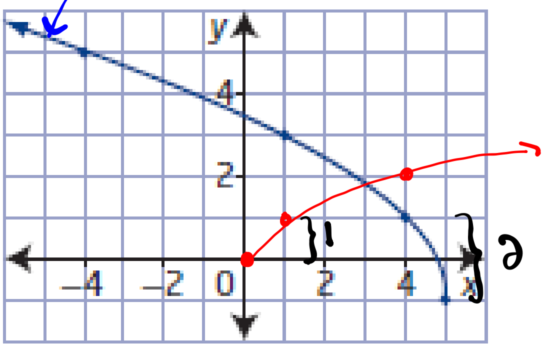
(10) Write equation in the form $y = a\sqrt{b(x-h)} + k$

transformed

c)



$(x, y) \rightarrow (-x+5, 2y-1)$

x	y
0	0
1	1
4	2
9	3

x	y
5	-1
4	1
1	3
-4	5

(1) Reflections: horizontal reflection in y-axis $(b<0)$

(2) VSF $= \frac{2}{1} = 2$ $(a=2)$

(3) HSF $= 1$ $(b = \frac{1}{1} = -1)$

(4) HT: $(0,0) \rightarrow (5,-1)$ 5 units right $(h=5)$

(5) VT: $(0,0) \rightarrow (5,-1)$ 1 unit down $(k=-1)$

(6) $y = a\sqrt{b(x-h)} + k$

$y = 2\sqrt{-1(x-5)} + (-1)$

$\boxed{y = 2\sqrt{-(x-5)} - 1}$ or $y = \sqrt{-4(x-5)} - 1$

Square Root of a Function

Focus on...

- sketching the graph of $y = \sqrt{f(x)}$ given the graph of $y = f(x)$
- explaining strategies for graphing $y = \sqrt{f(x)}$ given the graph of $y = f(x)$
- comparing the domains and ranges of the functions $y = f(x)$ and $y = \sqrt{f(x)}$, and explaining any differences

square root of a function

- the function $y = \sqrt{f(x)}$ is the square root of the function $y = f(x)$
- $y = \sqrt{f(x)}$ is only defined for $f(x) \geq 0$

The function $y = \sqrt{2x + 1}$ represents the **square root of the function** $y = 2x + 1$.

x	$y = 2x + 1$	$y = \sqrt{2x + 1}$
0	1	$\sqrt{1}$ = 1
4	9	$\sqrt{9}$ = 3
12	25	$\sqrt{25}$ = 5
24	49	$\sqrt{49}$ = 7
⋮	⋮	⋮

when $x = 0$

$y = 2x + 1$

$y = 2(0) + 1$

$y = 0 + 1$

$y = 1$

Example 1

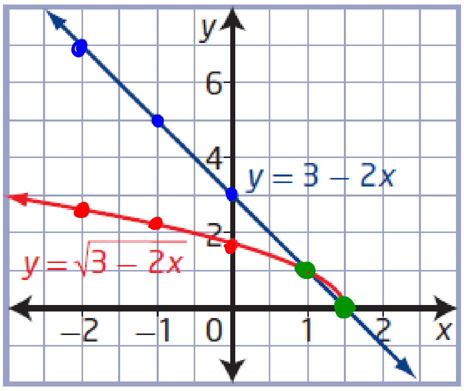
Compare Graphs of a Linear Function and the Square Root of the Function

linear

a) Given $f(x) = 3 - 2x$, graph the functions $y = f(x)$ and $y = \sqrt{f(x)}$.
b) Compare the two functions.

Use a table of values to graph $y = 3 - 2x$ and $y = \sqrt{3 - 2x}$.

x	$y = 3 - 2x$	$y = \sqrt{3 - 2x}$
−2	7	$\sqrt{7} = 2.7$
−1	5	$\sqrt{5} = 2.2$
0	3	$\sqrt{3} = 1.7$
1	1	$\sqrt{1} = 1$
1.5	0	$\sqrt{0} = 0$



Invariant points are
(1, 1) and (1.5, 0)

$y = 3 - 2x$:

D: $\{x \mid x \in R\}$ or $(-\infty, \infty)$

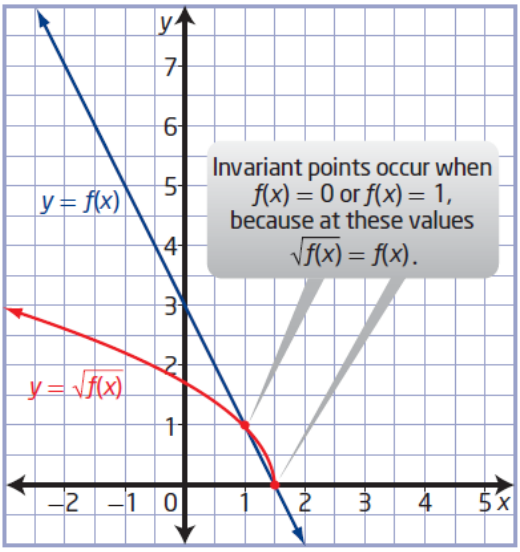
R: $\{y \mid y \in R\}$ or $(-\infty, \infty)$

$y = \sqrt{3 - 2x}$

D: $\{x \mid x \leq 1.5, x \in R\}$ or $(-\infty, 1.5]$

R: $\{y \mid y \geq 0, y \in R\}$ or $[0, \infty)$

b) Compare the graphs.



Why is the graph of $y = \sqrt{f(x)}$ above the graph of $y = f(x)$ for values of y between 0 and 1? Will this always be true?

For $y = f(x)$, the domain is $\{x \mid x \in \mathrm{R}\}$ and the range is $\{y \mid y \in \mathrm{R}\}$.
For $y = \sqrt{f(x)}$, the domain is $\{x \mid x \leq 1.5,\ x \in \mathrm{R}\}$ and the range is $\{y \mid y \geq 0,\ y \in \mathrm{R}\}$.

Invariant points occur at (1, 1) and (1.5, 0).

How does the domain of the graph of $y = \sqrt{f(x)}$ relate to the restrictions on the variable in the radicand? How could you determine the domain algebraically?

Relative Locations of $y = f(x)$ and $y = \sqrt{f(x)}$

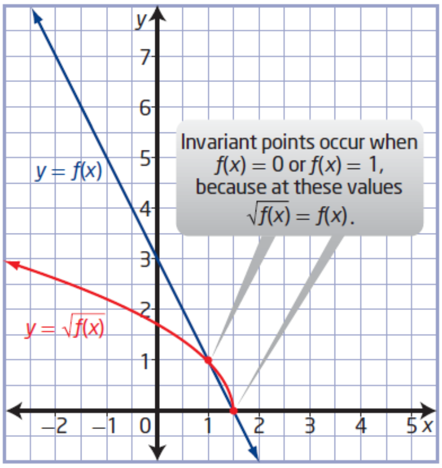
The domain of $y = \sqrt{f(x)}$ consists only of the values in the domain of $f(x)$ for which $f(x) \geq 0$.

The range of $y = \sqrt{f(x)}$ consists of the square roots of the values in the range of $y = f(x)$ for which $\sqrt{f(x)}$ is defined.

The graph of $y = \sqrt{f(x)}$ exists only where $f(x) \geq 0$. You can predict the location of $y = \sqrt{f(x)}$ relative to $y = f(x)$ using the values of $f(x)$.

	undefined	I.P.	Above f(x)	I.P.	Below f(x)
Value of $f(x)$	$f(x) < 0$	$f(x) = 0$	$0 < f(x) < 1$	$f(x) = 1$	$f(x) > 1$
Relative Location of Graph of $y = \sqrt{f(x)}$	The graph of $y = \sqrt{f(x)}$ is undefined.	The graphs of $y = \sqrt{f(x)}$ and $y = f(x)$ intersect on the x-axis.	The graph of $y = \sqrt{f(x)}$ is above the graph of $y = f(x)$.	The graph of $y = \sqrt{f(x)}$ intersects the graph of $y = f(x)$.	The graph of $y = \sqrt{f(x)}$ is below the graph of $y = f(x)$.

b) Compare the graphs.



Why is the graph of $y = \sqrt{f(x)}$ above the graph of $y = f(x)$ for values of y between 0 and 1? Will this always be true?

Your Turn

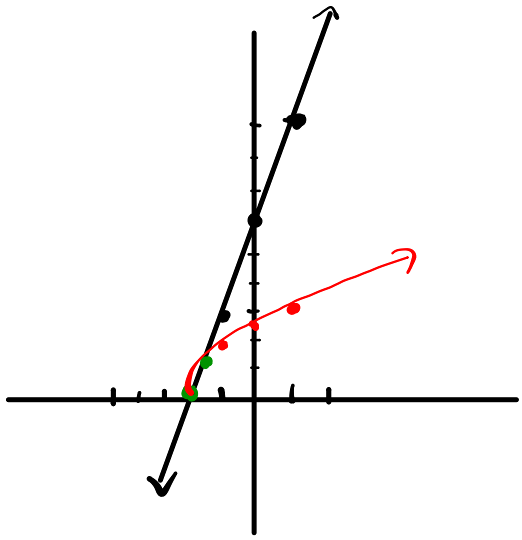
a) Given $g(x) = 3x + 6$, graph the functions $y = g(x)$ and $y = \sqrt{g(x)}$.
b) Identify the domain and range of each function and any invariant points.

$y = 3x+6$

x	y
-2	0
-1	3
0	6
1	9
2	12

$y = \sqrt{3x+6}$

x	y
-2	0
-1	1.7
0	2.4
1	3
2	3.5



D: $\{x \mid x \in R\}$
R: $\{y \mid y \in R\}$

D: $\{x \mid x \geq -2, x \in R\}$
R: $\{y \mid y \geq 0, y \in R\}$

Invariant Points: $(-2, 0)$ + $\left(-\frac{5}{3}, 1\right)$

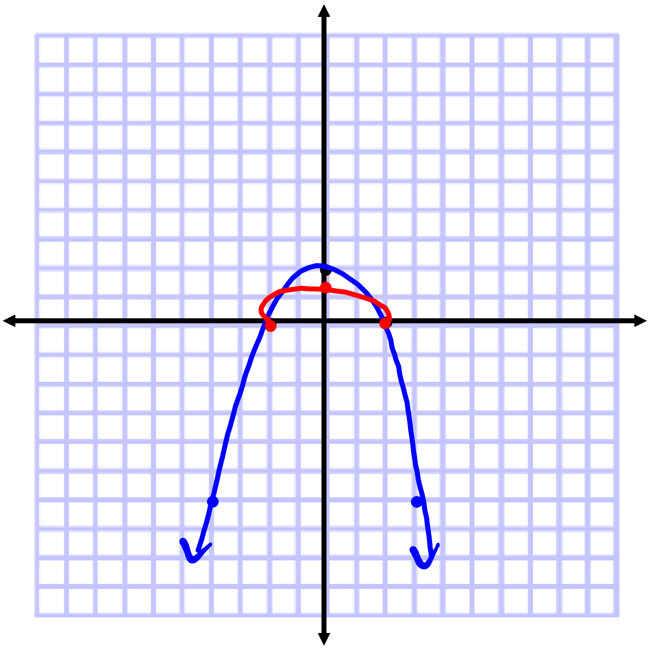
$3x+6=1$
$3x=-5$
$x=\frac{-5}{3}$

Example 2

Compare the Domains and Ranges of $y = f(x)$ and $y = \sqrt{f(x)}$

Identify and compare the domains and ranges of the functions in each pair.

a) $y = 2 - 0.5x^2$ and $y = \sqrt{2 - 0.5x^2}$



$y = 2 - 0.5x^2$

x	y
-4	-6
-2	0
0	2
2	0
4	-6

D: $\{x \mid x \in R\}$

R: $\{y \mid y \leq 2, y \in R\}$

$y = \sqrt{2 - 0.5x^2}$

x	y
-4	und.
-2	0
0	1.41
2	0
4	und.

D: $\{x \mid -2 \leq x \leq 2, x \in R\}$

R: $\{y \mid 0 \leq y \leq 1.41, y \in R\}$

Example 3

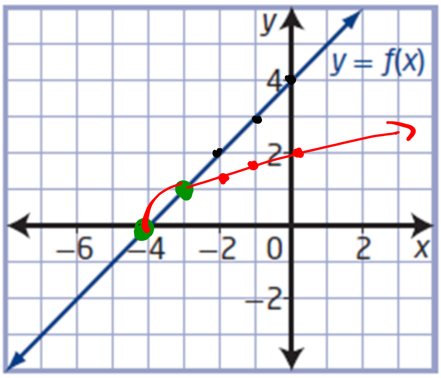
Graph the Square Root of a Function From the Graph of the Function

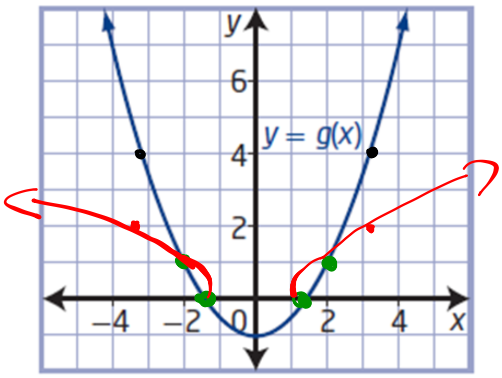
Step 1: Locate invariant points

Step 2: Draw the portion of each graph between the invariant points ($\sqrt{f(x)}$ above $f(x)$)

Step 3: Locate other key points on $y = f(x)$ and $y = g(x)$ where the values are greater than 1. Transform these points to locate image points on the graphs of $y = \sqrt{f(x)}$ and $y = \sqrt{g(x)}$. ($\sqrt{f(x)}$ below $f(x)$)

Using the graphs of $y = f(x)$ and $y = g(x)$, sketch the graphs of $y = \sqrt{f(x)}$ and $y = \sqrt{g(x)}$.





$y = f(x)$

D: $\{x \mid x \in R\}$ or $(-\infty, \infty)$

R: $\{y \mid y \in R\}$ or $(-\infty, \infty)$

$y = \sqrt{f(x)}$

D: $\{x \mid x \geq -4, x \in R\}$ or $[-4, \infty)$

R: $\{y \mid y \geq 0, y \in R\}$ or $[0, \infty)$

$y = g(x)$

D: $\{x \mid x \in R\}$ or $(-\infty, \infty)$

R: $\{y \mid y \geq -1, y \in R\}$ or $[-1, \infty)$

$y = \sqrt{g(x)}$

D: $\{x \mid x \leq -1.5, x \geq 1.5, x \in R\}$

or $(-\infty, -1.5] + [1.5, \infty)$

R: $\{y \mid y \geq 0, y \in R\}$ or $[0, \infty)$

Key Ideas

- You can use values of $f(x)$ to predict values of $\sqrt{f(x)}$ and to sketch the graph of $y = \sqrt{f(x)}$.
- The key values to consider are $f(x) = 0$ and $f(x) = 1$.
- The domain of $y = \sqrt{f(x)}$ consists of all values in the domain of $f(x)$ for which $f(x) \geq 0$.
- The range of $y = \sqrt{f(x)}$ consists of the square roots of all values in the range of $f(x)$ for which $f(x)$ is defined.
- The y-coordinates of the points on the graph of $y = \sqrt{f(x)}$ are the square roots of the y-coordinates of the corresponding points on the original function $y = f(x)$.

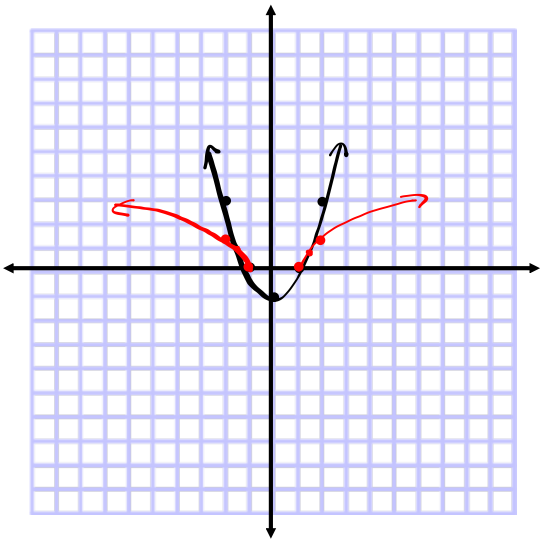
What do you know about the graph of $y = \sqrt{f(x)}$ at $f(x) = 0$ and $f(x) = 1$? How do the graphs of $y = f(x)$ and $y = \sqrt{f(x)}$ compare on either side of these locations?

Your Turn

1) Identify and compare the domains and ranges of the functions $y = x^2 - 1$ and $y = \sqrt{x^2 - 1}$. Verify your answers.

$y = x^2 - 1$

x	y
-2	3
-1	0
0	-1
1	0
2	3



$y = \sqrt{x^2 - 1}$

x	y
-2	$\sqrt{3} = 1.7$
-1	$\sqrt{0} = 0$
0	$\sqrt{-1}$ = undefined
1	$\sqrt{0} = 0$
2	$\sqrt{3} = 1.71$

D: $\{x \mid x \in R\}$ or $(-\infty, \infty)$

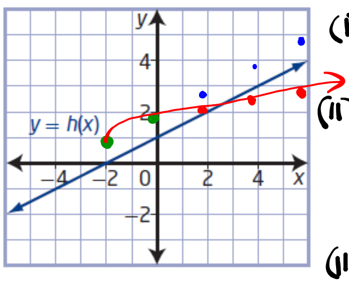
R: $\{y \mid y \geq -1, y \in R\}$ or $[-1, \infty)$

D: $\{x \mid x \leq -1$ and $x \geq 1, x \in R\}$

$(-\infty, -1]$ and $[1, \infty)$

R: $\{y \mid y \geq 0, y \in R\}$ or $[0, \infty)$

2) Using the graph of $y = h(x)$, sketch the graph of $y = \sqrt{h(x)}$.



(i) Locate invariant points $(-2, 0) + (0, 1)$

(ii) draw the portion of square root curve between invariant points. (above the original)

(iii) Locate key points on original and transform them (take square root of y-values)

$y = h(x)$

D: $\{x \mid x \in R\}$ or $(-\infty, \infty)$

R: $\{y \mid y \in R\}$ or $(-\infty, \infty)$

$y = \sqrt{h(x)}$

D: $\{x \mid x \geq -2, x \in R\}$ or $[-2, \infty)$

R: $\{y \mid y \geq 0, y \in R\}$ or $[0, \infty)$

Page 86-87
#1-6