \frac{1}{1048576} - \frac{3}{1}$

$y = \frac{1}{1048576} - \frac{3145728}{1048576}$

$y = -\frac{3145727}{1048576}$

$y = -2.\overline{9}$

$(0, -2.\overline{9})$

x-int ($y=0$)

$y = 4^{-2(x+5)} - 3$

$0 = 4^{-2(x+5)} - 3$

$3 = 4^{-2(x+5)}$

$(\cancel{4})^{0.7925} = \cancel{4}^{-2(x+5)}$

$\frac{0.7925}{-2} = \frac{-\cancel{2}(x+5)}{-\cancel{2}}$

$-0.39625 = x + 5$

$-5.39625 = x$

$(-5.4, 0)$

9. A bacterial culture starts with 2000 bacteria and doubles every 0.75 h. After how many hours will the bacteria count be 32 000?

Given:

Initial Amount = 2000

Base = 2

exp = $\frac{x}{0.75}$

A = 32000

$y = 2000(2)^{\frac{x}{0.75}}$

$A = 2000(2)^{\frac{t}{0.75}}$

$\frac{32000}{2000} = \frac{2000(2)^{t/0.75}}{2000}$

$16 = 2^{t/0.75}$

$\frac{\log 16}{\log 2} = 4 \rightarrow \cancel{2}^4 = \cancel{2}^{t/0.75}$

$0.75 \cdot 4 = \frac{t}{0.75} \cdot 0.75$

$\boxed{3h = t}$

$$16 = 2^{t/0.75}$$

$$\cancel{16}^{1} = \left(\cancel{16}^{0.25}\right)^{t/0.75}$$

$$(0.75)\ 1 = \frac{0.25t}{\cancel{0.75}}\ \cancel{(0.75)}$$

$$\frac{0.75}{0.25} = \frac{0.25t}{0.25}$$

$$\boxed{3 = t}$$

Given: $y = -3(2)^{2x+2} + 4$

$y = -3(2)^{2(x+1)} + 4$

i) state the parameters and describe the corresponding transformations
ii) create a table to show what happens to the given points under each transformation
iii) sketch the graph of the base function and the transformed function
iv) describe the effects on the domain, range, equation of the horizontal asymptote, and intercepts

(i) $y = -3(2)^{2(x+1)} + 4$ c = base = 2

a = -3 → a vertical stretch by a factor of 3 and a reflection in the x axis

b = 2 → a horizontal stretch by a factor of $\frac{1}{2}$

h = -1 → 1 unit left

k = 4 → 4 units up

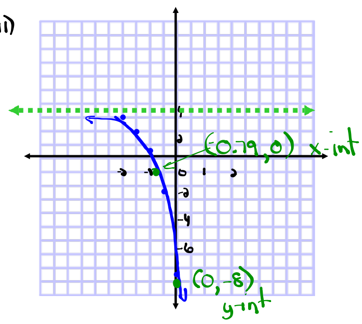
(ii) $(x, y) \rightarrow \left(\frac{1}{2}x - 1, -3y + 4\right)$

$y = 2^x$

x	y
-2	1/4
-1	1/2
0	1
1	2
2	4

x	y
-2	13/4 = 3.25
-1.5	5/2 = 2.5
-1	1
-0.5	-2
0	-8

(iii)



(iv) D: {x | x ∈ R} or (-∞, ∞)

R: {y < 4, y ∈ R} or (-∞, 4)

HA: y = 4

x int (y = 0)

$y = -3(2)^{2(x+1)} + 4$

$0 = -3(2)^{2(x+1)} + 4$

$\frac{-4}{-3} = \frac{-3(2)^{2(x+1)}}{-3}$

$1.\overline{3} = (2)^{2(x+1)}$

$(2)^{0.42} = (2)^{2(x+1)}$ $\frac{\log 1.3}{\log 2} = 0.42$

$\frac{0.42}{2} = \frac{2(x+1)}{2}$

$0.21 = x + 1$

$\boxed{-0.79 = x}$

(-0.79, 0)

y int (x = 0)

$y = -3(2)^{2(x+1)} + 4$

$y = -3(2)^{2(0+1)} + 4$

$y = -3(2)^{2(1)} + 4$

$y = -3(2)^2 + 4$

$y = -3(4) + 4$

$y = -12 + 4$

$\boxed{y = -8}$

(0, -8)

Check graph for these points

Ex: Exponential Equation

$$64^x = \left(\frac{1}{8}\right)^{x+1}\left(\sqrt{32}\right)$$

$$64^x = \left(\frac{1}{8}\right)^{x+1} \cdot (32)^{\frac{1}{2}}$$

$\frac{\log 64}{\log 2} = 6$

$\frac{\log(\frac{1}{8})}{\log 2} = -3$

$\frac{\log 32}{\log 2} = 5$

$$(2^6)^x = (2^{-3})^{x+1}\ (2^5)^{\frac{1}{2}}$$

$$2^{6x} = 2^{-3x-3} \cdot 2^{\frac{5}{2}}$$

$$2^{6x} = 2^{-3x-3+\frac{5}{2}}$$

$$2^{6x} = 2^{-3x-\frac{6}{2}+\frac{5}{2}}$$

$$\cancel{2}^{6x} = \cancel{2}^{-3x-\frac{1}{2}}$$

$$6x = -3x - \frac{1}{2}$$

$$\frac{9x}{9} = -\frac{1}{2} \div 9$$

$$x = -\frac{1}{2} \cdot \frac{1}{9}$$

$$\boxed{x = \frac{-1}{18}}$$

Base 2:   a) $\left(\frac{1}{4}\right)$ and $\sqrt{64}$

$\left(\frac{1}{4}\right)$ and 8

$\frac{\log(\frac{1}{4})}{\log 2} = -2 \quad (2)^{-2}$ and $(2)^{3} \quad \frac{\log 8}{\log 2} = 3$

$\sqrt{64}$

$(\underline{64})^{1/2}$

$(2^6)^{1/2}$

2^3

$\underline{4}^6$ or 4096

$(2^2)^6$ $\quad (2)^{12}$

2^{12}

$\frac{\log 4096}{\log 2} = \underline{12}$

base

exp

$$\left(\frac{1}{125}\right)^{2x} = 5^{3x+2} \cdot \sqrt{3125}$$

$$\left(\frac{1}{125}\right)^{2x} = (5)^{3x+2} \cdot (3125)^{1/2}$$

$$\left(5^{-3}\right)^{2x} = (5)^{3x+2} \left(5^{5}\right)^{1/2}$$

$$5^{-6x} = 5^{3x+2} \cdot 5^{5/2}$$

$$5^{-6x} = 5^{3x+2+\frac{5}{2}}$$

$$\cancel{5}^{-6x} = \cancel{5}^{3x+\frac{9}{2}}$$

$$-6x = 3x + \frac{9}{2}$$

$$\frac{-\cancel{9}x}{-\cancel{9}} = \frac{9}{2} \quad \div -9$$

$$x = \frac{9}{2} \cdot -\frac{1}{9}$$

$$\boxed{x = -\frac{9}{18} = -\frac{1}{2}}$$

Homework

Chapter 7 Review pg. 366-367 (Do all questions)

For $y = c^x$

D: $\{x \mid x \in R\}$

R: $\{y \mid y > \underline{0}, y \in R\}$

x int: none

y int: (0,1)

HA: $y = \underline{0}$

For $y = ac^{b(x-h)} + \underline{k}$

D: $\{x \mid x \in R\}$

R: $\{y \mid y > \underline{k}, y \in R\}$ (if $a < 0$ then $y < k$)

x int: sub 0 in for y

y int: sub 0 in for x

HA: $y = \underline{k}$

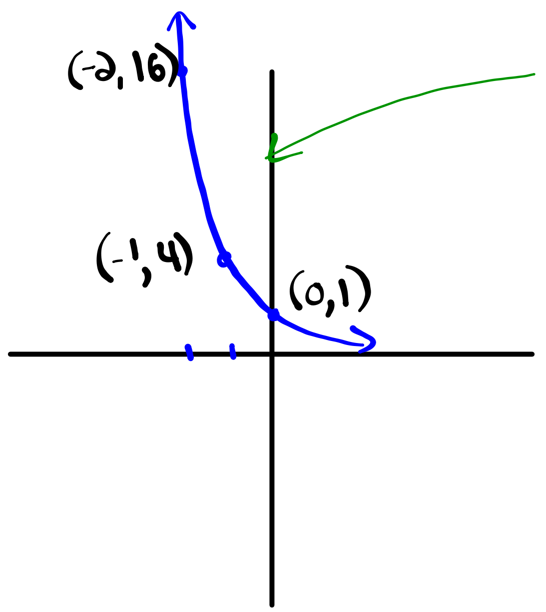
6 $2 = 1.07^x$

$\cancel{(1.07)}^{10.2} = \cancel{1.07}^x$

$\boxed{10.2 = x}$

$\frac{\log 2}{\underbrace{\log 1.07}_{\text{Base}}} = \underbrace{10.2}_{\text{exp}}$

3



decreasing

x	y	
-2	16	$\frac{1}{4}$
-1	4	$\frac{1}{4}$
0	1	

$\boxed{y = \left(\frac{1}{4}\right)^x}$

11 b)

$$2^{x-2} = 3^{x+1}$$

$$2^{x-2} = (2^{1.58})^{x+1}$$

$$\frac{\log 3}{\log 2} = 1.58$$

Base exp

$$\cancel{2}^{x-2} = \cancel{2}^{1.58x+1.58}$$

$$x-2 = 1.58x+1.58$$

$$-2-1.58 = 1.58x-x$$

$$\frac{-3.58}{0.58} = \frac{\cancel{0.58}x}{\cancel{0.58}}$$

$$\boxed{-6.17 = x}$$

(12) Ni - 65 half-life of 2.5 h

Base $= \frac{1}{2}$

exp $= \frac{t}{2.5}$

Initial Amount $= A_0$

$$A_F = A_0\left(\frac{1}{2}\right)^{\frac{t}{2.5}}$$

$$y = a(c)^{b(x-h)} + k$$

$$(x,y) \rightarrow \left[\frac{1}{b}x + h, ay + k\right]$$

$$y = \text{Initial Amount}(\text{Base})^{\text{exp.}}$$

Finding a common base

$$\frac{\log(\text{have})}{\log(\text{want})} = \text{exp.}$$

Base

Ex:

$$\sqrt{27} \cdot 9^{x} = 81^{5x-4} \div \left(\frac{1}{3}\right)^{x}$$

$$27^{1/2} \cdot 9^{x} = 81^{5x-4} \div \left(\frac{1}{3}\right)^{x}$$

$$(3^{3})^{1/2} \cdot (3^{2})^{x} = (3^{4})^{5x-4} \div (3^{-1})^{x}$$

$$3^{3/2} \cdot 3^{2x} = 3^{20x-16} \div 3^{-x}$$

$20x - 16 - (-x)$
$20x - 16 + x$

$$\cancel{3}^{2x+3/2} = \cancel{3}^{21x-16}$$

$$2x + 3/2 = 21x - 16$$

$$\frac{3}{2} + \frac{16}{1} = 21x - 2x$$

$$\frac{3}{2} + \frac{32}{2} = 19x$$

$$\frac{\frac{35}{2}}{19} = \frac{\cancel{19}x}{\cancel{19}}$$

$$\frac{35}{2} \times \frac{1}{19} = x$$

$$\boxed{\frac{35}{38} = x}$$

7.1 Characteristics of Exponential Functions, pages 334–345

1. Match each item in set A with its graph from set B.

Set A

a) The population of a country, in millions, grows at a rate of 1.5% per year. (Increasing) B

b) $y = 10^x$ (Increasing) D

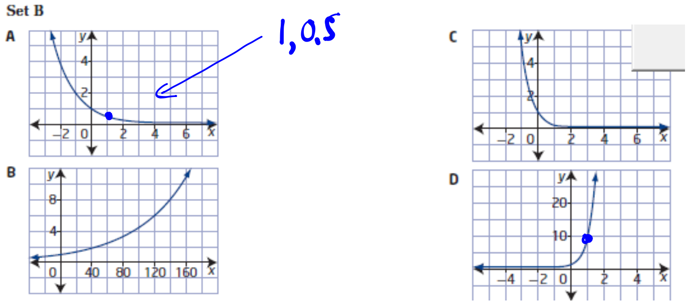
c) Tungsten-187 is a radioactive isotope that has a half-life of 1 day. (Decreasing) A

x	y
0	1
1	0.5

d) $y = 0.2^x$ (Decreasing)

Set B

1, 0.5



③ (x, y)

$-2, 16$ $\rangle \frac{1}{4}$

$-1, 4$ $\rangle \frac{1}{4}$

$0, 1$

$y = c^x$ (Find c)

$y = \left(\frac{1}{4}\right)^x$

$$16 \;\Big|\; \left(\frac{1}{4}\right)^{-2} \quad 4^2 \quad 16 \quad \checkmark$$

⑤ $y = -2(4)^{3(x-1)} + 2$ $a = -2$ $b = 3$ $h = 1$ $k = 2$

$y = 4^x$ $(x, y) \rightarrow \left(\frac{1}{3}x + 1, -2y + 2\right)$

x	y
-2	1/16
-1	1/4
0	1
1	4
2	16

x	y
1/3	15/8
2/3	3/2
1	0
4/3	-6
5/3	-30

9 c) $\left(\sqrt[3]{216}\right)^5$

$\left(216^{1/3}\right)^5$

$(216)^{5/3}$

$\frac{\log 216}{\log 6} = 3$ $\left(6^3\right)^{5/3}$

6^5

$$y = c^x$$

$$16 = c^{-2}$$

$$16 = \left(\frac{1}{c}\right)^2$$

$$16 = \frac{1}{c^2}$$

$$16c^2 = 1$$

$$c^2 = \frac{1}{16}$$

$$c = \frac{1}{4}$$

$y=a(c)^{b(x-h)}+k$

5) $y=-2(4)^{3(x-1)}+2$

a) $a=-2 \rightarrow$ vertical stretch by a factor of 2.
vertical reflection in x-axis

$b=3 \rightarrow$ horizontal stretch by a factor of $\frac{1}{3}$

$h=1 \rightarrow$ translate 1 unit right

$k=2 \rightarrow$ translate 2 units up

b) horizontal stretch $b=3$ $(x,y)\rightarrow(\frac{1}{3}x, y)$

c) $(x,y) \longrightarrow [\frac{1}{3}x+1, -2y+2]$

$y=4^x$

x	y
-2	1/16
-1	1/4
0	1
1	4
2	16

x	y
1/3	15/8
2/3	3/2
1	0
4/3	-6
5/3	-30

$-2(4)+2$
$-8+2$
-6

graph

d) D: $\{x \mid x \in \mathbb{R}\}$

R: $\{y \mid y<2, y \in \mathbb{R}\}$

HA: $y=2$

x int (y=0)

$y=-2(4)^{3(x-1)}+2$

$0=-2(4)^{3(x-1)}+2$

$\frac{-2}{-2}=\frac{-2(4)^{3x-3}}{-2}$

$1=4^{3x-3}$

$\cancel{4}^0=\cancel{4}^{3x-3}$

$0=3x-3$

$\frac{3}{3}=\frac{3x}{3}$

$\boxed{1=x}$

y int (x=0)

$y=-2(4)^{3(x-1)}+2$

$y=2(4)^{3(0-1)}+2$

$y=-2(4)^{-3}+2$

$y=-2\left(\frac{1}{64}\right)+2$

$y=\frac{-2}{64}+2$

$y=\frac{-1}{32}+\frac{64}{32}=\frac{63}{32}$

Test

Open Response:

→ Question like your assignment (Transforming exponential functions)

→ Word problem → $y = (\text{Initial Amount})(\text{Base})^{\overset{x}{\text{time it takes...}}}$

→ exponential equation.