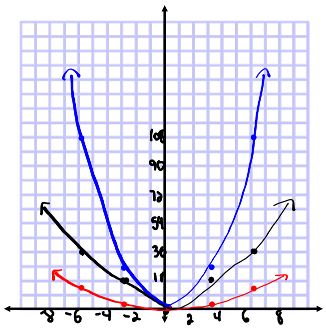
Questions from Homework

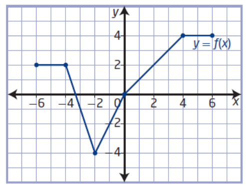
2. a) Copy and complete the table of values for the given functions.

x	$f(x) = x^2$	$g(x) = 3f(x)$	$h(x) = \frac{1}{3}f(x)$
−6	36	108	12
−3	9	27	3
0	0	0	0
3	9	27	3
6	36	108	12



6. The graph of the function $y = f(x)$ is vertically stretched about the x-axis by a factor of 2. $a = 2$

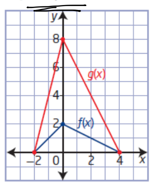
$(x,y) \rightarrow (x, 2y)$



$f(x)$	$g(x)$
D: $[-6,6]$	D: $[-6,6]$
R: $[-4,4]$	R: $[-8,8]$

7. Describe the transformation that must be applied to the graph of $f(x)$ to obtain the graph of $g(x)$. Then, determine the equation of $g(x)$ in the form $y = af(bx)$.

a)



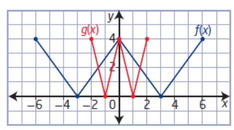
$(x,y) \rightarrow (x, 4y)$ A vertical stretch by a factor of 4

$a = 4$

$f(x)$	$g(x)$
$(-2,0)$	$(-2,0)$
$(0,2)$	$(0,8)$
$(4,0)$	$(4,0)$

$\boxed{y = 4f(x)}$

c)



$(x,y) \rightarrow (\frac{1}{3}x, y)$ A horizontal compression by a factor of $\frac{1}{3}$

$b = 3$

$f(x)$	$g(x)$
$(-6,4)$	$(-2,4)$
$(-3,0)$	$(-1,0)$
$(0,4)$	$(0,4)$
$(3,0)$	$(1,0)$
$(6,4)$	$(2,4)$

$\boxed{y = f(3x)}$

5 a) $y = 4f(x)$

$a = 4 \rightarrow$ A vertical stretch about the x-axis by a factor of 4

$(x,y) \rightarrow (x, 4y)$

b) $y = f(3x)$

$b = 3 \rightarrow$ A horizontal compression about the y-axis by a factor $\frac{1}{3}$

$(x,y) \rightarrow (\frac{1}{3}x, y)$

Warm-Up...

$y = af[b(x-h)]+k$

Given that (-2 , 5) is a point on the graph of $y = f(x)$, determine the coordinates of this point once the following transformations are applied...

(1) $y = 3f(x)$

$a=3 \rightarrow$ vertically stretched about the x-axis by a factor of 3

$b=1 \rightarrow$ no horizontal stretch.

$h=0 \rightarrow$ no horizontal trans

$k=0 \rightarrow$ no vertical trans.

$(x,y) \rightarrow (x, 3y)$

$(-2,5) \rightarrow \boxed{(-2,15)}$

(2) $y = f\left(-\frac{1}{3}x\right)$

$a=1 \rightarrow$ no vertical stretch

$b=-\frac{1}{3} \rightarrow$ horizontally stretched about the y-axis by a factor 3 and a reflection in the y-axis

$h=0 \rightarrow$ no horizontal trans.

$k=0 \rightarrow$ no vertical trans.

$(x,y) \rightarrow (-3x, y)$

$(-2,5) \rightarrow \boxed{(6,5)}$

(3) $y = 4f\left[\left(\frac{1}{2}\right)(x+5)\right]-3$

$a=4 \rightarrow$ vertically stretched about the x-axis by a factor of 4

$b=\frac{1}{2} \rightarrow$ horizontally stretched about the y-axis by a factor of 2.

$h=-5 \rightarrow$ horizontally translated 5 units left

$k=-3 \rightarrow$ vertically translated 3 units down

$(x,y) \rightarrow (2x-5, 4y-3)$

$(-2,5) \rightarrow \boxed{(-9,17)}$

(4) $y - 5 = -2f(-2x+6)$ factor

$y = -2f(-2x+6)+5$

$y = -2f[-2(x-3)]+5$

$a=-2 \rightarrow$ vertically stretched about the x-axis by a factor of 2 and reflected in the x-axis

$b=-2 \rightarrow$ horizontally stretched about the y-axis by a factor of $\frac{1}{2}$ and reflected in the y-axis

$h=3 \rightarrow$ horizontally trans 3 units right

$k=5 \rightarrow$ vertically trans. 5 units up

$(x,y) \rightarrow \left(-\frac{1}{2}x+3, -2y+5\right)$

$(-2,5) \rightarrow \boxed{(4,-5)}$

Transformations:

2. The function $y = f(x)$ is transformed to the function $g(x) = -3f(4x - 16) - 10$. Copy and complete the following statements by filling in the blanks.

The function $f(x)$ is transformed to the function $g(x)$ by a horizontal stretch about the **a** by a factor of **b**. It is vertically stretched about the **c** by a factor of **d**. It is reflected in the **e**, and then translated **f** units to the right and **g** units down.

a) y-axis
b) 1/4
c) x-axis
d) 3
e) x-axis
f) 4
g) 10

factor

$$g(x) = -3f(4x-16)-10$$

$$g(x) = -3f[4(x-4)]-10$$

$a=-3$ $b=4$ $h=4$ $k=-10$

Summary of Transformations...

Transformations of the graphs of functions			
$f(x)+k$	shift $f(x)$ up k units	vertical trans.	$(x,y)\to(x,y+k)$
$f(x)-k$	shift $f(x)$ down k units		$(x,y)\to(x,y-k)$
$f(x+h)$	shift $f(x)$ left h units	horizontal trans.	$(x,y)\to(x-h,y)$
$f(x-h)$	shift $f(x)$ right h units		$(x,y)\to(x+h,y)$
$f(-x)$	reflect $f(x)$ about the y-axis	horizontal ref.	$(x,y)\to(-x,y)$
$-f(x)$	reflect $f(x)$ about the x-axis	vertical ref	$(x,y)\to(x,-y)$
$af(x)$	When $0<a<1$ – vertical shrinking of $f(x)$ When $a>1$ – vertical stretching of $f(x)$ **Multiply the y values by a**		$(x,y)\to(x,ay)$
$f(bx)$	When $0<b<1$ – horizontal stretching of $f(x)$ When $b>1$ – horizontal shrinking of $f(x)$ **Divide the x values by b**	or multiply by $\frac{1}{b}$	$(x,y)\to(\frac{1}{b}x,y)$

Transformations:

$$y = f(x) \longrightarrow y = af\left(b(x-h)\right) + k$$

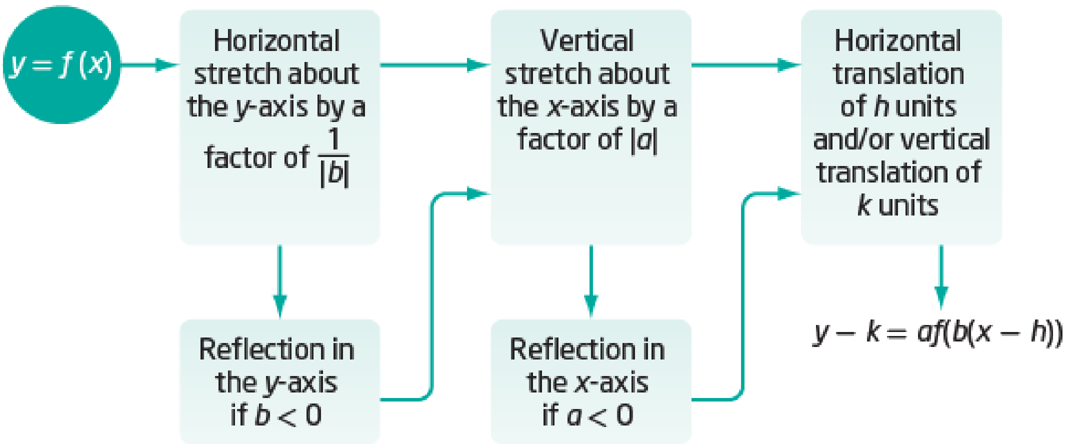
Mapping Rule: $\boxed{(x,y) \rightarrow \left(\frac{1}{b}x + h, ay + k\right)}$

Important note for sketching...

Transformations should be applied in following order:

1. Reflections
2. Stretches
3. Translations

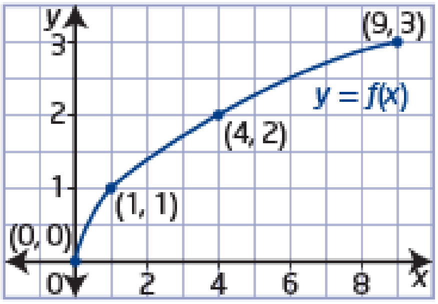
Remember....RST



Example 1

Graph a Transformed Function

Describe the combination of transformations that must be applied to the function $y = f(x)$ to obtain the transformed function. Sketch the graph, showing each step of the transformation.



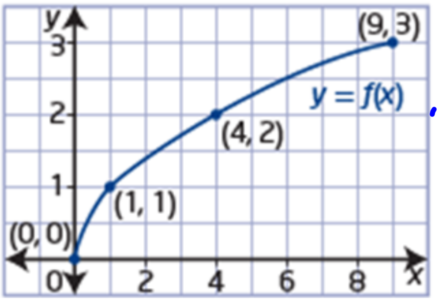
a) $y = 3f(2x)$

b) $y = f(3x + 6)$

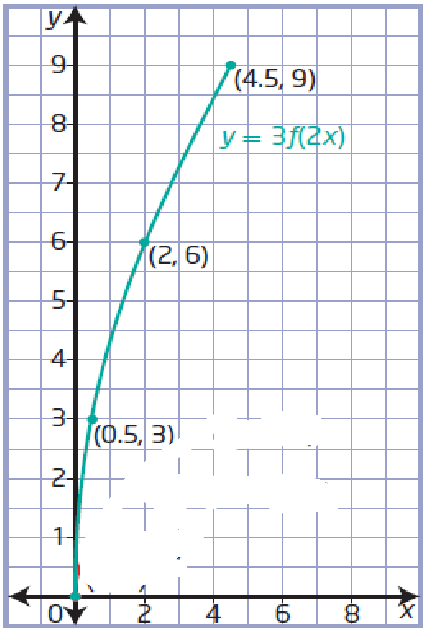
a) $y = 3f(2x)$ $a = 3$ $b = 2$ $h = 0$ $k = 0$

The graph of $y = f(x)$ is horizontally stretched about the y-axis by a factor of $\frac{1}{2}$ and then vertically stretched about the x-axis by a factor of 3.

$(x, y) \rightarrow \left[\frac{1}{2}x, 3y\right]$



$f(x)$	$g(x)$
$(0,0)$	$(0,0)$
$(1,1)$	$(\frac{1}{2}, 3)$
$(4,2)$	$(2,6)$
$(9,3)$	$(\frac{9}{2}, 9)$

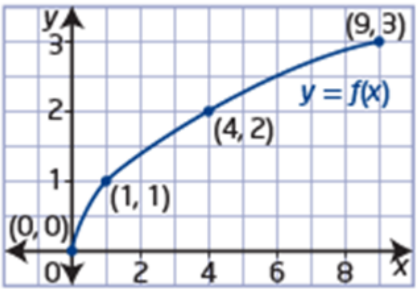


b) $y = f(3x + 6)$ $a = 1$ $b = 3$ $h = -2$ $k = 0$

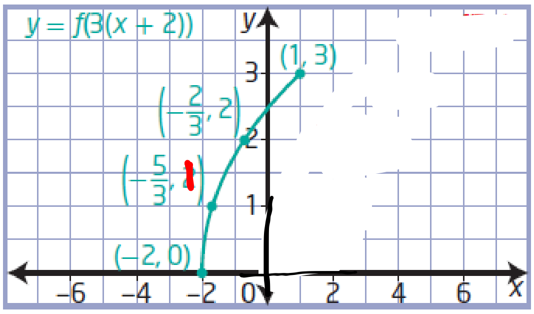
$y = f[3(x + 2)]$

The graph of $y = f(x)$ is horizontally stretched about the y-axis by a factor of $\frac{1}{3}$ and then horizontally translated 2 units to the left.

$(x, y) \rightarrow \left[\frac{1}{3}x - 2, y\right]$



$f(x)$	$g(x)$
(0,0)	(-2,0)
(1,1)	(-5/3,1)
(4,2)	(-2/3,2)
(9,3)	(1,3)



$\frac{1}{3}x - 2$ $\frac{1}{3}x - 2$

$\frac{1}{3}(1) - 2$ $\frac{1}{3}(4) - 2$

$\frac{1}{3} - \frac{2}{1}$ $\frac{4}{3} - \frac{2}{1}$

$\frac{1}{3} - \frac{6}{3}$ $\frac{4}{3} - \frac{6}{3}$

$-\frac{5}{3}$ $-\frac{2}{3}$

Questions From Homework

3. Copy and complete the table by describing the transformations of the given functions, compared to the function $y = f(x)$.

	Function	Reflections	Vertical Stretch Factor	Horizontal Stretch Factor	Vertical Translation	Horizontal Translation
(i)	$y - 4 = f(x - 5)$	–	–	–	4	5
(ii)	$y + 5 = 2f(3x)$	–	2	$\frac{1}{3}$	-5	-
(iii)	$y = \frac{1}{2}f\left(\frac{1}{2}(x - 4)\right)$	–	$\frac{1}{2}$	2	–	4
(iv)	$y + 2 = -3f(2(x + 2))$		3	$\frac{1}{2}$	-2	-2

vertical reflection in x-axis

(i) $y = f(x-5)+4$
$a=1 \quad b=1 \quad h=5 \quad k=4$

(ii) $y = 2f(3x)-5$
$a=2 \quad b=3 \quad h=0 \quad k=-5$

(iii) $y = \frac{1}{2}f\left(\frac{1}{2}(x-4)\right)$
$a=\frac{1}{2} \quad b=\frac{1}{2} \quad h=4 \quad k=0$

(iv) $y = -3f(2(x+2))-2$
$a=-3 \quad b=2 \quad h=-2 \quad k=-2$

6. The key point $(-12, 18)$ is on the graph of $y = f(x)$. What is its image point under each transformation of the graph of $f(x)$?

e) $y+3 = -\frac{1}{3}f[2(x+6)]$

$y = -\frac{1}{3}f[2(x+6)]-3$

$a=-\frac{1}{3} \quad b=2 \quad h=-6 \quad k=-3$

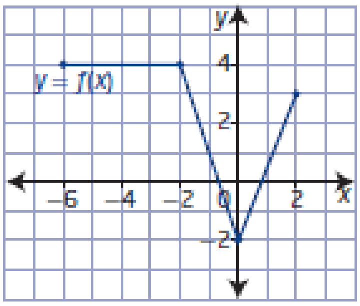
$(x,y) \rightarrow \left[\frac{1}{2}x-6,\ -\frac{1}{3}y-3\right]$

$(-12,18) \longrightarrow \boxed{[-12,-9]}$

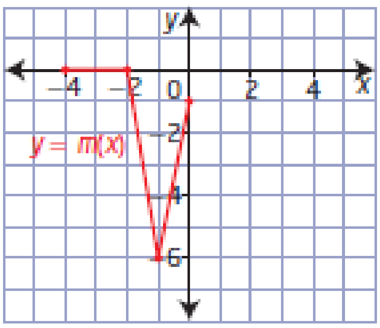
$\frac{1}{2}(-12)-6$
$= -6-6$
$= -12$

$-\frac{1}{3}(18)-3$
$= -6-3$
$= -9$

4. Using the graph of $y = f(x)$, write the equation of each transformed graph in the form $y = af(b(x - h)) + k$.



b)



$f(x)$	$m(x)$
$(-6,4)$	$(-4,0)$
$(-2,4)$	$(-2,0)$
$(0,-2)$	$(-1,-6)$
$(2,3)$	$(0,-1)$

$(x,y) \rightarrow \left(\frac{1}{2}x - 1,\ y - 4\right)$

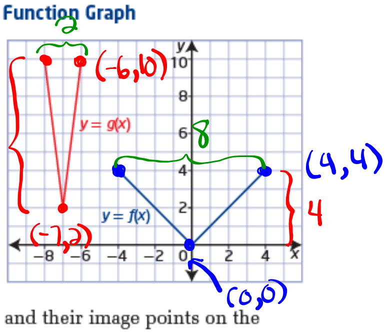
$a = 1 \quad b = 2 \quad h = -1 \quad k = -4$

$m(x) = 1f(2(x+1)) - 4$

Example 3

Write the Equation of a Transformed Function Graph

The graph of the function $y = g(x)$ represents a transformation of the graph of $y = f(x)$. Determine the equation of $g(x)$ in the form $y = af(b(x - h)) + k$. Explain your answer.



Solution

Locate key points on the graph of $f(x)$ and their image points on the graph of $g(x)$.

$(-4, 4) \rightarrow (-8, 10)$
$(0, 0) \rightarrow (-7, 2)$
$(4, 4) \rightarrow (-6, 10)$

The equation of the transformed function is $g(x) = 2f(4(x + 7)) + 2$.

① Reflections: none

② Vertical Stretch Factor: $VSF = \frac{8}{4} = 2$ $a = 2$

Range $\frac{\text{new}}{\text{old}}$

③ Horizontal Stretch Factor: $HSF = \frac{2}{8} = \frac{1}{4}$ $b = 4$

Domain $\frac{\text{new}}{\text{old}}$

④ Horizontal Translation: $(0,0) \rightarrow (-7,2)$ $h = -7$ (left 7)

Pick a point on the original where $x = 0$

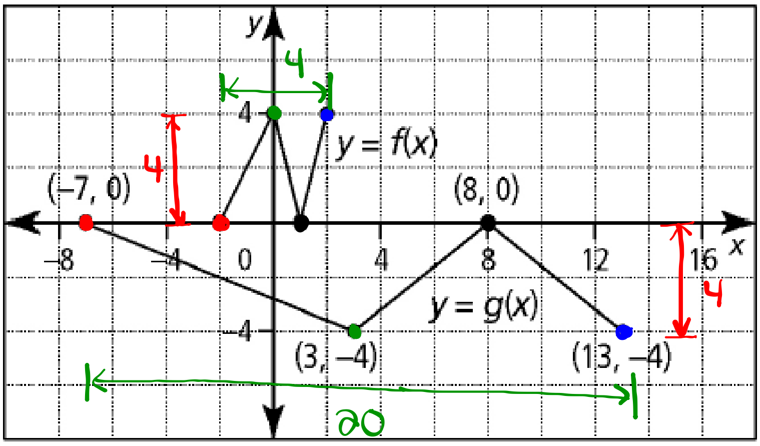
⑤ Vertical Translation: $(0,0) \rightarrow (-7,2)$ $k = 2$ (up 2)

Pick a point on the original where $y = 0$

⑥ Equation: $y = af[b(x-h)]+k$

$$\boxed{y = 2f[4(x+7)] + 2}$$

The graph of the function $y = g(x)$ represents a transformation of the graph of $y = f(x)$. Determine the equation of $g(x)$ in the form $y = af(b(x - h)) + k$.



① Reflections: vertical reflection in the x-axis $(a<0)$

② Vertical Stretch Factor: $VSF = \frac{4}{4} = 1$ $a = -1$
Range: $\left(\frac{new}{old}\right)$

③ Horizontal Stretch Factor: $HSF = \frac{20}{4} = 5$ $b = \frac{1}{5}$
Domain: $\left(\frac{new}{old}\right)$

④ Horizontal Translation $(0,4) \to (3,-4)$ $h = 3$
if possible find a point where $x=0$ on $f(x)$ (3 right)

⑤ Vertical Translation: $(-2,0) \to (-7,0)$ $k = 0$
if possible find a point where $y=0$ on $f(x)$

⑥ Equation: $y = af[b(x-h)]+k$

$y = -1f[\frac{1}{5}(x-3)] + 0$

$y = -f[\frac{1}{5}(x-3)]$

Homework

Page 38 # 3-6
Plus 7, 8, 9 (a, c, e) and 10

⑦ f) $3y - 6 = f(-2x+12)$

$\frac{3y}{3} = \frac{1}{3}f(-2x+12) + \frac{6}{3}$

when multiplying/dividing both sides of the equation only do it to a and k

Common Factor

$y = \frac{1}{3}f(-2x+12) + 2$

$y = \frac{1}{3}f[-2(x-6)] + 2$

$a = \frac{1}{3}$ $b = -2$ $h = 6$ $k = 2$

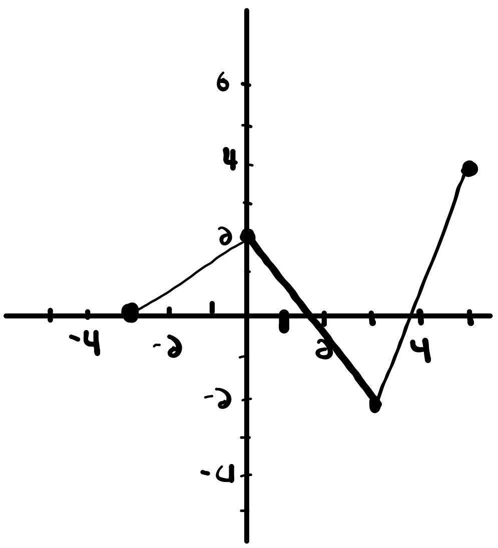
$(x,y) \rightarrow \left[-\frac{1}{2}x + 6, \frac{1}{3}y + 2\right]$

$a = \frac{1}{3} \rightarrow$ A vertical compression about the x-axis by a factor of $\frac{1}{3}$

$b = -2 \rightarrow$ A horizontal copression about the y-axis by a factor of $\frac{1}{2}$ and a horizontal reflection in the y-axis.

$h = 6 \rightarrow$ translated 6 units right

$k = 2 \rightarrow$ translated 2 units up.



Domain: $\{x \mid -3 \leq x \leq 5, x \in R\}$

or $[-3, 5]$

Range: $\{y \mid -2 \leq y \leq 4, y \in R\}$

or $[-2, 4]$

Page 39

⑥ 2) $y = -2f\left(-\frac{2}{3}x - 6\right) + 4$

$y = -2f\left(-\frac{2}{3}(x+9)\right) + 4$

$-6 \div -\frac{2}{3}$

$= -6 \times -\frac{3}{2}$

$= \frac{18}{2}$

$= 9$

$a = -2$ $b = -\frac{2}{3}$ $h = -9$ $k = 4$

$(x, y) \rightarrow \left[-\frac{3}{2}x - 9, -2y + 4\right]$