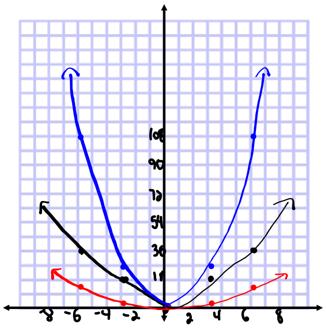
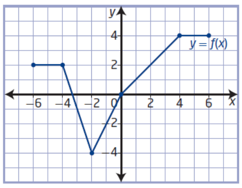
Questions from Homework

2. a) Copy and complete the table of values for the given functions.

x	$f(x) = x^2$	$g(x) = 3f(x)$	$h(x) = \frac{1}{3}f(x)$
−6	36	108	12
−3	9	27	3
0	0	0	0
3	9	27	3
6	36	108	12



6. The graph of the function $y = f(x)$ is vertically stretched about the x-axis by a factor of 2. $a = 2$



$(x,y) \rightarrow (x, 2y)$

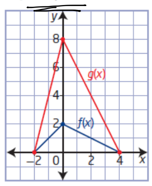
$f(x)$ — $g(x)$

D: $[-6,6]$ — D: $[-6,6]$

R: $[-4,4]$ — R: $[-8,8]$

7. Describe the transformation that must be applied to the graph of $f(x)$ to obtain the graph of $g(x)$. Then, determine the equation of $g(x)$ in the form $y = af(bx)$.

a)



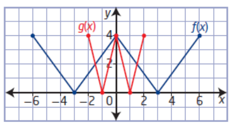
$(x,y) \rightarrow (x, 4y)$ A vertical stretch by a factor of 4

$f(x)$	$g(x)$
$(-2,0)$	$(-2,0)$
$(0,2)$	$(0,8)$
$(4,0)$	$(4,0)$

$a = 4$

$\boxed{y = 4f(x)}$

c)



$(x,y) \rightarrow (\frac{1}{3}x, y)$ A horizontal compression by a factor of $\frac{1}{3}$

$f(x)$	$g(x)$
$(-6,4)$	$(-2,4)$
$(-3,0)$	$(-1,0)$
$(0,4)$	$(0,4)$
$(3,0)$	$(1,0)$
$(6,4)$	$(2,4)$

$b = 3$

$\boxed{y = f(3x)}$

5 a) $y = 4f(x)$

$a = 4 \rightarrow$ A vertical stretch about the x-axis by a factor of 4

$(x,y) \rightarrow (x, 4y)$

b) $y = f(3x)$

$b = 3 \rightarrow$ A horizontal compression about the y-axis by a factor $\frac{1}{3}$

$(x,y) \rightarrow (\frac{1}{3}x, y)$

Warm-Up...

$y = af[b(x-h)]+k$

Given that (-2 , 5) is a point on the graph of $y = f(x)$, determine the coordinates of this point once the following transformations are applied...

(1) $y = 3f(x)$

$a=3 \rightarrow$ vertically stretched about the x-axis by a factor of 3

$b=1 \rightarrow$ no horizontal stretch.

$h=0 \rightarrow$ no horizontal trans

$k=0 \rightarrow$ no vertical trans.

$(x,y) \rightarrow (x, 3y)$

$(-2,5) \rightarrow \boxed{(-2,15)}$

(2) $y = f\left(-\frac{1}{3}x\right)$

$a=1 \rightarrow$ no vertical stretch

$b=-\frac{1}{3} \rightarrow$ horizontally stretched about the y-axis by a factor 3 and a reflection in the y-axis

$h=0 \rightarrow$ no horizontal trans.

$k=0 \rightarrow$ no vertical trans.

$(x,y) \rightarrow (-3x, y)$

$(-2,5) \rightarrow \boxed{(6,5)}$

(3) $y = 4f\left[\frac{1}{2}(x+5)\right] - 3$

$a=4 \rightarrow$ vertically stretched about the x-axis by a factor of 4

$b=\frac{1}{2} \rightarrow$ horizontally stretched about the y-axis by a factor of 2.

$h=-5 \rightarrow$ horizontally translated 5 units left

$k=-3 \rightarrow$ vertically translated 3 units down

$(x,y) \rightarrow (2x-5, 4y-3)$

$(-2,5) \rightarrow \boxed{(-9,17)}$

(4) $y - 5 = -2f(-2x+6)$ factor

$y = -2f(-2x+6)+5$

$y = -2f[-2(x-3)]+5$

$a=-2 \rightarrow$ vertically stretched about the x-axis by a factor of 2 and reflected in the x-axis

$b=-2 \rightarrow$ horizontally stretched about the y-axis by a factor of $\frac{1}{2}$ and reflected in the y-axis

$h=3 \rightarrow$ horizontally trans 3 units right

$k=5 \rightarrow$ vertically trans. 5 units up

$(x,y) \rightarrow \left(-\frac{1}{2}x+3, -2y+5\right)$

$(-2,5) \rightarrow \boxed{(4,-5)}$

Transformations:

2. The function $y = f(x)$ is transformed to the function $g(x) = -3f(4x - 16) - 10$. Copy and complete the following statements by filling in the blanks.

   The function $f(x)$ is transformed to the function $g(x)$ by a horizontal stretch about the a by a factor of b. It is vertically stretched about the c by a factor of d. It is reflected in the e, and then translated f units to the right and g units down.

Factor

$g(x) = -3f(4x-16)-10$

$g(x) = \underline{-3}f[\underline{4}(x\underline{-4})]-\underline{10}$

a =   b =   h =   k =

a) ______

b) ______

c) ______

d) ______

e) ______

f) ______

g) ______

Summary of Transformations...

Transformations of the graphs of functions	
$f(x) + k$	shift $f(x)$ up k units
$f(x) - k$	shift $f(x)$ down k units
$f(x + h)$	shift $f(x)$ left h units
$f(x - h)$	shift $f(x)$ right h units
$f(-x)$	reflect $f(x)$ about the y-axis
$-f(x)$	reflect $f(x)$ about the x-axis
$af(x)$	When $0 < a < 1$ – vertical shrinking of $f(x)$ When $a > 1$ – vertical stretching of $f(x)$ **Multiply the y values by a**
$f(bx)$	When $0 < b < 1$ – horizontal stretching of $f(x)$ When $b > 1$ – horizontal shrinking of $f(x)$ **Divide the x values by b**

Transformations:

$$y = f(x) \longrightarrow y = af\left(b(x-h)\right)+k$$

Mapping Rule:

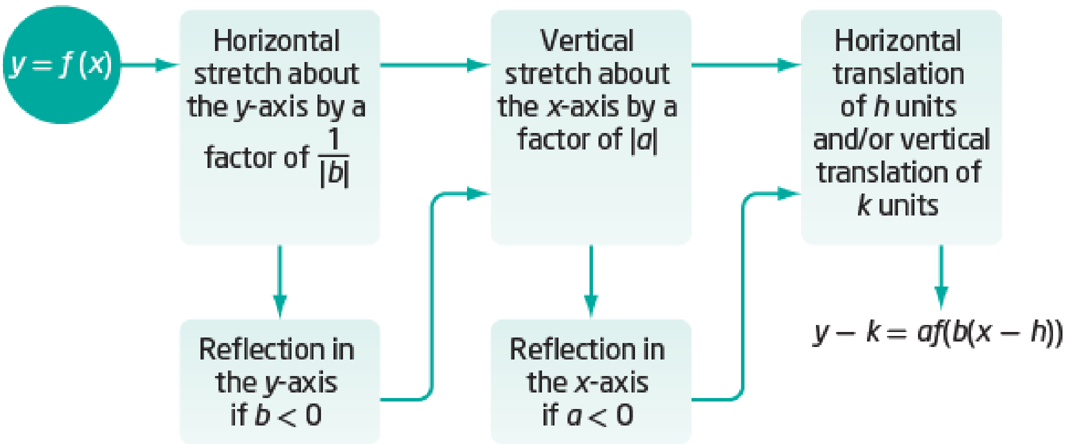
$$(x,y) \rightarrow \left(\frac{1}{b}x + h, ay + k\right)$$

Important note for sketching...

Transformations should be applied in following order:

1. Reflections
2. Stretches
3. Translations

Remember....RST



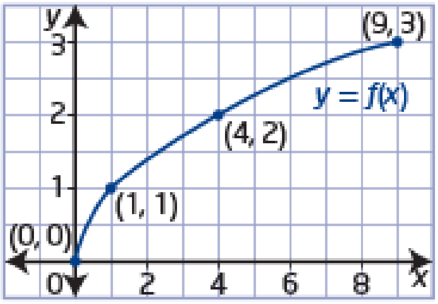
Example 1

Graph a Transformed Function

Describe the combination of transformations that must be applied to the function $y = f(x)$ to obtain the transformed function. Sketch the graph, showing each step of the transformation.

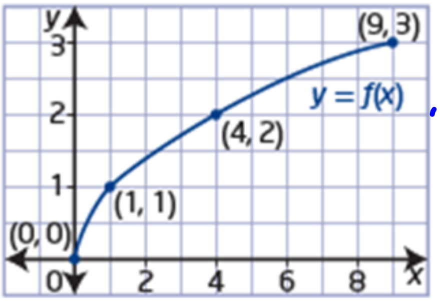
a) $y = 3f(2x)$

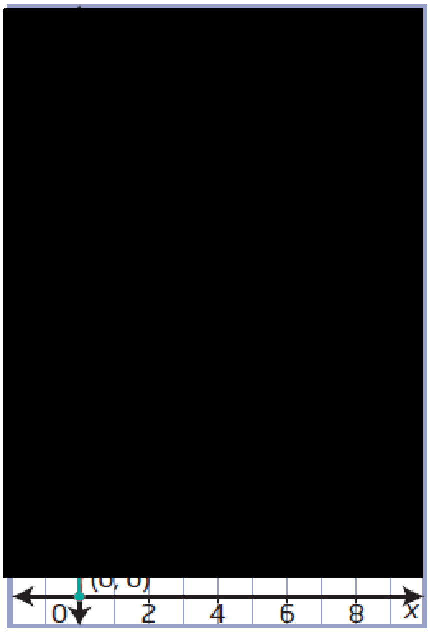
b) $y = f(3x + 6)$



a) $y = 3f(2x)$

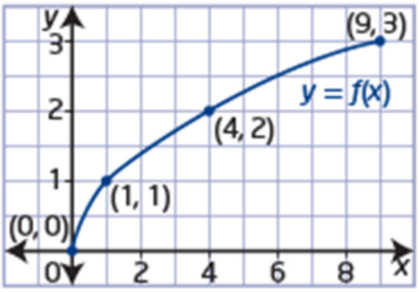
The graph of $y = f(x)$ is horizontally stretched about the y-axis by a factor of $\frac{1}{2}$ and then vertically stretched about the x-axis by a factor of 3.





b) $y = f(3x + 6)$

The graph of $y = f(x)$ is horizontally stretched about the y-axis by a factor of $\frac{1}{3}$ and then horizontally translated 2 units to the left.



[redacted]

Homework

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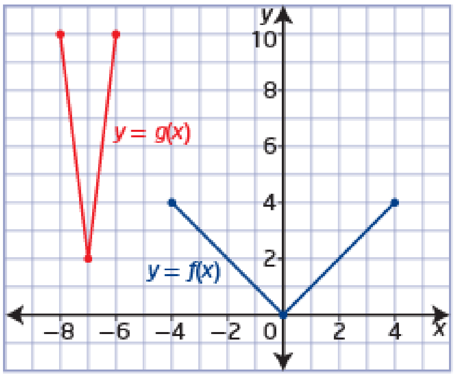
Example 3

Write the Equation of a Transformed Function Graph

The graph of the function $y = g(x)$ represents a transformation of the graph of $y = f(x)$. Determine the equation of $g(x)$ in the form $y = af(b(x - h)) + k$. Explain your answer.



Solution

Locate key points on the graph of $f(x)$ and their image points on the graph of $g(x)$.

$(-4, 4) \rightarrow (-8, 10)$
$(0, 0) \rightarrow (-7, 2)$
$(4, 4) \rightarrow (-6, 10)$

The equation of the transformed function is [redacted]

[redacted]

How could you use the mapping $(x, y) \rightarrow \left(\frac{1}{b}x + h, ay + k\right)$ to verify this equation?

17. The graph of the function $y = 2x^2 + x + 1$ is stretched vertically about the x-axis by a factor of 2, stretched horizontally about the y-axis by a factor of $\frac{1}{3}$, and translated 2 units to the right and 4 units down. Write the equation of the transformed function.

is stretched vertically about the x-axis by a factor of 2, stretched horizontally about the y-axis by a factor of $\frac{1}{3}$, and translated 2 units to the right and 4 units down. Write the equation of the transformed function.