Exercise 3.2

② minimize Perimeter:

Given:

$A = l \times w$

$64 = lw$

$\boxed{\frac{64}{l} = w}$

$w = \frac{64}{8} = \boxed{8\text{cm}}$

$P = 2l + 2w$

$P = 2(8) + 2(8)$

$P = 16 + 16 = 32\text{cm}$

$P = 2l + 2w$

$P = 2l + 2\left(\frac{64}{l}\right)$

$P = 2l + \frac{128}{l}$

$P = 2l + 128l^{-1}$

$P' = 2 - 128l^{-2}$

$P' = 2 - \frac{128}{l^2}$ or $\frac{2l^2 - 128}{l^2}$

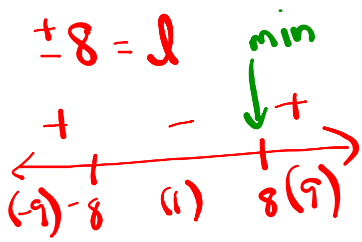
$0 = 2 - \frac{128}{l^2}$

$\frac{128}{l^2} = 2$

$128 = 2l^2$

$64 = l^2$

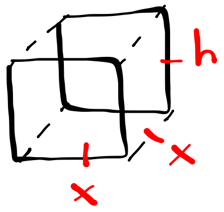
$\pm 8 = l$



$\boxed{l = 8\text{cm}}$

Exercise 3.2

5



$V = x^2 h$

$81 = x^2 h$

$\boxed{\dfrac{81}{x^2} = h}$

$A = 2x^2 + 4xh$

$A = 2x^2 + 4\cancel{x}\left(\dfrac{81}{x^{\cancel{2}}}\right)$

$A = 2x^2 + 324x^{-1}$

$A' = 4x - 324x^{-2}$

$A' = 4x - \dfrac{324}{x^2}$

$0 = 4x - \dfrac{324}{x^2}$

$x^2 \cdot \dfrac{324}{x^2} = 4x \cdot x^2$

$324 = 4x^3$

$81 = x^3$

$4.33 = x$

$h = \dfrac{81}{(4.33)^2} = 4.3$

Dimensions:

$\boxed{4.3\text{cm} \times 4.3\text{cm} \times 4.3\text{cm}}$

$A = 2x^2 + 4xh$

$A = 2(4.3)^2 + 4(4.3)(4.3)$

$A = 110.94\ \text{cm}^2$

Exercise 3.2

(10) $A = 2\pi r^2 + 2\pi rh$

$169.56 = 2\pi r^2 + 2\pi rh$

$\dfrac{169.56 - 2\pi r^2}{2\pi r} = \dfrac{2\pi rh}{2\pi r}$

$\dfrac{169.56 - 2\pi r^2}{2\pi r} = h$

$h = \dfrac{169.56 - 2\pi(3)^2}{2\pi(3)}$

$h = \dfrac{113.04}{18.84}$

$\boxed{h = 6\text{cm}}$

Max Volume:

$V = \pi r^2 h$

$V = \pi(3)^2(6)$

$V = 169.56 \text{ cm}^3$

$V = \pi r^2 h$

$V = \cancel{\pi r^2}\left[\dfrac{169.56 - 2\pi r^2}{2\cancel{\pi r}}\right]$

$V = \dfrac{169.56r - 2\pi r^3}{2}$

$V = 84.78r - \pi r^3$

$V = 84.78 - 3\pi r^2$

$0 = 84.78 - 3\pi r^2$

$3\pi r^2 = 84.78$

$r^2 = 9$

$r = \pm 3$

$\boxed{r = 3}$ cm

\+ | −

(2) 3 (4)

max

Extreme Values Final Review

① a) $f(x) = x^4 - 2x^2 + 16$

$f'(x) = 4x^3 - 4x$

$f'(x) = 4x(x^2 - 1)$

$f'(x) = 4x(x-1)(x+1)$

$- \quad + \quad - \quad +$

$(-2) \quad -1 \quad (-0.5) \quad 0 \quad (0.5) \quad 1 \quad (2)$

CV: $4x = 0$ | $x - 1 = 0$ | $x + 1 = 0$

$x = 0$ | $x = 1$ | $x = -1$

Increasing on $(-1, 0) + (1, \infty)$

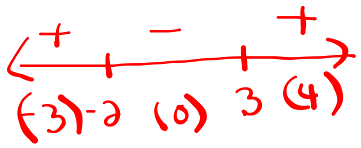
Decreasing on $(-\infty, -1) + (0, 1)$

① b) $y = 2x^3 - 3x^2 - 36x + 62$

$y' = 6x^2 - 6x - 36$

$y' = 6(x^2 - x - 6)$

$y' = 6(x-3)(x+2)$



CV: $x - 3 = 0$ | $x + 2 = 0$

$x = 3$ | $x = -2$

Increasing on $(-\infty, -2) + (3, \infty)$

Decreasing on $(-2, 3)$

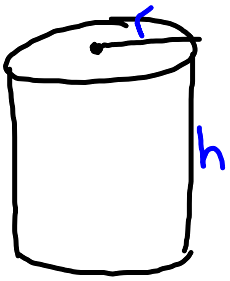
① c) $y = x^5 + 8x^3 + x$

$y' = 5x^4 + 24x^2 + 1$

y' is always positive. The function is always increasing

increasing on $(-\infty, \infty)$

②



Let r = the radius

$$A = 2\pi r^2 + 2\pi r h$$

Express with a single variable

$$V = \pi r^2 h$$

$$1000 = \pi r^2 h$$

$$\boxed{h = \frac{1000}{\pi r^2}}$$

$$A = 2\pi r^2 + 2\cancel{\pi}\cancel{r}\left[\frac{1000}{\cancel{\pi} r^2}\right]$$

$$A = 2\pi r^2 + 2000r^{-1}$$

$$A' = 4\pi r - \frac{2000}{r^2}$$

$$\frac{2000}{r^2} = 4\pi r$$

$$4\pi r^3 = 2000$$

$$r^3 = \frac{500}{\pi}$$

$$r = \sqrt[3]{\frac{500}{\pi}} \doteq 5.41 \text{ cm}$$

3 a) $f(x) = x^4 - 4x^3 - 8x^2 - 1$

$f'(x) = 4x^3 - 12x^2 - 16x$

$f'(x) = 4x(x^2 - 3x - 4)$

$f'(x) = 4x(x-4)(x+1)$

CV: $4x = 0$, $x = 0$ | $x - 4 = 0$, $x = 4$ | $x + 1 = 0$, $x = -1$

3 b) $f(x) = 2x^3 - 9x^2 - 60x + 82$

$f'(x) = 6x^2 - 18x - 60$

$f'(x) = 6(x^2 - 3x - 10)$

$f'(x) = 6(x-5)(x+2)$

CV: $x - 5 = 0$, $x = 5$ | $x + 2 = 0$, $x = -2$

④ a) $f(x) = 5x^4 + 20x^3 - 40x^2 + 8 \qquad -5 \le x \le 2$

$f'(x) = 20x^3 + 60x^2 - 80x$

$f'(x) = 20x(x^2 + 3x - 4)$

$f'(x) = 20x(x-1)(x+4)$

$0 = 20x(x-1)(x+4)$

CV: $x = -4, 0, 1$

$f(x) = 5x^4 + 20x^3 - 40x^2 + 8$

$f(-4) = 5(-4)^4 + 20(-4)^3 - 40(-4)^2 + 8 = -632 \quad \boxed{(-4, -632)}$

$f(0) = 5(0)^4 + 20(0)^3 - 40(0)^2 + 8 = 8 \quad (0, 8)$

$f(1) = 5(1)^4 + 20(1)^3 - 40(1)^2 + 8 = -7 \quad (1, -7)$

$f(-5) = 5(-5)^4 + 20(-5)^3 - 40(-5)^2 + 8 = -367 \quad (-5, -367)$

$f(2) = 5(2)^4 + 20(2)^3 - 40(2)^2 + 8 = 88 \quad \boxed{(2, 88)}$

④ b) $f(x) = x^4 - 4x^3 - 8x^2 - 1 \quad -3 \le x \le 5$

$f'(x) = 4x^3 - 12x^2 - 16x$

$f'(x) = 4x(x^2 - 3x - 4)$

$f'(x) = 4x(x+1)(x-4)$

cv: $4x=0$ | $x+1=0$ | $x-4=0$

$x=0$ | $x=-1$ | $x=4$

$f(x) = x^4 - 4x^3 - 8x^2 - 1$

$f(-3) = (-3)^4 - 4(-3)^3 - 8(-3)^2 - 1 = 81 + 108 - 72 - 1 = 116 \quad (-3, 116)$

abs max

$f(x) = x^4 - 4x^3 - 8x^2 - 1$

$f(-1) = (-1)^4 - 4(-1)^3 - 8(-1)^2 - 1 = 1 + 4 - 8 - 1 = -4 \quad (-1, -4)$

$f(x) = x^4 - 4x^3 - 8x^2 - 1$

$f(0) = (0)^4 - 4(0)^3 - 8(0)^2 - 1 = 0 - 0 - 0 - 1 = -1 \quad (0, -1)$

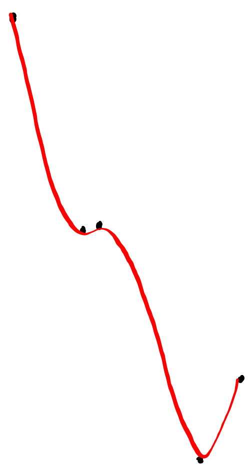
$f(x) = x^4 - 4x^3 - 8x^2 - 1$

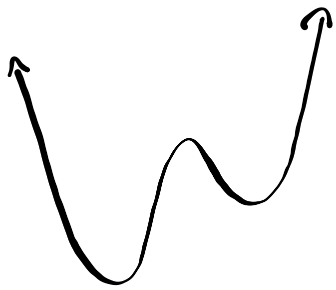
$f(4) = (4)^4 - 4(4)^3 - 8(4)^2 - 1 = 256 - 256 - 128 - 1 = -129 \quad (4, -129)$

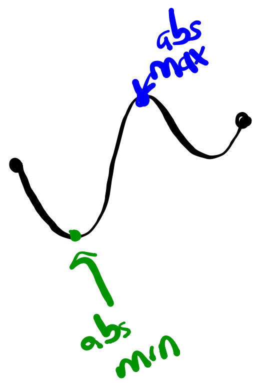
abs min

$f(x) = x^4 - 4x^3 - 8x^2 - 1$

$f(5) = (5)^4 - 4(5)^3 - 8(5)^2 - 1 = 625 - 500 - 200 - 1 = -76 \quad (5, -76)$






abs max
abs min

5a) $f(x) = 3x^4 - 16x^3 + 18x^2 + 1$

$f'(x) = 12x^3 - 48x^2 + 36x$

$f'(x) = 12x(x^2 - 4x + 3)$

$f'(x) = 12x(x-1)(x-3)$

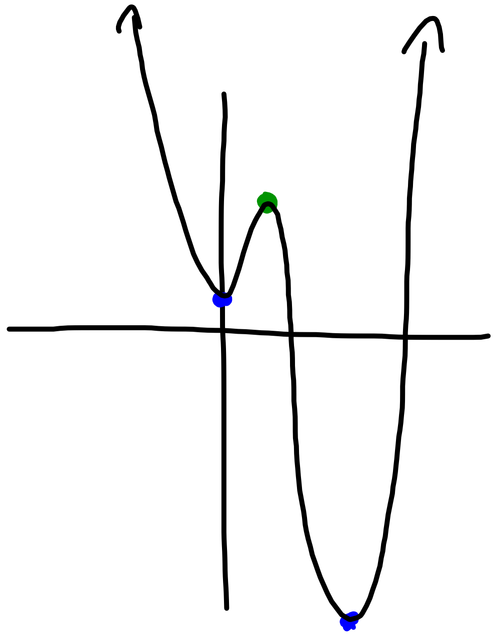
CV: $x = 0, 1, 3$

$f(0) = 1 \quad (0, 1)$

$f(1) = 6 \quad (1, 6)$

$f(3) = -26 \quad (3, -26)$

$$\begin{array}{ccccccc} - & & + & & - & & + \\ (-1) & 0 & (0.5) & 1 & (2) & 3 & (4) \\ & \text{min} & & \text{max} & & \text{min} & \end{array}$$



5) b) $f(x) = 1+3x^2-2x^3$

$f'(x) = 6x-6x^2$

$f'(x) = 6x(1-x)$

CV: $x=0,1$

$-\quad + \quad -$

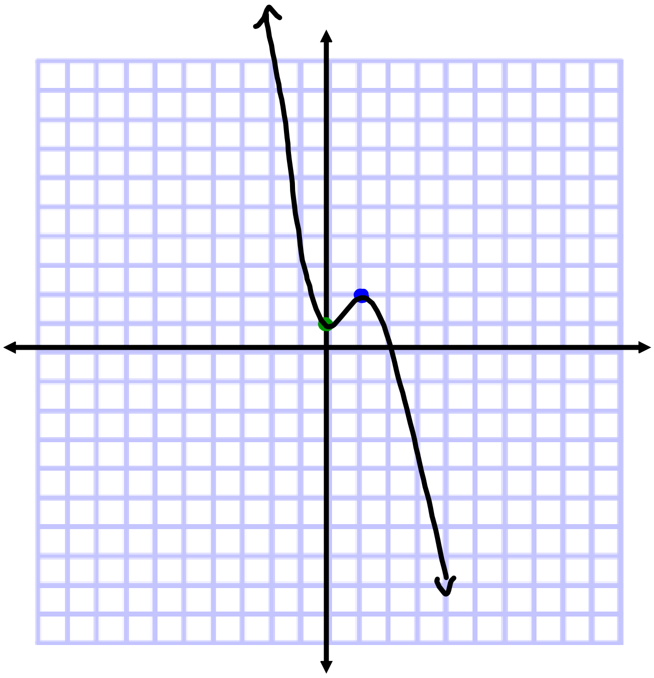
$(-1)\quad 0 \quad (\tfrac{1}{2}) \quad 1 \quad (2)$

$f(0) = 1+3(0)^2-2(0)^3 = 1$

$(0,1)$ local min

$f(1) = 1+3(1)^2-2(1) = 2$

$(1,2)$ local max



⑥ origin = $(0,0)$ $\qquad y=3x-2$

$$d=\sqrt{(x-x_1)^2+(y-y_1)^2}$$

$$d=\sqrt{(x-0)^2+(3x-2-0)^2}$$

$$d=\sqrt{x^2+(3x-2)^2}$$

$$d=\sqrt{x^2+9x^2-12x+4}$$

$$d=\sqrt{10x^2-12x+4}$$

$$* \quad f(x)=10x^2-12x+4$$

$$f'(x)=20x-12$$

$$0=20x-12$$

$$12=20x$$

$$\frac{12}{20}=x$$

$$\frac{3}{5}=x$$

or $f'(x)=20x-12$

$$f'(x)=4(5x-3)$$

CV: $5x-3=0$

$$5x=3$$

$$x=\frac{3}{5}$$

Find y:

$$y=3x-2$$

$$y=3\left(\frac{3}{5}\right)-2$$

$$y=\frac{9}{5}-\frac{10}{5}$$

$$y=\frac{-1}{5}$$

$-$ $\quad$ $+$

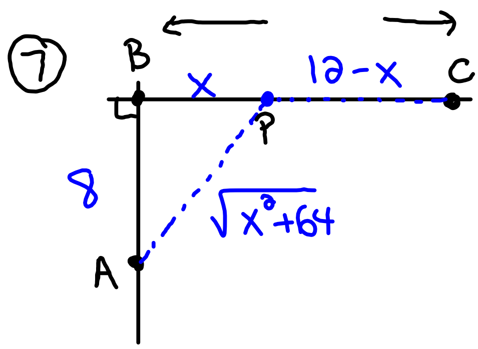
(0) $\quad \frac{3}{5}$ $\quad$ (1)

$\uparrow$

min

The point is $\left(\frac{3}{5},\frac{-1}{5}\right)$

7



Let x = the distance from B to P

$$T = \frac{d}{s}$$

$$T = \frac{\sqrt{x^2+64}}{2} + \frac{12-x}{6}$$

$$T = \frac{1}{2}(x^2+64)^{1/2} + \frac{12}{6} - \frac{1}{6}x$$

$$T' = \frac{1}{4}(x^2+64)^{-1/2}(2x) - \frac{1}{6}$$

$$T' = \frac{x}{2\sqrt{x^2+64}} - \frac{1}{6}$$

$$0 = \frac{x}{2\sqrt{x^2+64}} - \frac{1}{6}$$

$$\frac{1}{6} = \frac{x}{2\sqrt{x^2+64}}$$

$$2\sqrt{x^2+64} = 6x$$

$$4(x^2+64) = 36x^2$$

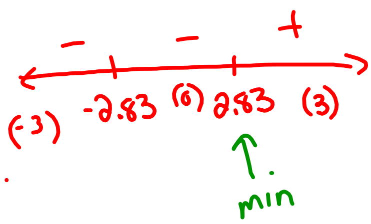
$$4x^2+256 = 36x^2$$

$$256 = 32x^2$$

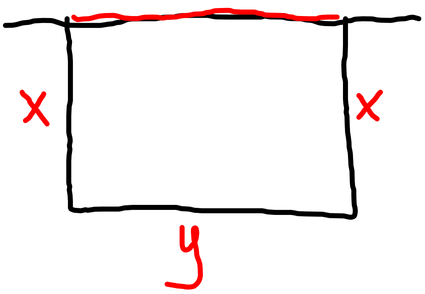
$$8 = x^2$$

$$x \doteq \pm 2.83$$

$\therefore$ Head to a point 2.83 Km East of A



8



$P = 2x + y$

$500 = 2x + y$

$\boxed{500 - 2x = y}$

$500 - 2(125) = y$

$500 - 250 = y$

$250\text{m} = y$

$\therefore 125\text{m} \times 250\text{m}$

$A = xy$ ← Express as a single variable

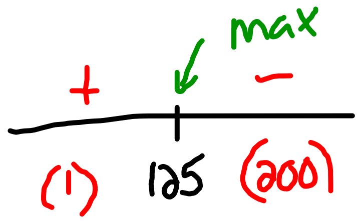
$A = x(500 - 2x)$

$A = 500x - 2x^2$

$A' = 500 - 4x$

$4x = 500$

$x = 125\text{m}$



⑨ $xy = 16$

$y = \frac{16}{x}$

$\boxed{y = \frac{16}{4} = 4}$

a) $S = x + y$

$S = x + \frac{16}{x}$

$S = x + 16x^{-1}$

$S' = 1 - 16x^{-2}$

$S' = 1 - \frac{16}{x^2}$

$0 = 1 - \frac{16}{x^2}$

$\frac{16}{x^2} = 1$

$x^2 = 16$

$x = \pm 4$

+ − +

(-5) -4 (1) 4 (5)

$\boxed{x = 4}$

b) $S = x + y^2$

$S = \frac{16}{y} + y^2$

$S = 16y^{-1} + y^2$

$S' = -16y^{-2} + 2y$

$S' = \frac{-16}{y^2} + 2y$

$0 = \frac{-16}{y^2} + 2y$

$\frac{16}{y^2} = 2y$

$16 = 2y^3$

$8 = y^3$

$\boxed{2 = y}$

− +

(0) 2 (3)

$x = \frac{16}{y}$

$\boxed{x = \frac{16}{2} = 8}$