Questions from Homework

④ $x > 2$

① Find x-int $(y=0)$

$x = 2$

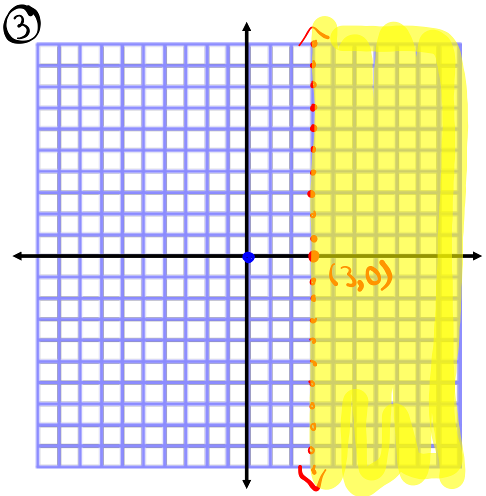
$(2,0)$

② y-int $(x=0)$

Skip

No y-intercept

③



④ $x > 2$

↑

Boundary is dashed

⑤ $x > 2$ Use $(0,0)$

$0 > 2$

False

Notes:

if $\left.\begin{array}{l} x < 3 \\ x > 3 \\ x \leq 3 \\ x \geq 3 \end{array}\right\}$ Vertical line $x = 3$
No y-intercept

if $\left.\begin{array}{l} y > 2 \\ y < 2 \\ y \geq 2 \\ y \leq 2 \end{array}\right\}$ Horizontal line $y = 2$
No x-intercept

5.1 Graphing Linear Inequalities in Two Variables

GOAL

Solve problems by modelling linear inequalities in two variables.

EXPLORE...

• For which inequalities is (3, 1) a possible solution? How do you know?

a) $13 - 3x > 4y$
b) $2y - 5 \le x$
c) $y + x < 10$
d) $y \ge 9$

a) $13-9>4$
$4>4$
False

b) $2-5 \le 3$
$-3 \le 3$
True

c) $1+3<10$
$4<10$
True

d) $1 \ge 9$
False

SAMPLE ANSWER

(3, 1) is a possible solution for parts b) and c). When (3, 1) is substituted for x and y into each inequality, they make a true statement.

Foundations of Math 11 **<u>Graphing Linear Inequalities</u>** **Sections 5.1-5.3**

A linear inequality is a relationship between two linear expressions in which one expression is less than(<), greater than(>), less than or equal to(≤), or greater than or equal to (≥) the other expression.

To graph linear inequalities in two variables (usually "x" and "y"), use the following steps.

<u>Preliminary Steps</u>:

- Determine whether the boundary line(s) will be solid or dashed.
  - ≥ or ≤ will represent **SOLID** lines. The boundary line(s) will be part of the solution.
  - > or < will represent **DASHED** lines. The boundary line(s) will not be part of the solution.

- Determine what number system the variables(x and y) represent. In other words, determine the Domain and Range. Usually the Domain and Range are given, however if they are not, or you cannot determine what they should be from context of the problem, assume $\{x \in R\}$ and $\{y \in R\}$.

  - Number Systems: ***R = REAL** Numbers (Negatives, Positives, Zero, Fractions/Decimals)
    ***W = WHOLE** Numbers (Zero and Positive Numbers Only)
    I = INTEGERS (Negative Numbers, Zero, and Positive Numbers)
    N = NATURAL Numbers (Positive Numbers Only)
    ***MOST COMMON**

  - If the number system you are given is **REAL**, you will use all **4 Quadrants** to draw your graph, and the ***SOLUTION SET*** will be ***<u>CONTINUOUS</u>***. In other words, all of the points in the colored region will be possible solutions to the problem.

  - If the number system you are given is **WHOLE**, you will only use the **first Quadrant** to draw your graph and the ***SOLUTION SET*** will be ***<u>DISCRETE</u>***. In other words, only the **WHOLE** numbers included in the colored region will be possible solutions to the problem. The solution region will need to be **STIPPLED** to show this. This means that you will place "dots" in the colored region only where the **WHOLE** numbers occur.

  - If the number system you are given involves **INTEGERS**, you will only use all **4 Quadrants** to draw your graph and the ***SOLUTION SET*** will be ***<u>DISCRETE</u>***. In other words, only the **INTEGER** values included in the colored region will be possible solutions to the problem. The solution region will need to be **STIPPLED** to show this. This means that you will place "dots" in the colored region only where the **INTEGER** numbers occur.

  - If the number system you are given is **NATURAL**, you will only use the **first Quadrant** to draw your graph and the ***SOLUTION SET*** will be ***<u>DISCRETE</u>***. In other words, only the **NATURAL** numbers included in the colored region will be possible solutions to the problem. The solution region will need to be **<u>STIPPLED</u>** to show this. This means that you will place "dots" in the colored region only where the **NATURAL** numbers occur.

STEP 1 – Determine the **<u>EQUATION OF THE BOUNDARY LINE(S).</u>**

- Replace each ≥ , ≤ , >, or < with an "=" sign.

STEP 2 – Determine the **"X" and "Y" INTERCEPT** on each boundary line.

- To determine the x-intercept, replace "y" with "0" and solve for "x".
- To determine the y-intercept, replace "x" with "0" and solve for "y".

STEP 3 – Use a **TEST POINT (0, 0)** to determine which side of the boundary line needs to be colored.

- Replace both "x" and "y" in the original inequality/inequalities with "0".
- If the inequality is "TRUE", color on the side of the boundary line(***HALF PLANE***) where (0, 0) is located.
- If the inequality is "FALSE", color on the side of the boundary line(***HALF PLANE***) where (0, 0) is not located.
- If a boundary line passes through the point (0, 0), you will need to use a different **TEST POINT**…try (0, 1). This will not happen very often ☺

STEP 4 – Complete your **GRAPH**.

- For each boundary line, use the "x" and "y" intercept that you found in **STEP 2** to create your line. Remember to check and see if the line should be solid or dashed.
- Using the information from **STEP 3**, color the side of the boundary line that satisfies each inequality. Use a different color each time.
- All possible solutions will be found in the colored region that you have created. If more than one color has been used, the ***SOLUTION REGION*** will be the location where all of the colored regions overlap. Remember to **STIPPLE** this area if the number system indicated in the Domain and Range is **W**, **I**, or **N**.

<u>QUICK GRAPHS</u>

✓ If the boundary line in **STEP 1** becomes **x = any number**, this will be a **VERTICAL line** that will pass through that number on the "x" axis. You can skip **STEP 2** and **STEP 3** as you should easily be able to tell what side of the boundary line needs to be colored from the inequality sign used.

✓ If the boundary line in **STEP 1** becomes **y = any number**, this will be a **HORIZONTAL line** that will pass through that number on the "y" axis. You can skip **STEP 2** and **STEP 3** as you should easily be able to tell what side of the boundary line needs to be colored from the inequality sign used.

APPLY the Math

EXAMPLE 1 Solving a linear inequality graphically when it has a continuous solution set in two variables

Graph the solution set for this linear inequality:

$-2x + 5y \geq 10$

Robert's Solution: Using graph paper

Linear equation that represents the boundary:

$-2x + 5y = 10$

I knew that the graph of the linear equation $-2x + 5y = 10$ would form the boundary of the linear inequality $-2x + 5y \geq 10$.

The variables represent numbers from the set of real numbers.

$x \in \mathrm{R}$ and $y \in \mathrm{R}$

The domain and range are not stated and no context is given, so I assumed that the domain and range are the set of real numbers. This means that the solution set is **continuous**.

continuous

A connected set of numbers. In a continuous set, there is always another number between any two given numbers. Continuous variables represent things that can be measured, such as time.

y-intercept:

$$-2x + 5y = 10$$
$$-2(0) + 5y = 10$$
$$\frac{5y}{5} = \frac{10}{5}$$
$$y = 2$$

The *y*-intercept is at (0, 2).

I knew that I needed to plot and join only two points to graph the linear equation. I decided to plot the two intercepts.

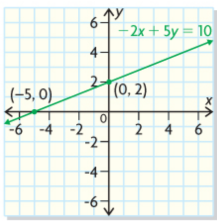
To determine the *y*-intercept, I substituted 0 for *x*.

x-intercept:

$$-2x + 5y = 10$$
$$-2x + 5(0) = 10$$
$$\frac{-2x}{-2} = \frac{10}{-2}$$
$$x = -5$$

The *x*-intercept is at $(-5, 0)$.

To determine the *x*-intercept, I substituted 0 for *y*.



Since the linear inequality has the possibility of equality ($\geq$), and the variables represent real numbers, I knew that the **solution region** includes all the points on its boundary. That's why I drew a solid green line through the intercepts.

solution region

The part of the graph of a linear inequality that represents the solution set; the solution region includes points on its boundary if the inequality has the possibility of equality.

Test (0, 0) in $-2x + 5y \geq 10$.

LS	RS
$-2(0) + 5(0)$	10
0	

Since 0 is not greater than or equal to 10, (0, 0) is not in the solution region.

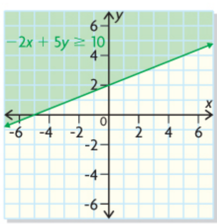
I needed to know which **half plane**, above or below the boundary, represents the solution region for the linear inequality.

To find out, I substituted the coordinates of a point in the half plane below the line. I used (0, 0) because it made the calculations simple.

I already knew that the solution region includes points on the boundary, so I didn't need to check a point on the line.

half plane

The region on one side of the graph of a linear relation on a Cartesian plane.



Since my test point below the boundary was not a solution, I shaded the half plane that did not include my test point. This was the region above the boundary.

I used green shading to show that the solution set belongs to the set of real numbers.

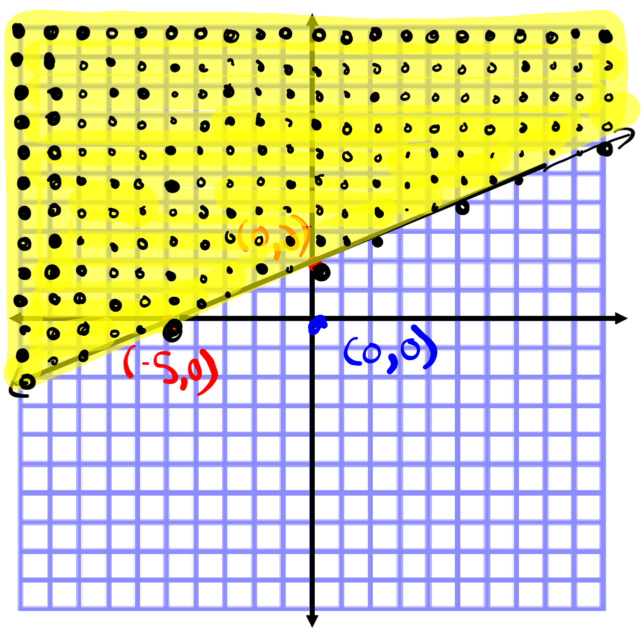
Since the domain and range are in the set of real numbers, I knew that the solution set is continuous. Therefore, the solution region includes all points in the shaded area and on the solid boundary.

Communication *Tip*

If the solution set to a linear inequality is continuous and the sign includes equality ($\leq$ or $\geq$), a solid green line is used for the boundary, and the solution region is shaded green.

Ex: $-2x+5y \geq 10$ Solid boundary line

x-int ($y=0$)	y-int ($x=0$)
$-2x+5y=10$	$-2x+5y=10$
$-2x+5(0)=10$	$-2(0)+5y=10$
$-2x=10$	$5y=10$
$x=-5$	$y=2$
$(-5,0)$	$(0,2)$



Test $(0,0)$

$-2x+5y \geq 10$

$-2(0)+5(0) \geq 10$

$0 \geq 10$

False

$-2x+5y=10$

$5y=2x+10$

$y=\frac{2}{5}x+2$

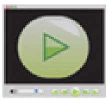
If $\left.\begin{array}{l} x \in I \\ y \in I \end{array}\right\}$ Answer is stipled

EXAMPLE 1 Solving a linear inequality graphically when it has a continuous solution set in two variables

Graph the solution set for this linear inequality:
$-2x + 5y \geq 10$

Your Turn

Compare the graphs of the following relations. What do you notice?
$-2x + 5y \geq 10$ $\qquad -2x + 5y = 10$ $\qquad -2x + 5y < 10$



Answer

They all have the same line or boundary, but the inequality $-2x + 5y \geq 10$ has a solution region that includes the boundary and the half plane above it. The equation $-2x + 5y = 10$ has a solution that includes only the values on the line. The inequality $-2x + 5y < 10$ has a solution region that is the half plane below the boundary and does not include values on the boundary.

EXAMPLE 2 Graphing linear inequalities with vertical or horizontal boundaries

Graph the solution set for each linear inequality on a Cartesian plane.

a) $\{(x, y) \mid x - 2 > 0, x \in \mathrm{R}, y \in \mathrm{R}\}$

b) $\{(x, y) \mid -3y + 6 \geq -6 + y, x \in \mathrm{I}, y \in \mathrm{I}\}$

Wynn's Solution

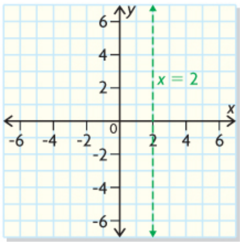
a) $x - 2 > 0$

$x > 2$

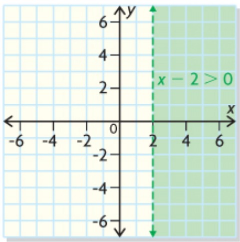
I isolated x so I could graph the inequality.

The variables represent numbers from the set of real numbers.
$x \in \mathrm{R}$ and $y \in \mathrm{R}$

The domain and range are stated as the set of real numbers. The solution set is continuous, so the solution region and its boundary will be green in my graph.



I drew the boundary of the linear inequality as a dashed green line because I knew that the linear inequality (>) does not include the possibility of x being equal to 2.



I needed to decide which half plane to shade. For x to be greater than 2, I knew that any point to the right of the boundary would work.

The solution region includes all the points in the shaded area because the solution set is continuous. The solution region does not include points on the boundary.

Communication Tip

If the solution set to a linear inequality is continuous and the sign does not include equality (< or >), a dashed green line is used for the boundary and the solution region is shaded green.

b) $-3y + 6 \geq -6 + y$

$-4y \geq -12$

$\frac{-4y}{-4} \leq \frac{-12}{-4}$

$y \leq 3$

Since the linear inequality has only one variable, y, I isolated the y.

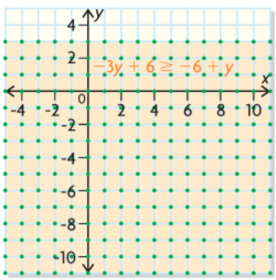
As I rearranged the linear inequality, I divided both sides by -4. That's why I reversed the sign from $\geq$ to $\leq$.

The variables represent integers.
$x \in \mathrm{I}$ and $y \in \mathrm{I}$

The domain and range are stated as being in the set of integers. I knew this means that the solution set is **discrete**.

discrete

Consisting of separate or distinct parts. Discrete variables represent things that can be counted, such as people in a room.



$\{(x, y) \mid -3y + 6 \geq -6 + y, x \in \mathrm{I}, y \in \mathrm{I}\}$

I knew that points with integer coordinates below the line $y = 3$ were solutions, so I shaded the half plane below it orange.

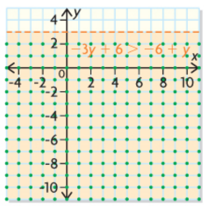
I knew the linear inequality includes 3, so points on the boundary with integer coordinates are also solutions to the linear inequality.

I stippled the boundary and the orange half plane with green points to show that the solution set is discrete.

The solution region includes only the points with integer coordinates in the shaded region and along the boundary.

Communication Tip

If the solution set to a linear inequality is discrete, the solution region is shaded orange and stippled with green points. If the sign includes equality (≥ or ≤), the boundary is also stippled. An example of this is shown to the left. If equality is not possible (< or >), the boundary is a dashed orange line. An example of this is shown below.



EXAMPLE 2 Graphing linear inequalities with vertical or horizontal boundaries

Graph the solution set for each linear inequality on a Cartesian plane.

a) $\{(x, y) \mid x - 2 > 0, x \in \mathrm{R}, y \in \mathrm{R}\}$

b) $\{(x, y) \mid -3y + 6 \geq -6 + y, x \in \mathrm{I}, y \in \mathrm{I}\}$

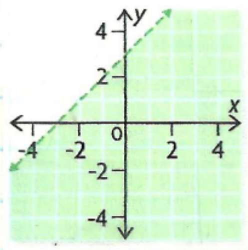
Your Turn

How can you tell if the boundary of a linear inequality is vertical or horizontal without graphing the linear inequality? Explain.

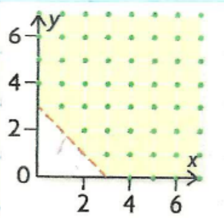
Answer

Assignment: pages 221- 222
Questions 4, 5abc, 6bd

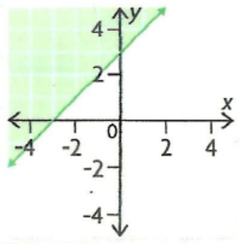
4. Match each graph with its linear inequality, and justify your match.

a) 

Match: ii) $\{(x,y) \mid x-y \underset{\substack{\downarrow \\ \text{dashed line}}}{>} -3, \underbrace{x \in R, y \in R}_{\substack{\downarrow \\ \text{Continuous (not stippled)}}}\}$

b) 

Match: i) $\{(x,y) \mid x-3 \underset{\substack{\downarrow \\ \text{dashed line}}}{>} -y, \underbrace{x \in W, y \in W}_{\substack{\downarrow \\ \text{Discrete (Stippled)}}}\}$

c) 

Match: iii) $\{(x,y) \mid y-3 \underset{\substack{\downarrow \\ \text{Solid line}}}{\geq} x, \underbrace{x \in R, y \in R}_{\substack{\downarrow \\ \text{Continuous (not stippled)}}}\}$

5. Graph the solution set for each linear inequality.

a) $y > -2x + 8$ (Dashed line)   Assume $X \in R$, $Y \in R$

① Equation of boundary: $y = -2x + 8$

② Boundary's x-int and y-int:

For $y = 0$,

$0 = -2x + 8$

$\frac{\cancel{2}x}{\cancel{2}} = \frac{8}{2}$

$x = 4$

x-int => (4,0)

For $x = 0$,

$y = -2(0) + 8$

$y = 0 + 8$

$y = 8$

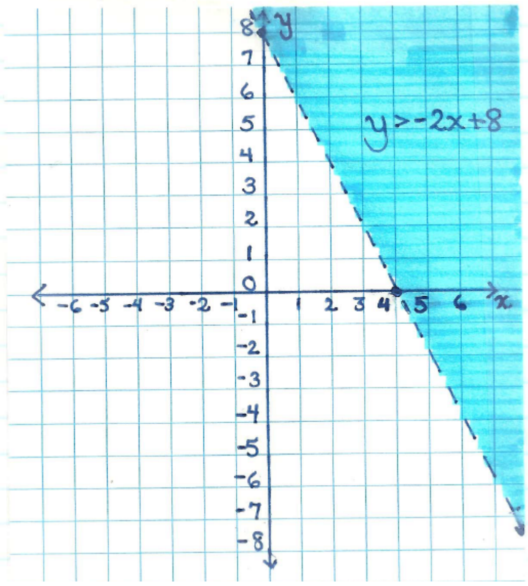
y-int => (0,8)

③ Test Point (0,0):

L.S	R.S.
y	$-2x + 8$
0	$-2(0) + 8$
	$0 + 8$
	8

Since $0 < 8$, (0,0) is not in the solution region.

④ Graph:



b) $-3y \leq 9x + 12$ Assume $X \in R$, $Y \in R$

Solid line

① Equation of boundary: $-3y = 9x + 12$

② Boundary's x-int and y-int:

For $y = 0$,
$-3(0) = 9x + 12$
$0 = 9x + 12$
$-\frac{12}{9} = \frac{\cancel{9}x}{\cancel{9}}$
$-\frac{4}{3} = x$
$-1.3 \doteq x$
x-int => $(-1.3, 0)$

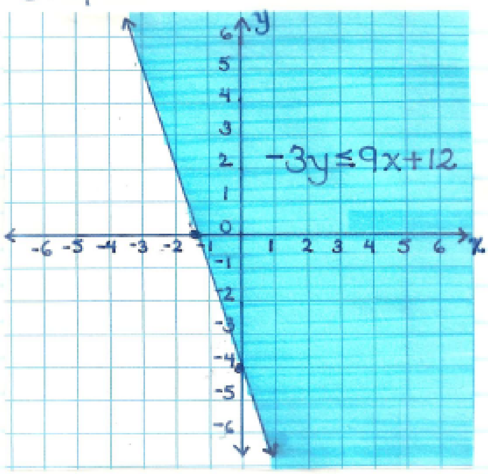
For $x = 0$,
$-3y = 9(0) + 12$
$-3y = 0 + 12$
$\frac{\cancel{-3}y}{\cancel{-3}} = \frac{12}{-3}$
$y = -4$
y-int => $(0, -4)$

③ Test Point $(0,0)$:

L.S.	R.S.
$-3y$	$9x+12$
$-3(0)$	$9(0)+12$
0	$0+12$
	12

Since $0 \leq 12$, $(0,0)$ is in the solution region.

④ Graph:



c) $y < 6$ Assume $X \in R$, $Y \in R$

Dashed Line

① Equation of boundary: $y = 6$

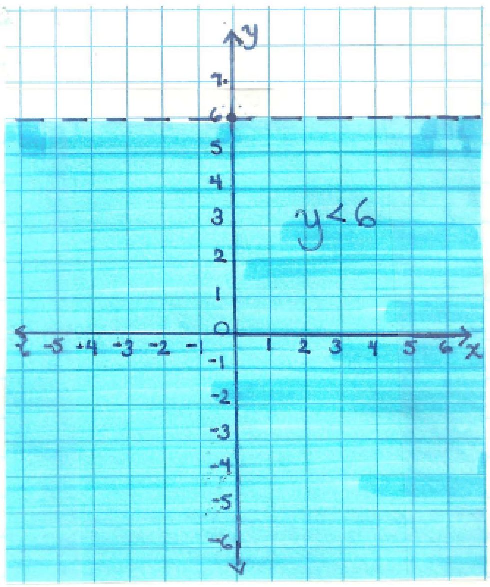
② Skip ⇒ Horizontal Line

③ Test Point $(0, 0)$:

L.S.	R.S.
y	6
0	

Since $0 < 6$, $(0,0)$ is in the solution region.

④ Graph:



6. Graph the solution set for each linear inequality.

b) $\{(x,y) \mid x+6y-14<0, x \in I, y \in I\}$ * Discrete (Stippled)

Dashed Line

① Equation of boundary: $x+6y-14=0$

② Boundary's x-int and y-int:

For $y=0$:
$x+6(0)-14=0$
$x+0-14=0$
$x-14=0$
$x=14$
x-int => (14, 0)

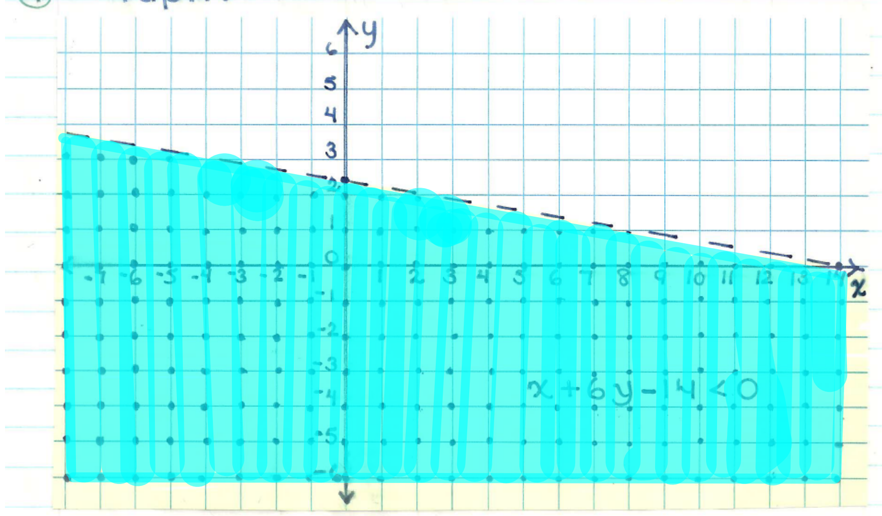
For $x=0$:
$0+6y-14=0$
$6y-14=0$
$\frac{6y}{6}=\frac{14}{6}$
$y=\frac{14}{6}$
$y=\frac{7}{3}$ or 2.3.
y-int => (0, 2.3)

③ Test Point (0,0):

L.S.	R.S.
$x+6y-14$	0
$0+6(0)-14$	
$0+0-14$	
-14	

Since $-14<0$, (0,0) is in the solution region.

④ Graph:



d) $\{(x,y) \mid 2x+2 \le 5+x,\ x \in I,\ y \in I\}$ * Discrete (Stippled)

Solid line

① Equation of boundary: $2x+2=5+x$

$2x-x=5-2$

$x=3$

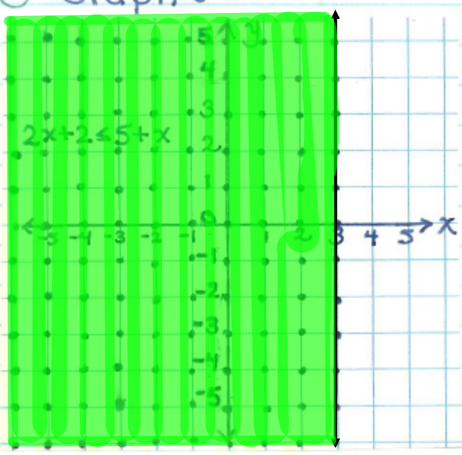
② Skip => Vertical Line

③ Test Point (0,0):

L.S	R.S.
$2x+2$	$5+x$
$2(0)+2$	$5+0$
$0+2$	5
2	

Since $2 \le 5$, (0,0) is located in the solution region.

④ Graph:



fm6s1-p5.tns

6Ws1e1.mp4

6Ws1e2.mp4

6Ws1e3.mp4

fm6s1-p9.tns