Foundations of Math 11 **Ambiguous Case of the Sine Law** **Section 4.3**

From now on when you use the Law of Sines to find an unknown angle, you must watch out for the ambiguous case. The ambiguous case occurs when two different triangles could be created when you are given the lengths of two sides and the measure of an angle that is not contained by the two sides (SSA). Depending on the measure of the given angle and the lengths of the given sides, it may be possible to construct and solve zero, one, or two triangles.

To determine the number of possible triangles, follow the steps below:

STEP 1 – Decide whether the description of a triangle that you are given involves the SSA (Side-Side-Angle) Situation. In other words, **check to see if you have a "matching pair"**.

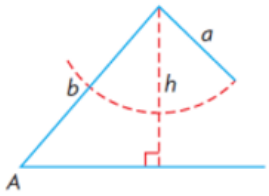
If a "matching pair" is given, label the side as "a" and the angle as "A".

****Label the other side that is given as "b".***

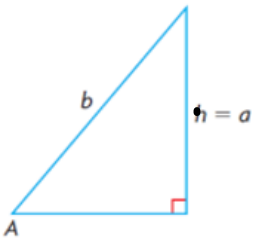
STEP 2 – Calculate the **height** of your triangle using: **h = bsinA.** ***SKIP TO STEP 3 IF "A" is OBTUSE.**

STEP 3 – If "A" is **ACUTE**, there are 4 possibilities to consider | If "A" is **OBTUSE**, there are 2 possibilities

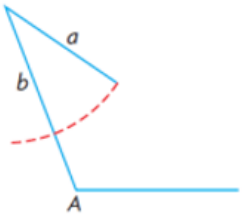
If $\angle A$ is acute and $a < h$, there is **no triangle.**



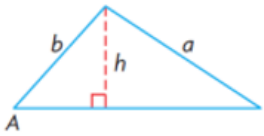
If $\angle A$ is acute and $a = h$, there is **one right triangle.**



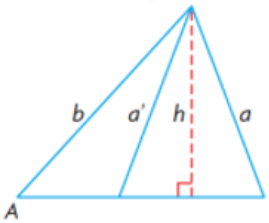
If $\angle A$ is obtuse and $a < b$ or $a = b$, there is **no triangle.**



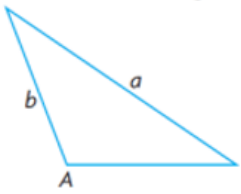
If $\angle A$ is acute and $a > b$ or $a = b$, there is **one triangle.**

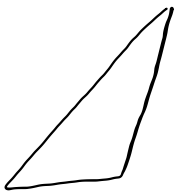


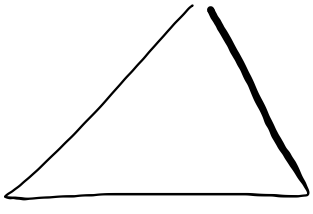
If $\angle A$ is acute and $h < a < b$, there are **two possible triangles.**



If $\angle A$ is obtuse and $a > b$, there is **one triangle.**







EXAMPLES:

Decide whether each description of a triangle involves the SSA situation. If it does, determine the number of triangles (zero, one, or two) are possible with the given measurements.

a) $\angle A = 30°$, $a = 4$ m, and $b = 12$ m

SOLUTION:

STEP 1: This is a SSA situation. (Matching Pair)

STEP 2: $h = b\sin A$ ($\angle A$ is acute)

$h = 12\sin 30°$

$h = 6$ m

STEP 3: Since $\angle A$ is acute and $a < h$, no triangle is possible.

Suppose you were asked to find $\angle B$:

a) $\frac{\sin B}{b} = \frac{\sin A}{a}$

$\frac{\sin B}{12} = \frac{\sin 30°}{4}$

$\sin B = \frac{12 \sin 30°}{4}$

$\sin B = 1.5$

$B = \sin^{-1}(1.5)$

error

No Solution because there is no triangl

b) $\angle D = 30°$, $d = 5$ m, and $e = 10$ m

SOLUTION:

* $\angle A = 30°$, $a = 5$ m, and $b = 10$ m

STEP 1: This is a SSA situation. (Matching Pair)

STEP 2: $h = b\sin A$ ($\angle A$ is acute)

$h = 10\sin 30°$

$h = 5$ m

STEP 3: Since $\angle A$ is acute and $a = h$, there will be one RIGHT triangle.

c) $\angle P = 45°$, $p = 25$ m, and $q = 16$ m

SOLUTION:

* $\angle A = 45°$, $a = 25$ m, and $b = 16$ m

STEP 1: This is a SSA situation. (Matching Pair)

STEP 2: $h = b\sin A$ ($\angle A$ is acute)

$h = 16\sin 45°$

$h = 11.3$ m

STEP 3: Since $\angle A$ is acute and $a > b$, there will be one triangle.

d) $\angle M = 45°$, n = 30 m, and m = 24 m

SOLUTION:

* $\angle A = 45°$, a = 24 m, and b = 30 m **WATCH ORDER** ☺

STEP 1: This is a SSA situation. (Matching Pair)

STEP 2: h = bsinA

h = 30sin45°

h = 21.2 m

STEP 3: Since $\angle A$ is acute and h < a < b, there will be two possible triangles.

e) $\angle Z = 120°$, z = 15 m, and y = 12 m

SOLUTION:

* $\angle A = 120°$, a = 15 m, and b = 12 m

STEP 1: This is a SSA situation. (Matching Pair)

STEP 2: ***SKIP*** – since $\angle A$ is OBTUSE (Skip)

STEP 3: Since $\angle A$ is obtuse and a > b, there will be one triangle.

f) $\angle A = 105°$, a = 50 m, and b = 75 m

SOLUTION:

STEP 1: This is a SSA situation. (Matching Pair)

STEP 2: ***SKIP*** – since $\angle A$ is OBTUSE (Skip)

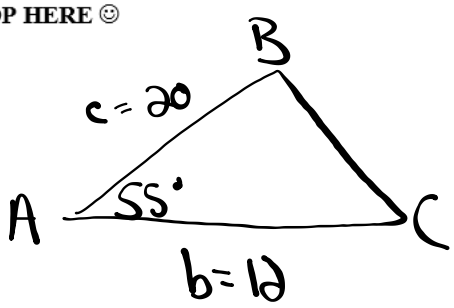
STEP 3: Since $\angle A$ is obtuse and a < b, there will be no triangle.

g) $\angle A = 55°$, b = 12 cm, and c = 20 cm.

SOLUTION:

STEP 1: This is a not a SSA situation. (No Matching Pair)

STOP HERE ☺



use cosine law

4.3 The Ambiguous Case of the Sine Law

Assignment: pgs. 183 - 184
2/3(Complete together), 4, 8

② a) In $\triangle ABC$, $\angle B = 100°$, $a = 8cm$, $b = 10cm$

$\angle A = 100°$, $b = 8cm$, $a = 10cm$

Step 1: This is SSA (Matching Pair)

Step 2: Skip ($\angle A = 100°$)

Step 3: Since $\angle A$ is obtuse and $a > b$ there is one triangle

b) In $\triangle DEF$, $\angle D = 81°$, $e = 9cm$, $f = 8cm$

$\angle A = 81°$, $b = 9cm$, $c = 8cm$

Step 1: This is <u>not</u> SSA (No matching pair)

SOLUTIONS => 4.3 The Ambiguous Case of the Sine Law

*Complete Together

2. Decide whether each description of a triangle involves the SSA situation.
3. Calculate the height of each triangle in question 2. Determine the number of triangles that are possible (zero, one, or two). Justify your answers.

a) In ΔABC, $\angle B = 100°$, $a = 8$cm, and $b = 10$cm.

* $\angle A = 100°$, $b = 8$cm, and $a = 10$cm

① This is a SSA situation. (Matching Pair)

② **We can skip step 2 since** $\angle A$ is obtuse.

③ Since $\angle A$ is obtuse and $a > b$, there is one triangle.

b) In ΔDEF, $\angle D = 81°$, $e = 9\,cm$, and $f = 8\,cm$.

① This is not a SSA situation. (No Matching Pair)
* Stop Here.

c) In ΔGHI, $\angle G = 40°$, $i = 5\,cm$, and $g = 4\,cm$.
* $\angle A = 40°$, $b = 5cm$, and $a = 4cm$

① This is a SSA situation. (Matching Pair)

② $h = b \sin A$

$h = 5 \sin 40°$
$h = 3.2\,cm$

③ Since $\angle A$ is acute and $h < a < b$, there are 2 possible triangles.

d) In ΔJKL, $\angle L = 15°$, $j = 71cm$, and $K = 36cm$.

① This is not a SSA situation. (No Matching Pair)
* Stop Here

e) In ΔMNO, $\angle O = 28°$, $m = 8.4$ cm, and $o = 4.0$ cm.

* $\angle A = 28°$, $b = 8.4$cm, and $a = 4.0$cm

① This is a SSA situation. (Matching Pair)

② $h = b \sin A$

$h = 8.4 \sin 28°$

$h = 3.9$ cm.

③ Since $\angle A$ is acute and $h < a < b$, there are 2 possible triangles.

f) In ΔPQR, $\angle Q = 95°$, $q = 1.0$ cm, and $r = 0.5$ cm.

* $\angle A = 95°$, $a = 1.0$cm, and $b = 0.5$cm

① This is a SSA situation. (Matching Pair)

② **We can skip step 2 since** $\angle A$ is obtuse.

③ Since $\angle A$ is obtuse, and $a > b$, there is one triangle.

4. Decide whether each description of a triangle involves the SSA situation. If it does, determine the number of triangles (zero, one, or two) that are possible with the given measurements. Draw the triangle(s), and justify your answer.

a) In ΔABC, $\angle A = 51°$, $a = 5m$, and $b = 14m$.

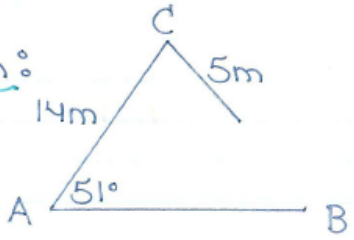
① This is a SSA situation. (Matching Pair)

* Since $\angle a$ is acute. To determine the number of triangles, you need to calculate the height, using $h = b\sin A$.

② $h = b\sin A$
$h = 14\sin 51°$
$h = 10.9m$

③ Since $\angle A$ is acute and $a < h$, there is no triangle.

Diagram:



b) In ΔABC, $\angle C = 30°$, $a = 6mm$, and $c = 12mm$.

* $\angle A = 30°$, b=6 mm, and a=12mm

① This is a SSA situation. (Matching Pair)

② $h = b \sin A$

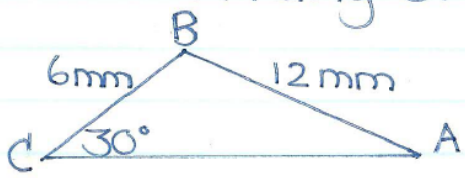
$h = 6 \sin 30$

$h = 3$ cm.

Not really necessary!

Diagram:

③ Since $\angle A$ is acute and $a > b$, there is one triangle.



c) In ΔABC, $\angle B = 40°$, $a = 12cm$, and $b = 10cm$.

* $\angle A = 40°$, $b = 12cm$, and $a = 10cm$

① This is a SSA situation. (Matching Pair)

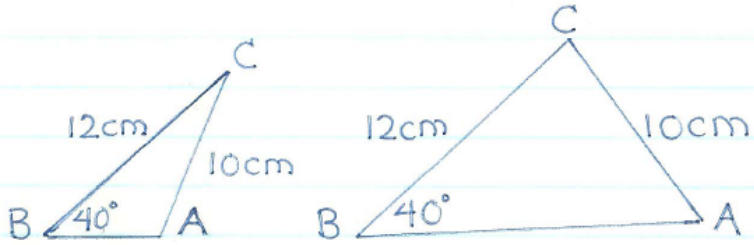
② $h = b \sin A$

$h = 12 \sin 40°$

$h = 7.7cm$

③ Since $\angle A$ is acute and $h < a < b$, there are two possible triangles.

Diagrams:



d) In ΔABC, $\angle A = 155°$, $b = 15m$, and $c = 12m$.

① This is not a SSA situation (No Matching Pair)

8. An obtuse triangle has two known side lengths: 4.0m and 4.2m. The angle that is opposite the shorter side measures 64.0°.

a) Calculate the obtuse angle is the triangle, to the nearest tenth of a degree.

$$\frac{\sin\theta}{4.2m} = \frac{\sin 64°}{4.0m}$$

$$\frac{4.0\sin\theta}{4.0} = \frac{4.2\sin 64°}{4.0}$$

$$\sin\theta = 0.9437$$

$$\theta = \sin^{-1}(0.9437)$$

$$\theta = 70.7°$$

Since the angle is obtuse:

$$\theta = 180° - 70.7°$$

$$\theta = 109.3°$$

b) Is there only one possible answer? Explain.

If $\angle A$ is 109.3°, $a = 4.2m$ and $b = 4.0m$, this is the only possible triangle. {Since $\angle A$ is obtuse and $a > b$}

Attachments

FM11-4s3.gsp

4Ws3e1.mp4

4Ws3e2.mp4

4Ws3e3.mp4