Questions From Homework

The sum of two numbers is 12. Find the numbers so that their product is a maximum?

A rectangle has a perimeter of 150cm. What length and width should it have so that its area is a maximum?

Find the point on the graph of $y = 2x + 6$ that is the minimum distance from the point (1, 2).

(minimize distance)

$x_1 = 1$
$y_1 = 2$

Remember ***d is smallest when d^2 is smallest***

$$d = \sqrt{(x-x_1)^2 + (y-y_1)^2}$$

$$d = \sqrt{(x-1)^2 + (y-2)^2}$$

(Express with a single variable)

$$d = \sqrt{(x-1)^2 + (2x+6-2)^2}$$

$$d = \sqrt{(x-1)^2 + (2x+4)^2}$$

$$d = \sqrt{x^2 - 2x + 1 + 4x^2 + 16x + 16}$$

$$d = \sqrt{5x^2 + 14x + 17} = (5x^2 + 14x + 17)^{\frac{1}{2}}$$

* $f(x) = 5x^2 + 14x + 17$

$f'(x) = 10x + 14$

min

$-$ | $+$

(-2) $\left(-\frac{7}{5}\right)$ (1)

CV: $0 = 10x + 14$

$-14 = 10x$

$\frac{-14}{10} = x$

$-\frac{7}{5} = x$

(ii) Find y:

$y = 2x + 6$

$y = 2\left(\frac{-7}{5}\right) + 6$

$y = \frac{-14}{5} + \frac{30}{5}$

$y = \frac{16}{5}$

$\therefore$ The point closest is $\left(-\frac{7}{5}, \frac{16}{5}\right)$

$d = \sqrt{(x-1)^2 + (2x+4)^2}$

$d = \sqrt{x^2 - 2x + 1 + 4x^2 + 16x + 16}$

$d = \sqrt{5x^2 + 14x + 17} = (5x^2 + 14x + 17)^{1/2}$

$d' = \frac{1}{2}(5x^2 + 14x + 17)^{-1/2}(10x + 14)$

$d' = \frac{10x + 14}{2(5x^2 + 14x + 17)^{1/2}} = \frac{5x + 7}{\sqrt{5x^2 + 14x + 17}}$ ← Always positive

CV: $5x + 7 = 0$

$5x = -7$

$x = \frac{-7}{5}$

$\cancel{\sqrt{5x^2 + 14x + 17} = 0}$

$\cancel{5x^2 + 14x + 17 = 0}$

If 2700 cm^2 of material is available to make a box with square base and an open top, find the largest possible volume of the box.
(max volume)

Let x = *the length of the base* (and width)
Let h = *the height*

We want to maximize the *volume*.

$$(1)\ A = x^2 + 4xh$$
$$2700 = x^2 + 4xh$$
$$2700 - x^2 = 4xh$$
$$\boxed{\frac{2700 - x^2}{4x} = h}$$

$V = lwh$

$V = x^2 h$

We want to eliminate h from the *volume* function and we do so by finding a relationship between x *and* h. We use the area of the available material

$$V = x^2 \left(\frac{2700 - x^2}{4x} \right)$$

$$V = \frac{2700x - x^3}{4} = \frac{2700x}{4} - \frac{x^3}{4} = \boxed{675x - \frac{1}{4}x^3}$$

$$V' = 675 - \frac{3}{4}x^2$$

CV: $0 = 675 - \frac{3}{4}x^2$

$$\frac{3}{4}x^2 = 675$$
$$3x^2 = 2700$$
$$x^2 = 900$$
$$x = \pm 30$$

$-$ $+$ $-$

(-31) -30 (0) 30 (31)

max

$x = 30\text{cm}$

Find h:

$$h = \frac{2700 - x^2}{4x}$$

$$h = \frac{2700 - (30)^2}{4(30)}$$

$$h = \frac{2700 - 900}{120} = 15\text{cm}$$

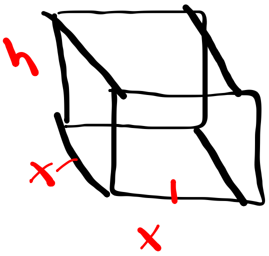
Max Volume:

$V = x^2 h$

$V = (30)^2 (15) = 13500 \text{ cm}^3$

$\therefore$ The dimensions of the box are

$30\text{cm} \times 30\text{cm} \times 15\text{cm}$



$A = x^2 + 4xh$ (open top)

$A = 2x^2 + 4xh$ (closed top)

Find the points on the parabola $y = 6 - x^2$ that are closest to the point (0, 3)

Homework