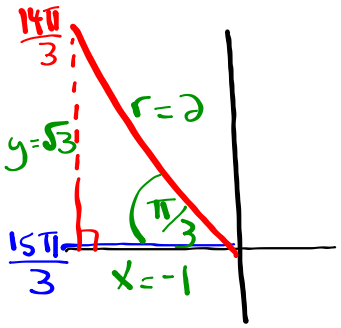
Questions from Homework

① $\sec\left(\frac{14\pi}{3}\right)\tan^2\left(\frac{37\pi}{6}\right) - \csc\left(\frac{17\pi}{2}\right)$

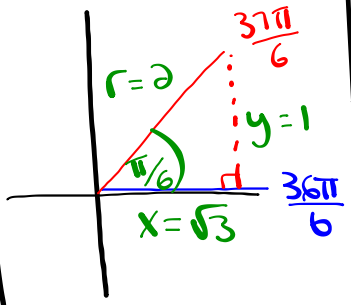
$\frac{13\pi}{3}, \frac{14\pi}{3}, \frac{15\pi}{3}$ → 5π

$\frac{36\pi}{6}, \frac{37\pi}{6}, \frac{38\pi}{6}$ → 6π

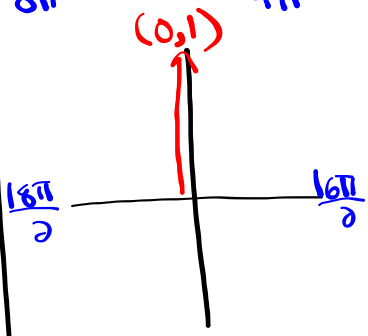
$\frac{16\pi}{2}, \frac{17\pi}{2}, \frac{18\pi}{2}$ → 8π, 9π



$\sec\left(\frac{14\pi}{3}\right) = \frac{2}{-1} = -2$



$\tan\left(\frac{37\pi}{6}\right) = \frac{1}{\sqrt{3}}$



$\csc\left(\frac{17\pi}{2}\right) = \frac{1}{1} = 1$

$\sec\left(\frac{14\pi}{3}\right)\tan^2\left(\frac{37\pi}{6}\right) - \csc\left(\frac{17\pi}{2}\right)$

$(-2)\left(\frac{1}{\sqrt{3}}\right)^2 - (1)$

$-2\left(\frac{1}{3}\right) - 1$

$-\frac{2}{3} - \frac{3}{3}$

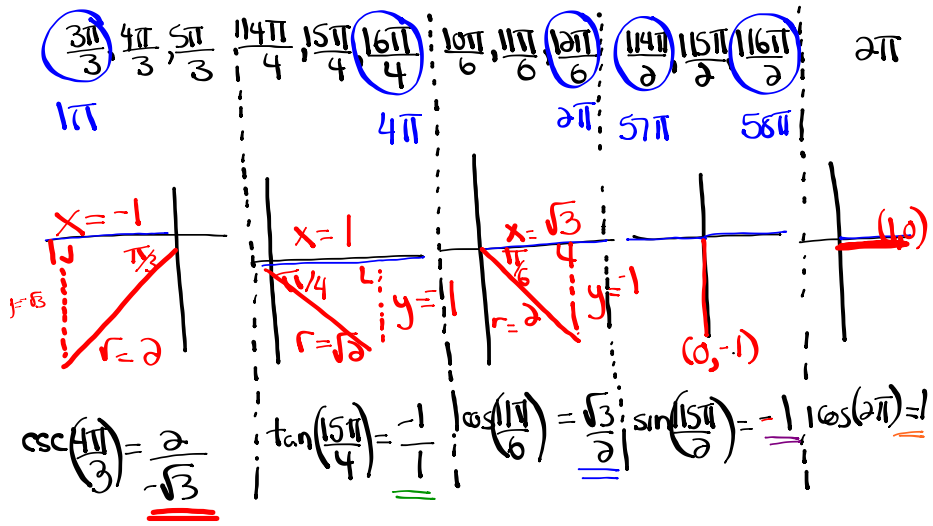
$-\frac{5}{3}$

Questions from Homework

$-\frac{13\pi}{6}+\frac{24\pi}{6}=\frac{11\pi}{6}$ $\quad -14\pi+16\pi=2\pi$

⑤ $\csc^2\left(\frac{4\pi}{3}\right)\tan\left(\frac{15\pi}{4}\right)+\cos\left(-\frac{13\pi}{6}\right)-\sin\left(\frac{115\pi}{2}\right)+\cos(-14\pi)$

$\csc^2\left(\frac{4\pi}{3}\right)\tan\left(\frac{15\pi}{4}\right)+\cos\left(\frac{11\pi}{6}\right)-\sin\left(\frac{115\pi}{2}\right)+\cos(2\pi)$



$\csc\left(\frac{4\pi}{3}\right)=\frac{2}{-\sqrt{3}}$ $\quad \tan\left(\frac{15\pi}{4}\right)=\frac{-1}{1}$ $\quad \cos\left(\frac{11\pi}{6}\right)=\frac{\sqrt{3}}{2}$ $\quad \sin\left(\frac{115\pi}{2}\right)=-1$ $\quad \cos(2\pi)=1$

$\csc^2\left(\frac{4\pi}{3}\right)\tan\left(\frac{15\pi}{4}\right)+\cos\left(\frac{11\pi}{6}\right)-\sin\left(\frac{115\pi}{2}\right)+\cos(2\pi)$

$\left(-\frac{2}{\sqrt{3}}\right)^2(-1)+\left(\frac{\sqrt{3}}{2}\right)-(-1)+(1)$

$\left(\frac{4}{3}\right)(-1)+\frac{\sqrt{3}}{2}+1+1$

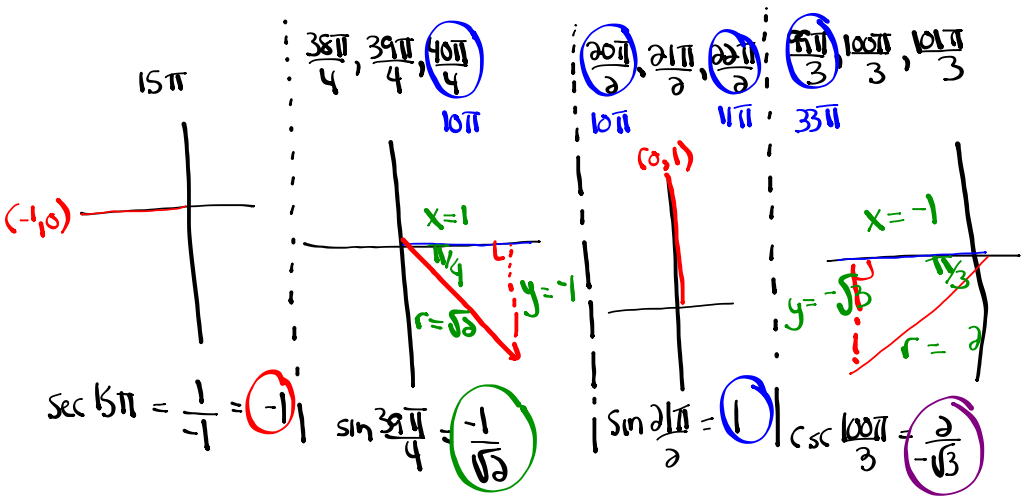
$-\frac{4}{3}+\frac{\sqrt{3}}{2}+\frac{2}{1}$

$-\frac{8}{6}+\frac{3\sqrt{3}}{6}+\frac{12}{6}$

$\boxed{\frac{4+3\sqrt{3}}{6}}$ or $\frac{3\sqrt{3}+4}{6}$

Questions from Homework

⑥ $\sec 15\pi + \sqrt{2}\sin\frac{39\pi}{4}\sin\frac{21\pi}{2} - \csc^2\frac{100\pi}{3}$



$\sec 15\pi = \frac{1}{-1} = -1$

$\sin\frac{39\pi}{4} = \frac{-1}{\sqrt{2}}$

$\sin\frac{21\pi}{2} = 1$

$\csc\frac{100\pi}{3} = \frac{2}{-\sqrt{3}}$

$$\sec 15\pi + \sqrt{2}\sin\frac{39\pi}{4}\sin\frac{21\pi}{2} - \csc^2\frac{100\pi}{3}$$

$$(-1) + \sqrt{2}\left(\frac{-1}{\sqrt{2}}\right)(1) - \left(\frac{2}{-\sqrt{3}}\right)^2$$

$$-1 + \sqrt{2}\left(\frac{-1}{\sqrt{2}}\right)(1) - \frac{4}{3}$$

$$-1 - \frac{\sqrt{2}}{\sqrt{2}} - \frac{4}{3}$$

$$-1 - 1 - \frac{4}{3}$$

$$-\frac{3}{3} - \frac{3}{3} - \frac{4}{3}$$

$$\boxed{-\frac{10}{3}}$$

Questions from Homework

⑧ $\dfrac{\tan\left(\frac{\pi}{4}\right)+\sec\left(\frac{301\pi}{3}\right)}{\csc\left(\frac{25\pi}{2}\right)+\cot\left(\frac{5\pi}{6}\right)}$

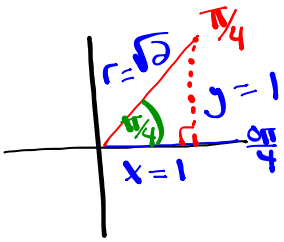
* $-\frac{15\pi}{4}+\frac{4\pi}{1}$ | $-\frac{31\pi}{6}+\frac{6\pi}{1}$

$-\frac{15\pi}{4}+\frac{16\pi}{4}$ | $-\frac{31\pi}{6}+\frac{36\pi}{6}$

$\frac{\pi}{4}$ | $\frac{5\pi}{6}$

① $\frac{9\pi}{4}, \frac{\pi}{4}, \frac{2\pi}{4}$

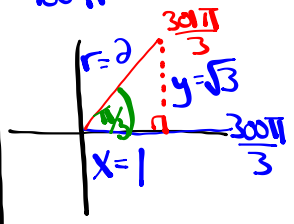
0



$\tan\frac{\pi}{4}=\frac{1}{1}=1$

② $\frac{3001\pi}{3}, \frac{301\pi}{3}, \frac{3021\pi}{3}$

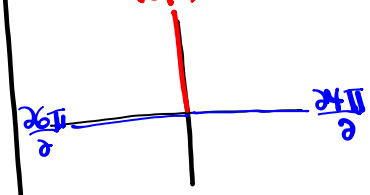
100π



$\sec\frac{301\pi}{3}=\frac{2}{1}=2$

③ $\frac{24\pi}{2}, \frac{25\pi}{2}, \frac{26\pi}{2}$

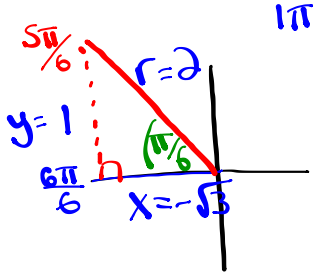
12π (0,1) 13π



$\csc\frac{25\pi}{2}=\frac{1}{1}=1$

④ $\frac{4\pi}{6}, \frac{5\pi}{6}, \frac{6\pi}{6}$

1π



$\cot\frac{5\pi}{6}=\frac{-\sqrt{3}}{1}=-\sqrt{3}$

$\dfrac{\tan\left(\frac{\pi}{4}\right)+\sec\left(\frac{301\pi}{3}\right)}{\csc\left(\frac{25\pi}{2}\right)+\cot\left(\frac{5\pi}{6}\right)}$

$\dfrac{1+2}{1-\sqrt{3}}$

$\dfrac{3(1+\sqrt{3})}{(1-\sqrt{3})(1+\sqrt{3})}$

$\dfrac{3+3\sqrt{3}}{1+\sqrt{3}-\sqrt{3}-3}$

$\dfrac{3+3\sqrt{3}}{-2}$ or $\boxed{\dfrac{-3-3\sqrt{3}}{2}}$

Introduction to Trigonometric Equations

trigonometric equation

- an equation involving trigonometric ratios

Focus on...

- algebraically solving first-degree and second-degree trigonometric equations in radians and in degrees
- verifying that a specific value is a solution to a trigonometric equation
- identifying exact and approximate solutions of a trigonometric equation in a restricted domain
- determining the general solution of a trigonometric equation

Did You Know?

In equations, mathematicians often use the notation $\cos^2 \theta$. This means the same as $(\cos \theta)^2$.

Let's start with basic LINEAR trigonometric equations...

Solve: $\sin\theta = 0.9659$ (Approx), $-360° < \theta < 720°$ Domain in degrees

...Pre-Calculus 110

- Reference angle?
- Which quadrants?
- Any co-terminal angles acceptable?

- If the domain is in degrees, give solutions in degrees.
- If the domain is in radians, give solutions in radians.

$\sin\theta = 0.9659$

① Find $\bar{\theta}$

② where is $\sin\theta > 0$

S	A
T	C

$\bar{\theta} = \sin^{-1}(0.9659)$

③ Find θ

$\bar{\theta} = 75°$

Q1	Q2
$\theta = \bar{\theta}$	$\theta = 180° - \bar{\theta}$
$\theta = 75°$	$\theta = 180° - 75° = 105°$
$\theta = 75° - 360° = -285°$	$\theta = 105° - 360° = -255°$
$\theta = 75° + 360° = 435°$	$\theta = 105° + 360° = 465°$

Solve: $\sec\theta = -1.3054\ ,\ -2\pi \le \theta \le 2\pi$ (Approx.) (Radians) $-6.28 \le \theta \le 6.28$

$\cos\theta = \frac{1}{-1.3054}$

$\cos\theta = -0.7660$

① Find $\bar{\theta}$

② where is $\cos\theta < 0$

S	A
T	C

$\bar{\theta} = \cos^{-1}(0.7660)$

$\bar{\theta} = 0.7$ rads

ignore negative

③ Find θ

Q2	Q3
$\theta = \pi - \bar{\theta}$	$\theta = \pi + \bar{\theta}$
$\theta = 3.14 - 0.7 = 2.44$	$\theta = 3.14 + 0.7 = 3.84$
$\theta = 2.44 - 6.28 = -3.84$	$\theta = 3.84 - 6.28 = -2.44$

Warm-up

Ex: $\tan\theta = -0.8524$ (Approx.), $-360^\circ \le \theta \le 360^\circ$ (Degrees)

① Find $\bar{\theta}$

$\bar{\theta} = \tan^{-1}(0.8524)$ ← ignore negative

$\bar{\theta} = 40.4^\circ$

② Where is $\tan\theta < 0$

S	A
T	C

③ Find θ

Q2	Q4
$\theta = 180^\circ - \bar{\theta}$	$\theta = 360^\circ - \bar{\theta}$
$\theta = 180^\circ - 40.4^\circ = 139.6^\circ$	$\theta = 360^\circ - 40.4^\circ = 319.6^\circ$
$\theta = 139.6^\circ - 360^\circ = -219.4^\circ$	$\theta = 319.6^\circ - 360^\circ = -40.4^\circ$

(Exact)

$-\frac{1}{\sqrt{2}} = -0.7071$

Ex. $\sqrt{2}\cos\theta + 1 = 0, \quad -360° \le \theta \le 720°$ (Degrees)

$\frac{\sqrt{2}\cos\theta}{\sqrt{2}} = \frac{-1}{\sqrt{2}}$

$\cos\theta = \frac{-1}{\sqrt{2}}$

① Find $\bar{\theta}$

$\bar{\theta} = \cos^{-1}\left(\frac{1}{\sqrt{2}}\right)$ (Triangle #1) $\cos\theta = \frac{a}{h} = \frac{x}{r}$

$\bar{\theta} = 45°$

② Where is $\cos\theta < 0$

S	A
T	C

③ Find θ

Q2	Q3
$\theta = 180° - \bar{\theta}$	$\theta = 180° + \bar{\theta}$
$\theta = 180° - 45° = 135°$	$\theta = 180° + 45° = 225°$
$\theta = 135° - 360° = -225°$	$\theta = 225° - 360° = -135°$
$\theta = 135° + 360° = 495°$	$\theta = 225° + 360° = 585°$

(Exact)

Ex. $\sin x + 1 = 0, \; -2\pi \le x \le 4\pi$ (Radians) $-\frac{4\pi}{2} \le x \le \frac{8\pi}{2}$

$\sin x = -1$

① Find x (Unit Circle)

$x = \frac{3\pi}{2}$

$x = \frac{3\pi}{2} - \frac{2\pi}{1} = \frac{3\pi}{2} - \frac{4\pi}{2} = -\frac{\pi}{2}$

$x = \frac{3\pi}{2} + \frac{2\pi}{1} = \frac{3\pi}{2} + \frac{4\pi}{2} = \frac{7\pi}{2}$

Your Turn

Solve each trigonometric equation in the specified domain.

a) $3\cos\theta - 1 = \cos\theta + 1,\ -2\pi \le \theta \le 2\pi$

b) $4\sec x + 8 = 0,\ 0° \le x < 360°$

(Exact)

a) $3\cos\theta - 1 = \cos\theta + 1$, $-2\pi \le \theta \le 2\pi$ (Radians)

$3\cos\theta - \cos\theta = 1 + 1$

$\frac{2\cos\theta}{2} = \frac{2}{2}$

$\cos\theta = 1$

① Find θ (unit circle)

$\theta = 0, 2\pi$

$\theta = 0 - 2\pi = -2\pi$

---

b) $4\sec x + 8 = 0$, $0° \le x < 360°$

$4\sec x = -8$

$\sec x = -2$

$\cos x = \frac{-1}{2}$

① Find $\bar{x}$

$\bar{x} = \cos^{-1}\left(\frac{1}{2}\right)$ (Triangle #2)

$\bar{x} = 60°$

② where is $\cos x < 0$

S	A
T	C

③ Find x

Q2	Q3
$x = 180° - \bar{x}$	$x = 180° + \bar{x}$
$x = 180° - 60° = 120°$	$x = 180° + 60° = 240°$

Homework

Finish worksheet and Page 211 #1-5

$\frac{1}{\sqrt{3}} = \frac{\sqrt{3}}{3}$

$\frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2}$

④ e) $\sec\theta = 2.77$ $0 \le \theta \le 2\pi$

$0 \le \theta \le 6.28$

$\cos\theta = \frac{1}{2.77}$

$\cos\theta = 0.3610$

① Find $\bar{\theta}$:

$\bar{\theta} = \cos^{-1}(0.3610)$

$\bar{\theta} = 1.20$

② Where is $\cos\theta > 0$

S	A ✓
T	C ✓

③

Q1	Q4
$\theta = 1.2$	$\theta = 6.28 - 1.2$
	$\theta = 5.08$

Worksheet Solutions

① $\sin\theta = -\frac{\sqrt{3}}{2}$ where is $\sin\theta < 0$ (S \| A / T \| C)

use positive

$\bar{\theta} = \sin^{-1}\left(\frac{\sqrt{3}}{2}\right)$

$\bar{\theta} = 60°$

Q3	Q4
$\theta = 180° + 60° = 240°$	$\theta = 360° - 60° = 300°$
$240° \pm 360°n, n \in \mathbb{N}$	$300° \pm 360°n, n \in \mathbb{N}$

② $\cos\theta = -\frac{1}{2}$ where is $\cos\theta < 0$ (S \| A / T \| C)

$\bar{\theta} = \cos^{-1}\left(\frac{1}{2}\right)$

$\bar{\theta} = 60°$

Q2	Q3
$\theta = 180° - 60° = 120°$	$\theta = 180° + 60° = 240°$
$120° \pm 360°n, n \in \mathbb{N}$	$240° \pm 360°n, n \in \mathbb{N}$

③ $\tan\theta = -\frac{\sqrt{3}}{3}$ where is $\tan\theta < 0$ (S \| A / T \| C)

$\tan\theta = -\frac{1}{\sqrt{3}}$

$\bar{\theta} = \tan^{-1}\left(\frac{1}{\sqrt{3}}\right)$

$\bar{\theta} = 30°$

Q2	Q4
$\theta = 180° - 30° = 150°$	$\theta = 360° - 30° = 330°$
$150° \pm 360°n, n \in \mathbb{N}$	$330° \pm 360°n, n \in \mathbb{N}$

④ $\tan\theta = \frac{\sqrt{3}}{1}$ Where is $\tan\theta > 0$ (S \| A / T \| C)

$\bar{\theta} = \tan^{-1}\left(\frac{\sqrt{3}}{1}\right)$

$\bar{\theta} = 60°$

Q1	Q4
$\theta = 60°$	$\theta = 360° - 60° = 300°$
$60° \pm 360°n, n \in \mathbb{N}$	$300° \pm 360°n, n \in \mathbb{N}$

⑥ $\sin\theta = -1$

* Unit Circle

$\theta = 270° \rightarrow 270° \pm 360°n, n \in \mathbb{N}$

⑬ $5\sin\theta - 4 = 0$ Where is $\sin\theta > 0$ (S \| A / T \| C)

$\sin\theta = \frac{4}{5}$

$\sin\theta = 0.8$ (approx. value)

$\bar{\theta} = \sin^{-1}(0.8)$

$\bar{\theta} = 53.1°$

Q1	Q2
$\theta = 53.1°$	$\theta = 180° - 53.1$ $= 126.9°$
$53.1° \pm 360°n, n \in \mathbb{N}$	$126.9° \pm 360°n, n \in \mathbb{N}$

Worksheet Solutions

⑧ $2\sin\theta + 1 = 0$

$\frac{2\sin\theta}{2} = \frac{-1}{2}$

$\sin\theta = -\frac{1}{2}$

① Find $\bar{\theta}$

$\bar{\theta} = \sin^{-1}\left(\frac{1}{2}\right)$ (Triangle #2)

$\bar{\theta} = 30°$

② where is $\sin\theta < 0$ S | A / T | C

③

Q3	Q4
$\theta = 180° + \bar{\theta}$	$\theta = 360° - \bar{\theta}$
$\theta = 180° + 30° = 210°$	$\theta = 360° - 30° = 330°$
$210° \pm 360°n, n \in \mathbb{N}$	$330° \pm 360°n, n \in \mathbb{N}$

⑨ $\sqrt{3} - 2\sin\theta = 0$

$\frac{-2\sin\theta}{-2} = \frac{-\sqrt{3}}{-2}$

$\sin\theta = \frac{\sqrt{3}}{2}$

① Find $\bar{\theta}$

$\bar{\theta} = \sin^{-1}\left(\frac{\sqrt{3}}{2}\right)$ (Triangle #2)

$\bar{\theta} = 60°$

② where is $\sin\theta > 0$ S | A / T | C

③

Q1	Q2
$\theta = \bar{\theta}$	$\theta = 180° - \bar{\theta}$
$\theta = 60°$	$\theta = 180° - 60° = 120°$
$60° \pm 360°n, n \in \mathbb{N}$	$120° \pm 360°n, n \in \mathbb{N}$

⑩ $\sqrt{\tan^2\theta} = \sqrt{3}$

$\tan\theta = \pm\sqrt{3}$

① Find $\bar{\theta}$

$\bar{\theta} = \tan(\sqrt{3})$ (Triangle #2)

$\bar{\theta} = 60°$

② where is $\tan\theta$ positive/negative S | A / T | C

③

Q1	Q2	Q3	Q4
$\theta = \bar{\theta}$	$\theta = 180° - \bar{\theta}$	$\theta = 180° + \bar{\theta}$	$\theta = 360° - \bar{\theta}$
$\theta = 60°$	$\theta = 120°$	$\theta = 240°$	$\theta = 300°$
$60° \pm 360°n$	$120° \pm 360°n$	$240° \pm 360°n$	$300° \pm 360°n$

⑪ $5\sin\theta - 4 = 0$

$\frac{5\sin\theta}{5} = \frac{4}{5}$

$\sin\theta = \frac{4}{5} = 0.8$ (Approx)

① Find $\bar{\theta}$

$\bar{\theta} = \sin^{-1}(0.8)$

$\bar{\theta} = 53.1°$

② where is $\sin\theta > 0$ S | A / T | C

③

Q1	Q2
$\theta = \bar{\theta}$	$\theta = 180° - \bar{\theta}$
$\theta = 53.1°$	$\theta = 126.9°$
$53.1° \pm 360°n, n \in \mathbb{N}$	$126.9 \pm 360°n, n \in \mathbb{N}$

Worksheet Solutions

Backside of sheet $0 \le \theta \le 2\pi$

③ $\tan\theta = \frac{\sqrt{3}}{3} = \frac{1}{\sqrt{3}}$

① Find $\bar{\theta}$

$\bar{\theta} = \tan^{-1}\left(\frac{1}{\sqrt{3}}\right)$ (Triangle #4)

$\bar{\theta} = \frac{\pi}{6}$

② where is $\tan\theta > 0$

S	A ✓
T ✓	C

③ Find θ

Q1	Q3
$\theta = \bar{\theta}$	$\theta = \pi + \bar{\theta}$
$\theta = \frac{\pi}{6}$	$\theta = \pi + \frac{\pi}{6} = \frac{6\pi}{6} + \frac{\pi}{6} = \frac{7\pi}{6}$

Factoring trinomials:

① Hard Trinomial

$2x^2+7x+6$ $\quad 2\times 6=12$, $\ 3+4=7$

$(x+\frac{3}{2})(x+\frac{4}{2})$

$(2x+3)(x+2)$

② Simple trinomial

x^2+7x+6 $\quad 6\times 1=6$, $\ 6+1=7$

$(x+1)(x+6)$

③ Difference of Squares

x^2-16

$(x-4)(x+4)$

④ Common Factor

$3x^2-9x^3+21x$

$3x(x-3x^2+7)$

Let's move onto QUADRATIC trigonometric equations...

...Pre-Calculus 110

- What strategies can we use to solve quadratic equations? (Factor)
- Quadratic trigonometric equations will ultimately become TWO linear trigonometric equations.

Solve: $2\sin^2 x + \sin x = 1 \quad 0 \le x \le 4\pi$ (Radians)

$2\sin^2 x + \sin x - 1 = 0$ Hard Trinomial

$(\sin x - 1)(\sin x + 2) = 0$ $-1 + 2 = 1$, $-1 \times 2 = -2$ (2, -1)

$(2\sin x - 1)(\sin x + 1) = 0$

$2\sin x - 1 = 0$

$\frac{2\sin x}{2} = \frac{1}{2}$

$\sin x = \frac{1}{2}$

① Find $\bar{x}$:

$\bar{x} = \sin^{-1}\left(\frac{1}{2}\right)$ (Triangle #4)

$\bar{x} = \frac{\pi}{6}$

② where is $\sin x > 0$

S	A
T	C

③ Find x:

Q1	Q2
$x = \bar{x}$	$x = \pi - \bar{x}$
$x = \pi/6$	$x = \pi - \frac{\pi}{6}$
	$x = \frac{6\pi}{6} - \frac{\pi}{6} = \frac{5\pi}{6}$
$x = \frac{\pi}{6} + 2\pi$	$x = \frac{5\pi}{6} + 2\pi$
$x = \frac{\pi}{6} + \frac{12\pi}{6} = \frac{13\pi}{6}$	$x = \frac{5\pi}{6} + \frac{12\pi}{6} = \frac{17\pi}{6}$

$\sin x + 1 = 0$

$\sin x = -1$ (Unit Circle)

① Find x:

$x = \frac{3\pi}{2}$

$x = \frac{3\pi}{2} + 2\pi$

$x = \frac{3\pi}{2} + \frac{4\pi}{2} = \frac{7\pi}{2}$

Ex. $\cos^2\theta - \frac{1}{2}\cos\theta = 0, \ -2\pi \le \theta \le 4\pi$ (Common Factor)

(Radians)

$\cos\theta\left(\cos\theta - \frac{1}{2}\right) = 0$

$\cos\theta = 0$ (Unit Circle)

① Find θ:

$\theta = \frac{\pi}{2}$ $\quad \theta = \frac{3\pi}{2}$

$\theta = \frac{\pi}{2} - 2\pi$ $\quad \theta = \frac{3\pi}{2} - 2\pi$

$\theta = \frac{\pi}{2} - \frac{4\pi}{2} = \frac{-3\pi}{2}$ $\quad \theta = \frac{3\pi}{2} - \frac{4\pi}{2} = \frac{-\pi}{2}$

$\theta = \frac{\pi}{2} + 2\pi$ $\quad \theta = \frac{3\pi}{2} + 2\pi$

$\theta = \frac{\pi}{2} + \frac{4\pi}{2} = \frac{5\pi}{2}$ $\quad \theta = \frac{3\pi}{2} + \frac{4\pi}{2} = \frac{7\pi}{2}$

$\cos\theta - \frac{1}{2} = 0$

$\cos\theta = \frac{1}{2}$

① Find $\bar{\theta}$

$\bar{\theta} = \cos^{-1}\left(\frac{1}{2}\right)$ (Triangle)

$\bar{\theta} = \frac{\pi}{3}$

② where is $\cos\theta > 0$

S	~~A~~
T	C

③ Find θ:

Q1

$\theta = \bar{\theta}$ $\quad \theta = 2\pi - \bar{\theta}$

$\theta = \frac{\pi}{3}$ $\quad \theta = \frac{6\pi}{3} - \frac{\pi}{3} = \frac{5\pi}{3}$

$\theta = \frac{\pi}{3} - 2\pi$ $\quad \theta = \frac{5\pi}{3} - 2\pi$

$\theta = \frac{\pi}{3} - \frac{6\pi}{3} = \frac{-5\pi}{3}$ $\quad \theta = \frac{5\pi}{3} - \frac{6\pi}{3} = \frac{-\pi}{3}$

$\theta = \frac{\pi}{3} + 2\pi$ $\quad \theta = \frac{5\pi}{3} + 2\pi$

$\theta = \frac{\pi}{3} + \frac{6\pi}{3} = \frac{7\pi}{3}$ $\quad \theta = \frac{5\pi}{3} + \frac{6\pi}{3} = \frac{11\pi}{3}$

$-\frac{12\pi}{6} \le \theta \le \frac{24\pi}{6}$

Ex. $6\sin^2 x - \sin x = 2 \quad -2\pi \le \theta \le 4\pi$ (Radians)

(Hard Trinomial)

$3 + -4 = -1$

$3 \times -4 = -12$

$6\sin^2\theta - \sin\theta - 2 = 0$

$(\sin\theta + \frac{3}{6})(\sin\theta - \frac{4}{6}) = 0$

$(\sin\theta + \frac{1}{2})(\sin\theta - \frac{2}{3}) = 0$

$(2\sin\theta + 1)(3\sin\theta - 2) = 0$

$2\sin\theta + 1 = 0$

$\frac{2\sin\theta}{2} = \frac{-1}{2}$

$\sin\theta = -\frac{1}{2}$

① Find $\bar{\theta}$:

$\bar{\theta} = \sin^{-1}(\frac{1}{2})$ (Triangle #4)

$\bar{\theta} = \frac{\pi}{6}$ (Exact)

② where is $\sin\theta < 0$ S | A / T | C

② Find θ:

Q3	Q4
$\theta = \pi + \bar{\theta}$	$\theta = 2\pi - \bar{\theta}$
$\theta = \pi + \frac{\pi}{6}$	$\theta = 2\pi - \frac{\pi}{6}$
$\theta = \frac{6\pi}{6} + \frac{\pi}{6} = \frac{7\pi}{6}$	$\theta = \frac{12\pi}{6} - \frac{\pi}{6} = \frac{11\pi}{6}$
$\theta = \frac{7\pi}{6} - 2\pi$	$\theta = \frac{11\pi}{6} - 2\pi$
$\theta = \frac{7\pi}{6} - \frac{12\pi}{6} = \frac{-5\pi}{6}$	$\theta = \frac{11\pi}{6} - \frac{12\pi}{6} = \frac{-\pi}{6}$
$\theta = \frac{7\pi}{6} + 2\pi$	$\theta = \frac{11\pi}{6} + 2\pi$
$\theta = \frac{7\pi}{6} + \frac{12\pi}{6} = \frac{19\pi}{6}$	$\theta = \frac{11\pi}{6} + \frac{12\pi}{6} = \frac{23\pi}{6}$

$3\sin\theta - 2 = 0$

$\frac{3\sin\theta}{3} = \frac{2}{3}$

$\sin\theta = \frac{2}{3} = 0.\overline{6}$

① Find $\bar{\theta}$

$\bar{\theta} = \sin^{-1}(0.\overline{6})$

$\bar{\theta} = 0.73$ (Approx)

② where is $\sin\theta > 0$ ~~S~~ | ~~A~~ / T | C

③ Find θ:

Q1	Q2
$\theta = \bar{\theta}$	$\theta = 3.14 - \bar{\theta}$
$\theta = 0.73$	$\theta = 3.14 - 0.73 = 2.41$
$\theta = 0.73 - 6.28$	$\theta = 2.41 - 6.28$
$\theta = -5.55$	$\theta = -3.87$
$\theta = 0.73 + 6.28$	$\theta = 2.41 + 6.28$
$\theta = 7.01$	$\theta = 8.69$

Your Turn

Solve for θ.

$\cos^2\theta - \cos\theta - 2 = 0,\ 0° \le \theta < 360°$ (Simple Trinomial)

Give solutions as exact values (Degrees) where possible. Otherwise, give approximate measures to the nearest thousandth of a degree.

$\cos^2\theta - \cos\theta - 2 = 0$

$(\cos\theta - 2)(\cos\theta + 1) = 0$

$-2 + 1 = -1$

$-2 \times 1 = -2$

$\cos\theta - 2 = 0$

$\cos\theta = 2$ (No diagram)

$\cos\theta + 1 = 0$

$\cos\theta = -1$ (Unit Circle)

① Find $\bar{\theta}$:

$\bar{\theta} = \cos^{-1}(2)$

$\bar{\theta}$ = Math Error

Not Possible

(Extraneous)

① Find θ:

$\theta = 180°$

General Solution of a Trigonometric Equation

↖ All angles

Solve: $3\cos^2\theta - \cos\theta = 2; \ \theta \in \mathbb{R}$ (Assume Degrees)

$3\cos^2\theta - 1\cos\theta - 2 = 0$ Hard Trinomial

$-3 + 2 = -1$

$-3 \times 2 = -6$

$(\cos\theta - 3)(\cos\theta + 2) = 0$

$(\cos\theta - 1)(3\cos\theta + 2) = 0$

$\cos\theta - 1 = 0$

$\cos\theta = 1$ (unit circle)

① Find θ:

$\theta = 0°$ or $\theta = 360°$

$0° \pm 360°n, n \in \mathbb{N}$

$360° \pm 360°n, n \in \mathbb{N}$

not necessary

$3\cos\theta + 2 = 0$

$\frac{3\cos\theta}{3} = \frac{-2}{3}$

$\cos\theta = \frac{-2}{3} = -0.\overline{6}$

① Find $\bar{\theta}$

$\bar{\theta} = \cos^{-1}(0.\overline{6})$ (Approx)

$\theta = 48.2°$

② where is $\cos\theta < 0$ — S | A / T | C

③ Find θ:

Q2	Q3
$\theta = 180° - \bar{\theta}$	$\theta = 180° + \bar{\theta}$
$\theta = 180° - 48.2° = 131.8°$	$\theta = 180° + 48.2° = 228.2°$
$131.8° \pm 360°n, n \in \mathbb{N}$	$228.2° \pm 360°n, n \in \mathbb{N}$

Determine the general solution for $\sin^2 x - 1 = 0$ over the real numbers if x is measured in radians.

(Diff. of squares)

$\sin^2 x - 1 = 0$

$(\sin x + 1)(\sin x - 1) = 0$

$\sin x + 1 = 0$ $\qquad$ $\sin x - 1 = 0$

$\sin x = -1$ ← (Unit circle) → $\sin x = 1$

① $x = 270°$ $\qquad$ ① $x = 90°$

$270° \pm 360°n, n \in \mathbb{N}$ $\qquad$ $90° \pm 360°n, n \in \mathbb{N}$

General Solution is all angles

Did You Know?

$2n$, where $n \in \mathrm{I}$, represents all even integers.

$2n + 1$, where $n \in \mathrm{I}$, is an expression for all odd integers.

$$x = \frac{\pi}{2} + 2\pi n, \text{ where } n \in \mathrm{I}$$

$$x = \frac{3\pi}{2} + 2\pi n, \text{ where } n \in \mathrm{I}$$

or

$$x = \frac{\pi}{2} + \pi n, \text{ where } n \in \mathrm{I}$$

or

$$(2n + 1)\left(\frac{\pi}{2}\right), n \in \mathrm{I}$$

Determine the general solution for $\cos^2 x - 1 = 0$, where the domain is real numbers measured in degrees.

$\cos^2 x - 1 = 0$

$(\cos x - 1)(\cos x + 1) = 0$

$\cos x - 1 = 0$

$\cos x = 1$ (unit circle)

$x = 0^\circ, 360^\circ$

$x = 0^\circ \pm 360^\circ n, n \in N$

$\cos x + 1 = 0$

$\cos x = -1$ (unit circle)

$x = 180^\circ$

$x = 180^\circ \pm 360^\circ n, n \in N$

$x = 0^\circ \pm 180^\circ n, n \in N$

Chapter 4 Test

Open Response:

1. Trig Expression (# 2 on Review)
2. Trig Equation (# 1 e) on Review)
3. Problem involving Trig Ratios (# 3 or 4 on Review)

Ch. 4 Review

① a) $\cos\theta = \frac{\sqrt{3}}{2}$, $0° < \theta < 360°$

① Find $\bar{\theta}$:

$\bar{\theta} = \cos^{-1}\left(\frac{\sqrt{3}}{2}\right)$ (Triangle #2)

$\bar{\theta} = 30°$

② where is $\cos\theta > 0$

S	A
T	C

③ Find θ:

Q1	Q4
$\theta = \bar{\theta}$	$\theta = 360° - \bar{\theta}$
$\theta = 30°$	$\theta = 360° - 30° = 330°$

b) $\sin\theta = -\frac{\sqrt{2}}{2} = -\frac{1}{\sqrt{2}}$ $0 < \theta < 2\pi$

① Find $\bar{\theta}$

$\bar{\theta} = \sin^{-1}\left(\frac{1}{\sqrt{2}}\right)$ (Triangle #3)

$\bar{\theta} = \frac{\pi}{4}$

② where is $\sin\theta < 0$

S	A
T	C

③ Find θ:

Q3	Q4
$\theta = \pi + \bar{\theta}$	$\theta = 2\pi - \bar{\theta}$
$\theta = \pi + \frac{\pi}{4}$	$\theta = 2\pi - \frac{\pi}{4}$
$\theta = \frac{4\pi}{4} + \frac{\pi}{4} = \frac{5\pi}{4}$	$\theta = \frac{8\pi}{4} - \frac{\pi}{4} = \frac{7\pi}{4}$

Ch. 4 Review

① c) $\cot\theta = \text{undefined}$, $0° \le \theta \le 270°$ $\cot\theta = \frac{x}{y}$

(Unit Circle)

* anywhere $y=0$

(i) Find θ:

$\theta = 0°$

$\theta = 180°$

~~$\theta = 360°$~~

① d) $2\sin\theta - 1 = 0$ $-2\pi \le \theta \le 2\pi$

$\frac{2\sin\theta}{2} = \frac{1}{2}$

$\sin\theta = \frac{1}{2}$ (Triangle #4)

(i) Find $\bar{\theta}$:

$\bar{\theta} = \sin^{-1}\left(\frac{1}{2}\right)$

$\bar{\theta} = \frac{\pi}{6}$

(ii) where is $\sin\theta > 0$

S	A
T	C

(iii) Find θ:

Q1	Q2
$\theta = \bar{\theta}$	$\theta = \pi - \bar{\theta}$
$\theta = \frac{\pi}{6}$	$\theta = \pi - \frac{\pi}{6}$
$\theta = \frac{\pi}{6} - 2\pi$	$\theta = \frac{6\pi}{6} - \frac{\pi}{6} = \frac{5\pi}{6}$
$\theta = \frac{\pi}{6} - \frac{12\pi}{6} = \frac{-11\pi}{6}$	$\theta = \frac{5\pi}{6} - 2\pi$
	$\theta = \frac{5\pi}{6} - \frac{12\pi}{6} = \frac{-7\pi}{6}$

Ch.4 Review

① e) $\cos^2\theta + \frac{1}{2}\cos\theta = 0$, $0° \le \theta \le 360°$

$\cos\theta\left(\cos\theta + \frac{1}{2}\right) = 0$

$\cos\theta = 0$ (Unit Circle) $\theta = 90°, 270°$

$\cos\theta + \frac{1}{2} = 0$

$\cos\theta = -\frac{1}{2}$ where is $\cos\theta < 0$

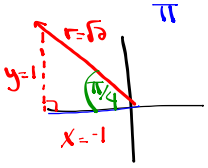
(Triangle #2) $\bar{\theta} = 60°$

Q2	Q3
$\theta = 180° - 60°$	$\theta = 180° + 60°$
$\theta = 120°$	$\theta = 240°$

$\frac{2\pi}{4}, \frac{3\pi}{4}, \frac{4\pi}{4}$

π

② a) $\frac{3}{1 - 2\sin\frac{3\pi}{4}}$



$\sin\frac{3\pi}{4} = \frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2}$

$\frac{3}{1 - 2\left(\frac{\sqrt{2}}{2}\right)}$

$\frac{3}{(1-\sqrt{2})(1+\sqrt{2})}$ Rationalize

$\frac{3+3\sqrt{2}}{1+\sqrt{2}-\sqrt{2}-2}$

$\frac{3+3\sqrt{2}}{-1}$ or $-3-3\sqrt{2}$

② d) $\frac{2\cos 3\pi + \sin 11\pi/4}{\cos^2 \pi/6}$

$\cos 3\pi = -1$

$\frac{2(-1) + \frac{\sqrt{2}}{2}}{\left(\frac{\sqrt{3}}{2}\right)^2}$

$\frac{-2 + \frac{\sqrt{2}}{2}}{\frac{3}{4}}$

$\left(-\frac{4}{2} + \frac{\sqrt{2}}{2}\right) \cdot \frac{4}{3}$ multiply by reciprocal

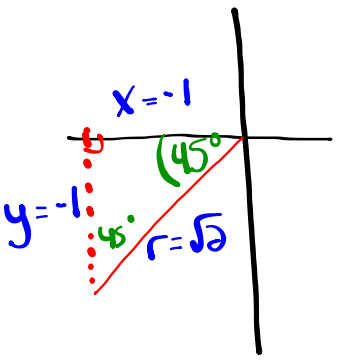
$\left(\frac{-4+\sqrt{2}}{2}\right) \cdot \frac{4}{3}$

$\frac{-16+4\sqrt{2}}{6} \rightarrow \frac{-8+2\sqrt{2}}{3}$ Reduce

Ch. 4 Review

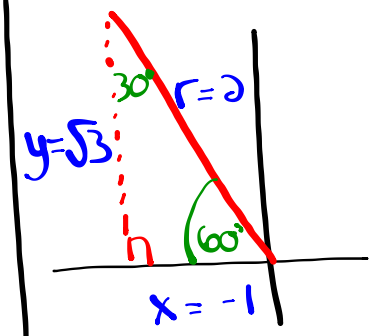
② b) $\dfrac{\sin^2 225^\circ}{8\sin 120^\circ}$

Sketch 225°



$\sin 225^\circ = \dfrac{-1}{\sqrt{2}}$

Sketch 120°



$\sin 120^\circ = \dfrac{\sqrt{3}}{2}$

$\dfrac{\left(\frac{-1}{\sqrt{2}}\right)^2}{8\left(\frac{\sqrt{3}}{2}\right)}$

$\dfrac{\frac{1}{2}}{\frac{8\sqrt{3}}{2}}$

$\dfrac{\frac{1}{2}}{4\sqrt{3}}$

$\frac{1}{2} \times \frac{1}{4\sqrt{3}}$

$\frac{1}{8\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}}$

$\dfrac{\sqrt{3}}{8(3)}$

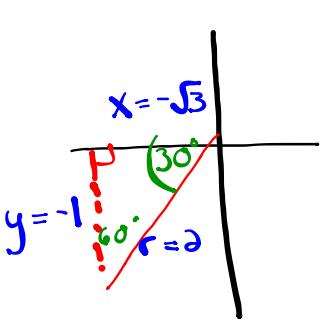
$\boxed{\dfrac{\sqrt{3}}{24}}$

Ch.4 Review:

② c) $\cos^2 210^\circ + \sin^2(-30^\circ) - \sin 90^\circ + \tan 480^\circ$

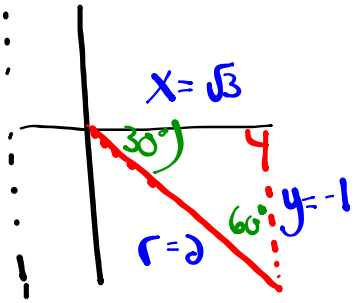
$-30^\circ = 330^\circ$ $\quad 480^\circ = 120^\circ$

Sketch 210°



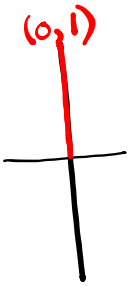
$\cos 210^\circ = -\frac{\sqrt{3}}{2}$

Sketch 330°



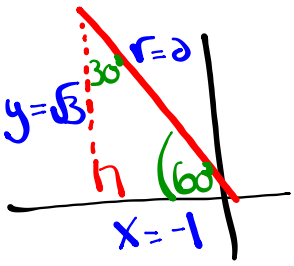
$\sin 330^\circ = -\frac{1}{2}$

Sketch 90°



$\sin 90^\circ = 1$

Sketch 120°



$\tan 120^\circ = \frac{\sqrt{3}}{-1} = -\sqrt{3}$

$\cos^2 210^\circ + \sin^2(-30^\circ) - \sin 90^\circ + \tan 480^\circ$

$\left(-\frac{\sqrt{3}}{2}\right)^2 + \left(\frac{-1}{2}\right)^2 - (1) + (-\sqrt{3})$

$\frac{3}{4} + \frac{1}{4} - 1 - \sqrt{3}$

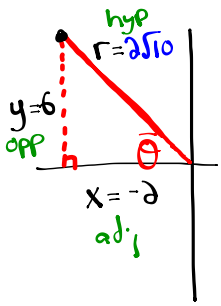
$\frac{3}{4} + \frac{1}{4} - \frac{4}{4} - \frac{4\sqrt{3}}{4}$

$-\frac{4\sqrt{3}}{4}$

$\boxed{-\sqrt{3}}$

Ch. 4 Review

③



(i) Find r:

$x^2+y^2=r^2$

$(-2)^2+(6)^2=r^2$

$4+36=r^2$

$40=r^2$

$\sqrt{40}=r$

$\sqrt{2\cdot2\cdot10}=r$

$2\sqrt{10}=r$

(ii) $\sin\theta=\frac{6}{2\sqrt{10}}=\frac{3}{\sqrt{10}}=\frac{3\sqrt{10}}{10}$

$\cos\theta=\frac{-2}{2\sqrt{10}}=\frac{-1}{\sqrt{10}}=-\frac{\sqrt{10}}{10}$

$\tan\theta=\frac{6}{-2}=-3$

$\csc\theta=\frac{\sqrt{10}}{3}$

$\sec\theta=-\sqrt{10}$

$\cot\theta=-\frac{1}{3}$

} Reciprocal Ratios

④ $\sec\theta=-\frac{\sqrt{11}}{3}$ hyp / adj and $\tan\theta>0$

$\cos\theta<0$

Given:

$r=\text{hyp}=\sqrt{11}$

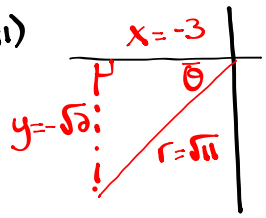
$x=\text{adj}=-3$

(i) • $\cos\theta<0$
• $\tan\theta>0$

S	A
T	C

θ is in Q3

(ii)



$x^2+y^2=r^2$

$(-3)^2+y^2=(\sqrt{11})^2$

$9+y^2=11$

$y^2=2$

$y=\pm\sqrt{2}$

$y=-\sqrt{2}$ Q3

(iii) $\sin\theta=\frac{-\sqrt{2}}{\sqrt{11}}=-\frac{\sqrt{22}}{11}$

$\cos\theta=\frac{-3}{\sqrt{11}}=-\frac{3\sqrt{11}}{11}$

$\tan\theta=\frac{-\sqrt{2}}{-3}=\frac{\sqrt{2}}{3}$

$\csc\theta=-\frac{\sqrt{11}}{\sqrt{2}}=-\frac{\sqrt{22}}{2}$

$\cot\theta=\frac{3}{\sqrt{2}}=\frac{3\sqrt{2}}{2}$

⑤ Find one positive and one negative co-terminal angle:

a) $\frac{\pi}{9}$

$\frac{\pi}{9} - \pi$

$\frac{\pi}{9} - \frac{18\pi}{9} = \boxed{\frac{-16\pi}{9}}$

$\frac{\pi}{9} + \pi$

$\frac{\pi}{9} + \frac{18\pi}{9} = \boxed{\frac{20\pi}{9}}$

b) -900°

$-900^\circ - 360^\circ = \boxed{-1260^\circ}$

$-900^\circ + 1080^\circ = \boxed{180^\circ}$

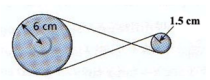
-900°
-180°

c) 300°

$300^\circ - 360^\circ = \boxed{-60^\circ}$

$300^\circ + 360^\circ = \boxed{660^\circ}$

If the belt in the pulley system below travels 30 cm, what is the angle of rotation of the smaller pulley?



[A] $\frac{\pi}{9}$ radians [B] 20°

[C] 20 radians [D] 5°

Given:
$a = 30cm$
$r = 1.5cm$

$\theta = \frac{a}{r} = \frac{30cm}{1.5cm} = 20\ rads$

Nibbles the hamster is running at 0.02 m/s on an exercise wheel of radius 8 cm. What is the angular velocity of this wheel?

[A] 0.15 rad/minute [B] 240 rad/minute [C] 0.25 rad/minute [D] 15 radians/minute

Given:
$r = 8cm = 0.08m$
$a = 0.02m$ (after 1 sec)

① $\theta = \frac{a}{r} = \frac{0.02m}{0.08m} = 0.25\ rads$

② $v_a = \frac{\theta}{t} = \frac{0.25rads}{1\ sec} = \frac{15\ rads}{60sec}$

Solve: $2(1-\sin\theta)^2 + \sin\theta = 2(3-4\sin^2\theta),\ -360° \le \theta \le 720°$ (Degrees)

$2(1-\sin\theta)(1-\sin\theta) + \sin\theta = 6 - 8\sin^2\theta$

$2(1-\sin\theta-\sin\theta+\sin^2\theta) + \sin\theta = 6-8\sin^2\theta$

$2(1-2\sin\theta+\sin^2\theta) + \sin\theta = 6-8\sin^2\theta$

$2-4\sin\theta+2\sin^2\theta+\sin\theta = 6-8\sin^2\theta$

$10\sin^2\theta - 3\sin\theta - 4 = 0$ $\quad -8+5=-3$, $-8\times 5 = -40$

$(\sin\theta - \frac{8}{10})(\sin\theta + \frac{5}{10}) = 0$

$(\sin\theta - \frac{4}{5})(\sin\theta + \frac{1}{2}) = 0$

$(5\sin\theta - 4)(2\sin\theta + 1) = 0$

$5\sin\theta - 4 = 0$

$\sin\theta = \frac{4}{5}$

① Find $\bar{\theta}$:

$\bar{\theta} = \sin^{-1}(\frac{4}{5})$

$\bar{\theta} = 53°$

② where is $\sin\theta > 0$

S | A
T | C

③ Find θ:

Q1	Q2
$\theta = \bar{\theta}$	$\theta = 180° - \bar{\theta}$
$\theta = 53°$	$\theta = 180° - 53° = 127°$
$\theta = 53° - 360° = -307°$	$\theta = 127° - 360° = -233°$
$\theta = 53° + 360° = 413°$	$\theta = 127° + 360° = 487°$

$2\sin\theta + 1 = 0$

$\sin\theta = -\frac{1}{2}$

① Find $\bar{\theta}$:

$\bar{\theta} = \sin^{-1}(\frac{1}{2})$

$\bar{\theta} = 30°$

② where is $\sin\theta < 0$

S | A
T | C

③ Find θ:

Q3	Q4
$\theta = 180° + \bar{\theta}$	$\theta = 360° - \bar{\theta}$
$\theta = 180° + 30° = 210°$	$\theta = 330°$
$\theta = -150°$	$\theta = -30°$
$\theta = 570°$	$\theta = 690°$

Solve: $6\sin^2\theta - 3\sin\theta = 0\ ,\ 0 \le \theta \le 360°$ (Open Response)

[A] $0°, 30°, 180°, 330°, 360°$
[B] $0°, 30°, 180°, 150°, 360°$
[C] $30°, 90°, 120°, 270°$
[D] $0°, 180°, 210°, 330°, 360°$

$6\sin^2\theta - 3\sin\theta = 0$ (Common Factor)

$3\sin\theta(2\sin\theta - 1) = 0$

$\frac{3\sin\theta}{3} = \frac{0}{3}$

$\sin\theta = 0$ (Unit Circle)

(i) Find θ

$\theta = 0°$
$\theta = 180°$
$\theta = 360°$

$2\sin\theta - 1 = 0$

$\frac{2\sin\theta}{2} = \frac{1}{2}$

$\sin\theta = \frac{1}{2}$ (Triangle #2)

(i) Find $\bar{\theta}$:

$\bar{\theta} = \sin^{-1}\left(\frac{1}{2}\right)$

$\bar{\theta} = 30°$

(ii) where is $\sin\theta > 0$ ~~S~~ A / ~~T~~ C

(iii) Find θ:

Q1	Q2
$\theta = \bar{\theta}$	$\theta = 180° - \bar{\theta}$
$\theta = 30°$	$\theta = 180° - 30° = 150°$

If $\csc\theta < 0$ and $\tan\theta > 0$, then which of the following could be a possible measure of angle θ?

[A] $\frac{11\pi}{6}$ $\frac{10\pi}{6}, \frac{11\pi}{6}, \frac{12\pi}{6}$ 2π Q4
[B] $\frac{4\pi}{3}$ $\frac{3\pi}{3}, \frac{4\pi}{3}, \frac{5\pi}{3}$ 1π Q3
[C] $\frac{3\pi}{4}$ Q2
[D] $\frac{\pi}{2}$

Given:
$\csc\theta < 0$ (negative)
$\tan\theta > 0$ (positive)

S	~~A~~
~~T~~	C

Angle θ is in Q3

What is the principal angle of $-\frac{25\pi}{4}$?

[A] $\frac{3\pi}{4}$
[B] $\frac{\pi}{4}$
$\frac{2\pi}{1} = \frac{8\pi}{4}$ [C] $-\frac{\pi}{4}$
[D] $\frac{7\pi}{4}$

① $-\frac{25\pi}{4} \div 2\pi = -\frac{25\pi}{4} \times \frac{1}{2\pi} = -\frac{25}{8} = -3\frac{1}{8}$

② $-3\frac{1}{8} + 3 = -\frac{1}{8}$

③ $-\frac{1}{8} \times 2\pi = -\frac{2\pi}{8} = -\frac{\pi}{4}$

④ $-\frac{\pi}{4} + 2\pi = -\frac{\pi}{4} + \frac{8\pi}{4} = \frac{7\pi}{4}$

$$\frac{1}{2} \div 4\sqrt{3}$$

$$\frac{1}{2} \cdot \frac{1}{4\sqrt{3}}$$

$$\frac{1}{8\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}}$$

$$\frac{\sqrt{3}}{8\sqrt{9}}$$

$$\frac{\sqrt{3}}{24}$$

Attachments

Worksheet - Sketching Angles in Radians.doc