

Questions from Homework

Ex 10.4

ⓐ c) 36, 18, 9, ..., $\frac{9}{128}$

$a = 36$
 $r = \frac{18}{36} = \frac{1}{2}$
 $t_n = \frac{9}{128}$
 $t_n = ar^{n-1}$
 $\frac{9}{128} = 36 \left(\frac{1}{2}\right)^{n-1}$
 $\frac{1}{512} = \left(\frac{1}{2}\right)^{n-1}$
 $\left(\frac{1}{2}\right)^9 = \left(\frac{1}{2}\right)^{n-1}$ * $\frac{\log(\frac{1}{512})}{\log(\frac{1}{2})}$
 $9 = n-1$
 $10 = n$

ⓐ d) $2^{50}, 2^{48}, 2^{46}, \dots$

$a = 2^{50}$
 $r = \frac{2^{48}}{2^{50}} = 2^{-2} = \frac{1}{4}$
 $t_{15} = (2^{50}) \left(\frac{1}{4}\right)^{14}$
 $t_{15} = (2^{50}) (2^{-28})$
 $t_{15} = 2^{22}$

ⓐ $t_3 = \frac{1}{9} \quad | \quad t_7 = 9$
 $t_3 = ar^{3-1} \quad | \quad t_7 = ar^{7-1}$
 $t_3 = ar^2 \quad | \quad t_7 = ar^6$
 $ar^2 = \frac{1}{9} \quad | \quad ar^6 = 9$
 $\frac{ar^6}{ar^2} = \frac{9}{\frac{1}{9}}$
 $r^4 = 81$
 $r = \pm 3$
 $a(3)^2 = \frac{1}{9}$
 $9a = \frac{1}{9}$
 $a = \frac{1}{81}$

$\frac{1}{81}, \frac{1}{27}, \frac{1}{9}, \frac{1}{3}$
 $t_4 = \pm \frac{1}{3}$

ⓐ b) 16, -8, 4, ..., $\frac{1}{4}$

$n = ?$
 $a = 16$
 $r = \frac{-8}{16} = -\frac{1}{2}$
 $t_n = \frac{1}{4}$
 $t_n = ar^{n-1}$
 $\frac{1}{4} = (16) \left(-\frac{1}{2}\right)^{n-1}$
 $\frac{1}{64} = \left(-\frac{1}{2}\right)^{n-1}$
 $\left(-\frac{1}{2}\right)^6 = \left(-\frac{1}{2}\right)^{n-1}$ ← Get common base.
 $\frac{\log(\frac{1}{64})}{\log(\frac{1}{2})} = 6$
 $6 = n-1$
 $7 = n$

ⓐ $t_{10} = 2560 \quad | \quad t_5 = 80$
 $t_{10} = ar^{10-1} \quad | \quad t_5 = ar^{5-1}$
 $t_{10} = ar^9 \quad | \quad t_5 = ar^4$
 $2560 = ar^9 \quad | \quad 80 = ar^4$
 $ar^9 = 2560 \quad | \quad ar^4 = 80$
 $\frac{ar^9}{ar^4} = \frac{2560}{80}$
 $r^5 = 32$
 $r = 2$
 $a(2)^4 = 80$
 $16a = 80$
 $a = 5$

$t_{10} = (5)(2)^{10-1}$
 $t_{10} = (5)(2)^9$
 $t_{10} = (5)(512) = 2560$
 $t_{10} = (5)(2048) = 10240$

Questions from Homework

① a) $3, 6, 12, \dots$ geometric $r = \frac{6}{3} = \frac{12}{6} = 2$

c) $-5, -7, -9, \dots$ arithmetic $d = -7 - (-5) = -2$

② c) $(-\frac{1}{6}), \frac{1}{3}, -\frac{2}{3}, \dots$

Given:

$a = -\frac{1}{6}$

$r = \frac{1}{3} \div -\frac{1}{6}$

$= \frac{1}{3} \times -\frac{6}{1}$

$= -\frac{6}{3}$

$= -2$

$t_n = ar^{n-1}$

$t_7 = (-\frac{1}{6})(-2)^{7-1}$

$t_7 = (-\frac{1}{6})(-2)^6$

$t_7 = (-\frac{1}{6})(64)$

$t_7 = \frac{-64}{6} = -\frac{32}{3}$

③ c) $(\frac{p^3}{q}), \frac{p^3}{2q}, \frac{p^4}{4q}$

$a = \frac{p^3}{q}$

$r = \frac{p^3}{2q} \div \frac{p^3}{q}$

$= \frac{p^3}{2q} \cdot \frac{q}{p^3}$

$= \frac{p^3 q}{2 p^3 q} = \frac{1}{2}$

$t_n = ar^{n-1}$

$t_{10} = (\frac{p^3}{q})(\frac{1}{2})^{10-1}$

$t_{10} = (\frac{p^3}{q})(\frac{1}{2})^9$

$t_{10} = (\frac{p^3}{q})(\frac{1}{512}) = \frac{p^3}{512q}$

④ b) $t_5 = 8$

$t_n = ar^{n-1}$

$t_5 = ar^{5-1}$

$8 = ar^4$

$t_0 = \frac{1}{4}$

$t_n = ar^{n-1}$

$t_{10} = ar^{10-1}$

$\frac{1}{2} = ar^9$

$\frac{1}{4} = ar^9$

Elimination

$\frac{1}{4} = ar^9$

$8 = ar^4$

$8 = a(\frac{1}{2})^4$

$8 = \frac{1}{16} a^{16}$

$128 = a$

$t_3 = (128)(\frac{1}{2})^{3-1}$

$= 128(\frac{1}{2})^2$

$= 128(\frac{1}{4})$

$= 32$

④ b) $16, 8, 4, \dots, \frac{1}{4}$ (Solving for n)

$a = 16$

$r = -\frac{1}{2}$

$t_n = \frac{1}{4}$

$t_n = ar^{n-1}$ $\frac{1}{4} \times \frac{1}{16}$

$\frac{1}{4} = (16)(-\frac{1}{2})^{n-1}$

$\frac{1}{64} = (-\frac{1}{2})^{n-1}$

$(-\frac{1}{2})^6 = (-\frac{1}{2})^{n-1}$

$6 = n - 1$

$7 = n$

$\frac{\log(\frac{1}{64})}{\log(-\frac{1}{2})} = 6$

Questions from Homework

$$\begin{array}{l|l} \textcircled{6} & \\ t_{10} = \underline{\underline{2560}} & t_5 = \underline{80} \\ t_n = ar^{n-1} & t_n = ar^{n-1} \\ t_{10} = ar^9 & t_5 = ar^4 \\ \underline{\underline{2560}} = ar^9 & \underline{\underline{80}} = ar^4 \\ ar^9 = 2560 & ar^4 = 80 \end{array}$$

Elimination
by division

$$\frac{ar^9 = 2560}{ar^4 = 80}$$

$$r^5 = 32$$

$$r = (32)^{\frac{1}{5}} = 2$$

$$ar = 2$$

$$r = \sqrt[5]{32} = 2$$

$$ar^4 = 80$$

$$a(2)^4 = 80$$

$$\frac{16a = 80}{16 \quad 16}$$

$$a = 5$$

$$t_n = (5)(2)^{n-1}$$

$$t_{12} = (5)(2)^{12-1}$$

$$t_{12} = (5)(2)^{11}$$

$$t_{12} = (5)(2048)$$

$$t_{12} = 10240$$

Arithmetic Series

Series: The sum of the terms of a sequence. The sum is usually finite: $1+2+3+4+5$. However it could be infinite: $2+4+8+16+\dots$. You can find the sum of many finite series and certain types of infinite series by using formulas.

$$S_n = \frac{n}{2}(2a + (n-1)d)$$

$$S_n = \frac{n}{2}(a + t_n)$$

Ex: $2+5+8+11+\underline{14}$

$$t_5 = \underline{14}$$

$$S_5 = 40$$



Sum of the first five terms

$$t_4 = 11$$

$$S_4 = 26$$

:

Find the sum of the first 100 terms of the arithmetic series $1+4+7+10+\dots$

$$a = 1 \quad S_n = \frac{n}{2}(2a + (n-1)d)$$

$$d = t_2 - t_1 = 3$$

$$n = 100$$

$$S_{100} = \frac{100}{2} [2(1) + (100-1)3]$$

$$S_{100} = 50(2 + 297)$$

$$S_{100} = 50(299)$$

$$S_{100} = 14950$$

Find the sum of the following series

$$\left(\frac{1}{2}\right) + 1 + \frac{3}{2} + 2, \dots + \underline{\underline{20}} \quad d = 1 - \frac{1}{2} = \frac{2}{2} - \frac{1}{2} = \frac{1}{2}$$

Hint: How many terms are there?

$$\begin{aligned} a &= \frac{1}{2} \\ d &= \frac{1}{2} \\ t_n &= 20 \\ n &= \end{aligned}$$

$$t_n = a + (n-1)d$$

$$20 = \frac{1}{2} + (n-1)\left(\frac{1}{2}\right)$$

$$20 = \frac{1}{2} + \frac{n}{2} - \frac{1}{2}$$

$$2 \cdot 20 = \frac{n}{2} \cdot 2$$

$$40 = n$$

$$S_{40} = \frac{40}{2} \left(\frac{1}{2} + \frac{20}{1} \right)$$

$$S_{40} = 20 \left(\frac{1}{2} + \frac{40}{2} \right)$$

$$S_{40} = \left(\frac{20}{1}\right) \left(\frac{41}{2}\right) = 410$$

$$S_{40} = \frac{820}{2}$$

$$S_{40} = 410$$

$$20 = \frac{1}{2} + (n-1)\left(\frac{1}{2}\right)$$

$$\cancel{\frac{2}{1}} \cdot \frac{39}{\cancel{2}} = (n-1) \left(\cancel{\frac{1}{2}} \right) \div \cancel{\frac{1}{2}}$$

$$39 = n-1$$

$$40 = n$$

(Solve for n)
 How many terms are in the series:
 $3+8+13+\dots+248$ if its sum is 6275?

$\underbrace{\quad}_5 \quad \underbrace{\quad}_5$

$$a = 3$$

$$d = 5$$

$$S_n = 6275$$

$$t_n = 248$$

$$S_n = \frac{n}{2}(a + t_n)$$

$$6275 = \frac{n}{2}(3 + 248)$$

$$\cancel{2} \cdot 6275 = \cancel{2} \cdot \frac{n}{\cancel{2}}(251)$$

$$\frac{12550}{251} = \frac{251n}{251}$$

$$50 = n$$

Find the indicated sums of the following series:

$$S_{15} \text{ of } 2+6+10\dots \quad S_{15} = \frac{15}{2} [2(2) + (15-1)(4)]$$

$$n=15$$

$$a=2$$

$$d=4$$

$$= \frac{15}{2} [4 + 14(4)]$$

$$= \frac{15}{2} [4 + 56]$$

$$= \frac{15}{2} (60)$$

$$= 450$$

$$S_{20} \text{ of } \underline{-15-10-5+\dots}$$

$$-10 - (-15) = 5$$

$$-5 - (-10) = 5$$

$$n=20$$

$$a=-15$$

$$d=5$$

$$S_{20} = \frac{20}{2} [2(-15) + (20-1)(5)]$$

$$= 10 [-30 + 19(5)]$$

$$= 10 [-30 + 95]$$

$$= 10 (65)$$

$$= 650$$

Homework

#1-8

$$\begin{array}{l}
 \textcircled{7} \quad t_3 = \underline{-1} \\
 t_3 = a + (3-1)d \\
 \underline{t_3} = a + d \\
 -1 = a + d \\
 a + d = -1
 \end{array}
 \quad
 \left.
 \begin{array}{l}
 t_{10} = \underline{19} \\
 t_1 = a + (10-1)d \\
 \underline{t_1} = a + 11d \\
 19 = a + 11d \\
 a + 11d = 19
 \end{array}
 \right\}
 \begin{array}{l}
 a + 11d = 19 \\
 \Leftrightarrow a + d = -1 \\
 \hline
 10d = 20 \\
 \boxed{d = 2}
 \end{array}
 \quad
 \begin{array}{l}
 a + d = -1 \\
 a + \textcircled{2} = -1 \\
 \boxed{a = -3}
 \end{array}$$

