Questions From Homework

(2) Let $x = 1^{st}$ number
Let $y = 2^{nd}$ number

$x + y = 8$

$x = \boxed{8 - y}$

$x = 8 - 2$

$x = 6$

$S = x^2 + y^3$

$S = (8-y)^2 + y^3$

$S = 64 - 16y + y^2 + y^3$

$S' = -16 + 2y + 3y^2$

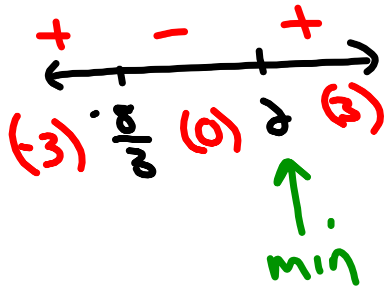
$S' = 3y^2 + 2y - 16$ decomp.

$S' = 3y^2 - 6y + 8y - 16$

$S' = 3y(y-2) + 8(y-2)$

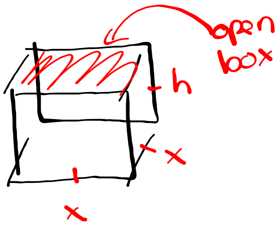
$S' = (3y+8)(y-2)$

CV: $y = -\frac{8}{3}$, $\boxed{2}$



Questions From Homework

⑤ A box with a square base and open top must have a volume of 4000 cm^3. Find the dimensions of the box that minimizes the amount of material used. minimize the surface area!



$V = x^2 h$

$4000 = x^2 h$

$\boxed{\frac{4000}{x^2} = h}$

$A = x^2 + 4xh$

$A = x^2 + 4\cancel{x}\left[\frac{4000}{x^{\cancel{2}}}\right]$

$A = x^2 + \frac{16000}{x}$

$A = x^2 + 16000x^{-1}$

$A' = 2x - 16000x^{-2}$

$A' = 2x - \frac{16000}{x^2}$

$A' = \frac{2x^3}{x^2} - \frac{16000}{x^2}$

$A' = \frac{2x^3 - 16000}{x^2}$

CV: $2x^3 - 16000 = 0$ | $x^2 = 0$

$2x^3 = 16000$ | $x = 0$ cm

$x^3 = 8000$

$\boxed{x = 20\text{cm}}$

$h = \frac{4000}{(20)^2}$

$h = \frac{4000}{400}$

$\boxed{h = 10\text{cm}}$

∴ The dimensions that minimize the surface area are 20x20x10

$-$ $-$ $+$

(-1) 0 (1) 20 (21)

↑ min

Questions From Homework

(4) Find the point on the line $y = 5x+4$ that is closest to the origin.

minimize the distance

$x_1 = 0$ (0,0)

$y_1 = 0$

$$d = \sqrt{(x-x_1)^2 + (y-y_1)^2}$$

$$d = \sqrt{(x-0)^2 + (5x+4-0)^2}$$

$$d = \sqrt{x^2 + (5x+4)^2}$$

$$d = \sqrt{x^2 + 25x^2 + 40x + 16}$$

$$d = \sqrt{26x^2 + 40x + 16}$$

$$d = (26x^2 + 40x + 16)^{\frac{1}{2}}$$

$$d' = \frac{1}{\cancel{2}}(26x^2+40x+16)^{-\frac{1}{2}}\,(\cancel{52x+40})$$

$26x + 20$

$$d' = \frac{26x+20}{\sqrt{26x^2+40x+16}}$$

min

$-$ $+$

(-1) $\frac{-10}{13}$ (1)

CV: $26x + 20 = 0$

$26x = -20$

$x = \frac{-20}{26}$

$x = \frac{-10}{13}$

$\sqrt{26x^2+40x+16} = 0$

$26x^2 + 40x + 16 = 0$

Always positive

Does not factor

when $x = \frac{-10}{13}$

$$y = 5\left(\frac{-10}{13}\right) + 4$$

$$y = \frac{-50}{13} + \frac{52}{13}$$

$$y = \frac{2}{13}$$

$\therefore$ The point on the line $y=5x+4$ that is closest to the origin is $\left(\frac{-10}{13}, \frac{2}{13}\right)$

since d is smallest when d^2 is smallest . . . .

$$d = \sqrt{26x^2 + 40x + 16}$$

$$f(x) = 26x^2 + 40x + 16$$

$$f'(x) = 52x + 40$$

CV: $52x + 40 = 0$

$$52x = -40$$

$$x = \frac{-40}{52}$$

$$x = \frac{-10}{13}$$

Questions From Homework

6 Find the point on the parabola $2y = x^2$ that is closest to the point (-4, 1)

$y = \frac{x^2}{2}$ $\quad x_1 \quad y_1$

$d = \sqrt{(x-x_1)^2+(y-y_1)^2}$

$d = \sqrt{(x+4)^2+(y-1)^2}$

$d = \sqrt{(x+4)^2+\left(\frac{1}{2}x^2-1\right)^2}$

$f(x) = (x+4)^2+\left(\frac{1}{2}x^2-1\right)^2$

$f'(x) = 2(x+4)(1) + 2\left(\frac{1}{2}x^2-1\right)(x)$

$f'(x) = 2x+8+x^3-2x$

$f'(x) = x^3+8$

$x^3 = -8$

$\boxed{x=-2}$

$y = \frac{x^2}{2}$

$y = \frac{(-2)^2}{2}$

$y = 2$

(-2, 2)

$-$ $+$

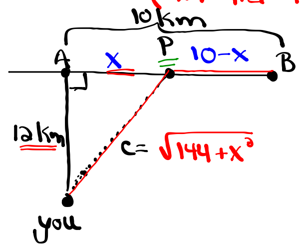
(-3) -2 (1)

min

You are in a dune buggy in the desert 12km due south of the nearest point A on a straight east-west road. You wish to get to point B on the road 10km east of point A. If your dune buggy can average 15km/h traveling over the desert, and 39km/h traveling on the road, toward what point on the road should you head to in order to minimize your travel time to B?

(minimize time)

$t = \frac{d}{s}$



(1) Find the length of the hypotenuse "c"

$a^2+b^2=c^2$

$(12)^2+(x)^2=c^2$

$144+x^2=c^2$

$\pm\sqrt{144+x^2}=c$

$\sqrt{144+x^2}=c$ (distance)

let x = the distance from A to P

(2) minimize t:

$t = \frac{d}{s}$

$t = \frac{\sqrt{144+x^2}}{15} + \frac{10-x}{39}$

travel time (desert)

travel time (road)

$t = \frac{1}{15}(144+x^2)^{1/2} + \frac{10}{39} - \frac{1}{39}x$

$t' = \frac{1}{\cancel{30}\,15}(144+x^2)^{-1/2}(\cancel{2}x) + 0 - \frac{1}{39}$

$t' = \frac{x}{15(144+x^2)^{1/2}} - \frac{1}{39}$

CV: $0 = \frac{x}{15(144+x^2)^{1/2}} - \frac{1}{39}$

$\frac{1}{39} = \frac{x}{15(144+x^2)^{1/2}}$

$15(144+x^2)^{1/2} = 39x$

(Square both sides)

$(15\sqrt{144+x^2})(15\sqrt{144+x^2})$

$225(144+x^2) = 1521x^2$

$32400 + 225x^2 = 1521x^2$

$\frac{32400}{1296} = \frac{\cancel{1296}x^2}{\cancel{1296}}$

$25 = x^2$

$\pm 5 = x$

$\boxed{5\text{km} = x}$

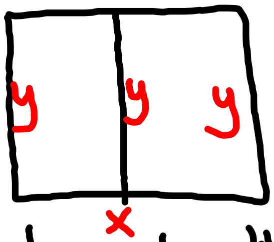
min

$-\ \ +$

(−6) −5 (1) 5 (6)

Drive to a point 5 Km due east of Point A

You have 400 m of fencing to construct a rectangular pen that will be divided into 2 sections of equal size. Find the dimensions that would maximize the area of the whole pen.



Let x = length

Let y = width

$$P = 2x + 3y$$

$$400 = 2x + 3y$$

$$400 - 2x = 3y$$

$$\boxed{\frac{400-2x}{3} = y}$$

$$A = xy$$

$$A = x\left[\frac{400-2x}{3}\right]$$

$$A = \frac{400x - 2x^2}{3}$$

$$A = \frac{400}{3}x - \frac{2}{3}x^2$$

$$A' = \frac{400}{3} - \frac{4x}{3}$$

$$0 = \frac{400}{3} - \frac{4}{3}x$$

$$\frac{4x}{3} = \frac{400}{3}$$

$$12x = 1200$$

$$x = 100\text{m}$$

$$y = \frac{400 - 2(100)}{3}$$

$$y = \frac{200}{3}$$

$$y = 66.\overline{6}\text{ m}$$

Find the points on the parabola $y = 6 - x^2$ that are closest to the point (0, 3)

$$d = \sqrt{(x-x_1)^2+(y-y_1)^2}$$

$$d = \sqrt{(x-0)^2+(6-x^2-3)^2}$$

$$d = \sqrt{x^2+(3-x^2)^2}$$

$$d = \sqrt{x^2+9-6x^2+x^4}$$

$$d = \sqrt{x^4-5x^2+9}$$

$$f(x) = x^4-5x^2+9$$

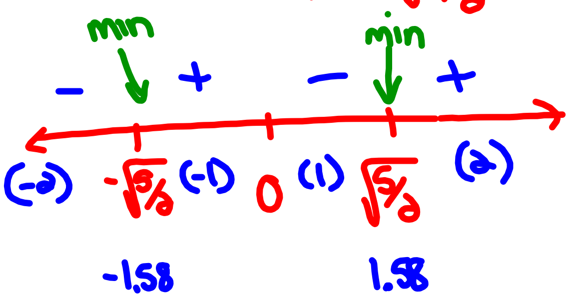
$$f'(x) = 4x^3-10x$$

$$f'(x) = 2x(2x^2-5)$$

$2x=0$ | $2x^2-5=0$

$x=0$ | $x^2 = 5/2$

$x = \pm\sqrt{5/2}$



$$y = 6-\left(-\sqrt{5/2}\right)^2$$

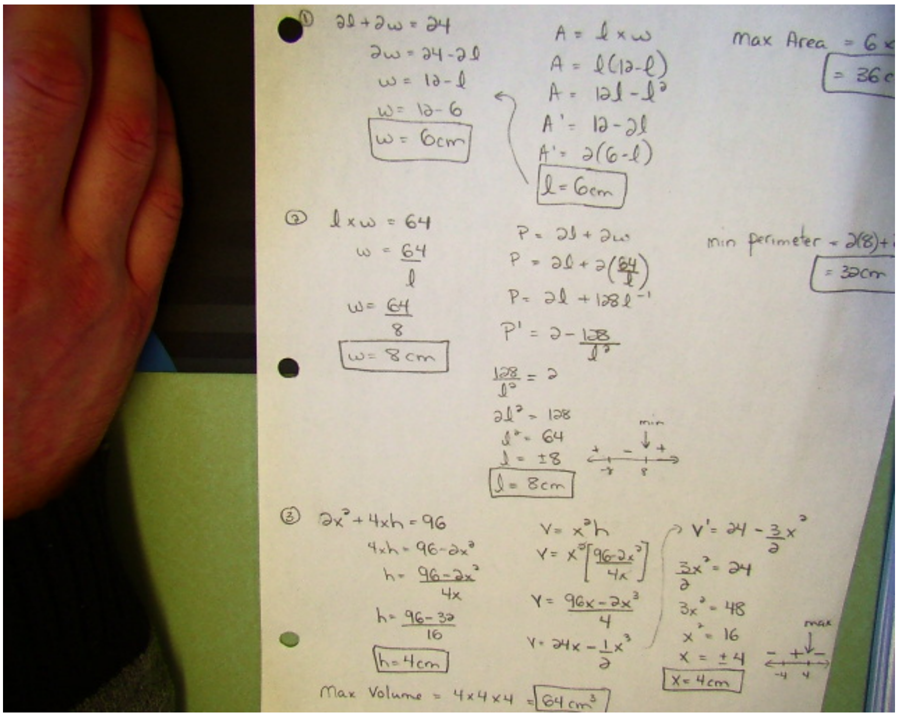
$$y = 6-\frac{5}{2} = \frac{7}{2}$$

$$y = 6-\left(\sqrt{5/2}\right)^2$$

$$y = 6-\frac{5}{2} = \frac{7}{2}$$

The points are $\left(-\sqrt{5/2}, 7/2\right)$ and $\left(\sqrt{5/2}, 7/2\right)$

Homework



① $2l + 2w = 24$

$2w = 24 - 2l$

$w = 12 - l$

$w = 12 - 6$

$\boxed{w = 6cm}$

$A = l \times w$

$A = l(12 - l)$

$A = 12l - l^2$

$A' = 12 - 2l$

$A' = 2(6 - l)$

$\boxed{l = 6cm}$

Max Area $= 6 \times$

$\boxed{= 36c}$

② $l \times w = 64$

$w = \frac{64}{l}$

$w = \frac{64}{8}$

$\boxed{w = 8cm}$

$P = 2l + 2w$

$P = 2l + 2\left(\frac{64}{l}\right)$

$P = 2l + 128l^{-1}$

$P' = 2 - \frac{128}{l^2}$

$\frac{128}{l^2} = 2$

$2l^2 = 128$

$l^2 = 64$

$l = \pm 8$

$\boxed{l = 8cm}$

min

$+ \quad - \quad +$

$-8 \quad 8$

min perimeter $= 2(8)+$

$\boxed{= 32cm}$

③ $2x^2 + 4xh = 96$

$4xh = 96 - 2x^2$

$h = \frac{96 - 2x^2}{4x}$

$h = \frac{96 - 32}{16}$

$\boxed{h = 4cm}$

$V = x^2 h$

$V = x^2\left[\frac{96 - 2x^2}{4x}\right]$

$V = \frac{96x - 2x^3}{4}$

$V = 24x - \frac{1}{2}x^3$

$V' = 24 - \frac{3}{2}x^2$

$\frac{3}{2}x^2 = 24$

$3x^2 = 48$

$x^2 = 16$

$x = \pm 4$

$\boxed{x = 4cm}$

max

$- \quad + \quad -$

$-4 \quad 4$

Max Volume $= 4 \times 4 \times 4 = \boxed{64 cm^3}$

4. $x^2+4xh=108$

$4xh=108-x^2$

$h=\frac{108-x^2}{4x}$

$h=\frac{108-36}{24}$

$\boxed{h=3cm}$

$V=x^2h$

$V=x^2\left[\frac{108-x^2}{4x}\right]$

$V=\frac{108x-x^3}{4}$

$V=27x-\frac{1}{4}x^3$

$V'=27-\frac{3}{4}x^2$

$\frac{3}{4}x^2=27$

$3x^2=108$

$x^2=36$

$x=\pm 6$

$\boxed{x=6cm}$

Max Volume $=6\times 6\times 3=\boxed{108cm^3}$

5. $x^2h=81$

$h=\frac{81}{x^2}$

$h=\frac{81}{18.72}$

$\boxed{h=4.326cm}$

$A=2x^2+4xh$

$A=2x^2+4x\left[\frac{81}{x^2}\right]$

$A=2x^2+324x^{-1}$

$A'=4x-\frac{324}{x^2}$

$\frac{324}{x^2}=4x$

$4x^3=324$

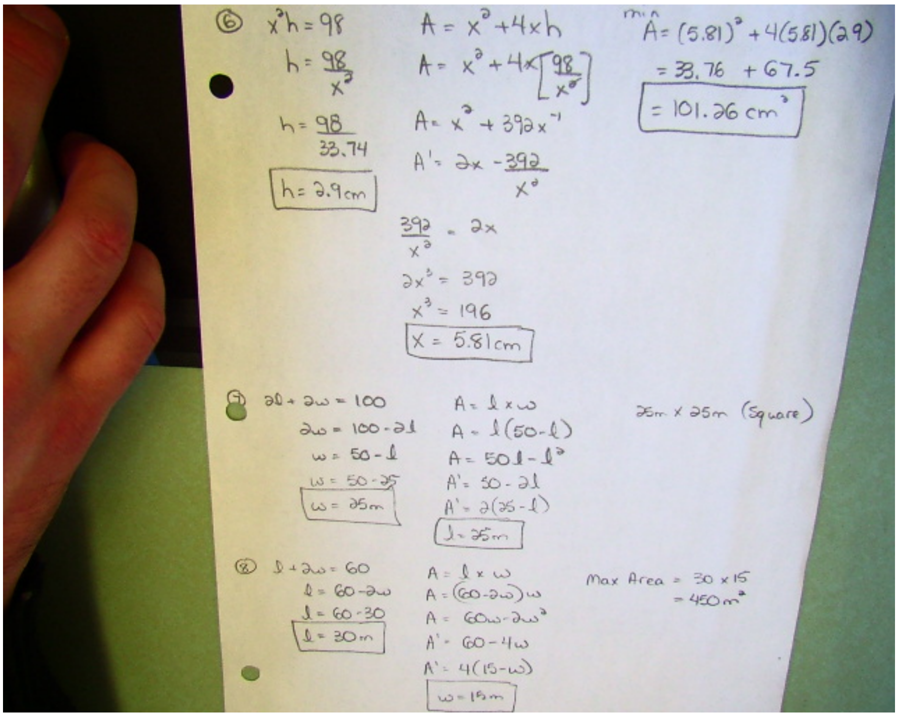
$x^3=81$

$\boxed{x=4.326cm}$

Min Area $=2(4.326)^2+4(4.326)^2$

$=37.44+74.88$

$\boxed{=112.32cm^2}$



⑥ $x^2h = 98$

$h = \frac{98}{x^2}$

$h = \frac{98}{33.74}$

$\boxed{h = 2.9\text{cm}}$

$A = x^2 + 4xh$

$A = x^2 + 4x\left[\frac{98}{x^2}\right]$

$A = x^2 + 392x^{-1}$

$A' = 2x - \frac{392}{x^2}$

$\frac{392}{x^2} = 2x$

$2x^3 = 392$

$x^3 = 196$

$\boxed{X = 5.81\text{cm}}$

$^{\min}A = (5.81)^2 + 4(5.81)(2.9)$

$= 33.76 + 67.5$

$\boxed{= 101.26\text{ cm}^2}$

⑦ $2l + 2w = 100$

$2w = 100 - 2l$

$w = 50 - l$

$w = 50 - 25$

$\boxed{w = 25m}$

$A = l \times w$

$A = l(50 - l)$

$A = 50l - l^2$

$A' = 50 - 2l$

$A' = 2(25 - l)$

$\boxed{l = 25m}$

25m × 25m (Square)

⑧ $l + 2w = 60$

$l = 60 - 2w$

$l = 60 - 30$

$\boxed{l = 30m}$

$A = l \times w$

$A = (60 - 2w)w$

$A = 60w - 2w^2$

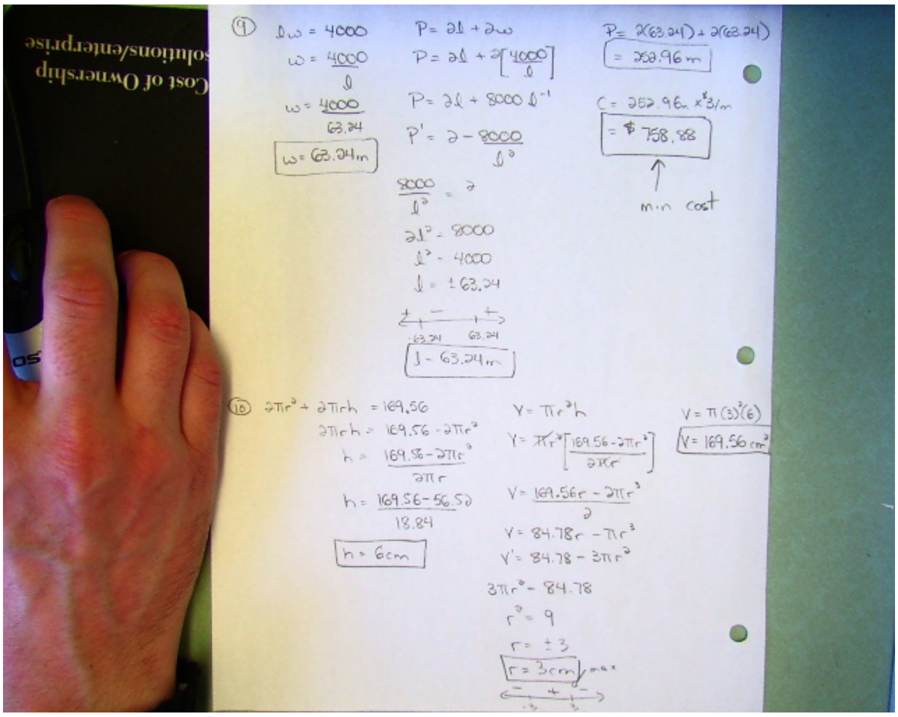
$A' = 60 - 4w$

$A' = 4(15 - w)$

$\boxed{w = 15m}$

Max Area $= 30 \times 15$

$= 450m^2$



9. $lw = 4000$

$w = \frac{4000}{l}$

$w = \frac{4000}{63.24}$

$\boxed{w = 63.24\text{m}}$

$P = 2l + 2w$

$P = 2l + 2\left[\frac{4000}{l}\right]$

$P = 2l + 8000l^{-1}$

$P' = 2 - \frac{8000}{l^2}$

$\frac{8000}{l^2} = 2$

$2l^2 = 8000$

$l^2 = 4000$

$l = \pm 63.24$

$+ \quad - \quad +$

$-63.24 \quad 63.24$

$\boxed{l = 63.24\text{m}}$

$P = 2(63.24) + 2(63.24)$

$\boxed{= 252.96\text{m}}$

$C = 252.96\text{m} \times \$3/\text{m}$

$\boxed{= \$758.88}$

min cost

10. $2\pi r^2 + 2\pi rh = 169.56$

$2\pi rh = 169.56 - 2\pi r^2$

$h = \frac{169.56 - 2\pi r^2}{2\pi r}$

$h = \frac{169.56 - 56.52}{18.84}$

$\boxed{h = 6\text{cm}}$

$V = \pi r^2 h$

$V = \cancel{\pi} r^{\cancel{2}}\left[\frac{169.56 - 2\pi r^2}{2\cancel{\pi} \cancel{r}}\right]$

$V = \frac{169.56r - 2\pi r^3}{2}$

$V = 84.78r - \pi r^3$

$V' = 84.78 - 3\pi r^2$

$3\pi r^2 = 84.78$

$r^2 = 9$

$r = \pm 3$

$\boxed{r = 3\text{cm}}$ max

$- \quad + \quad -$

$-3 \quad 3$

$V = \pi(3)^2(6)$

$\boxed{V = 169.56\text{ cm}^3}$