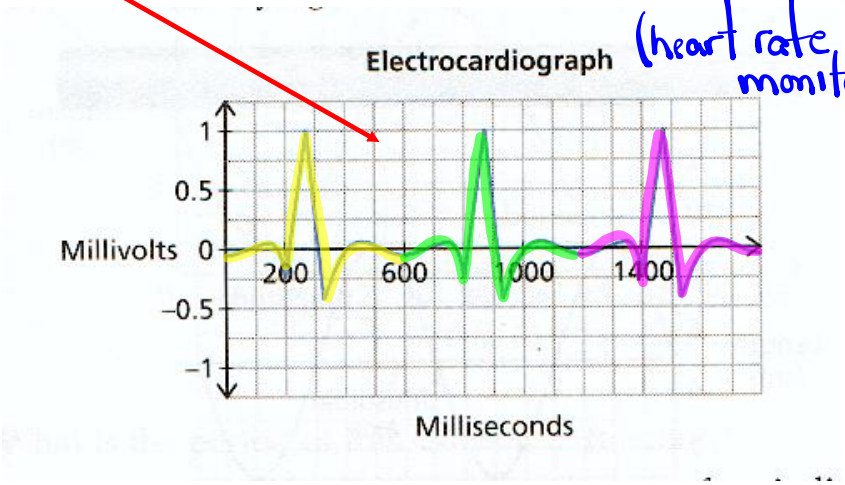
Sinusoidal Relations (Trig Graphs)

$y = \sin x$
$y = \cos x$

Periodic Function: A function for which the dependent variable takes on the same set of values over and over again as the independent variable changes.

(a function that repeats)

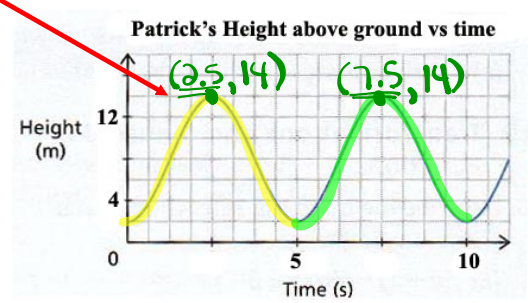
Example of periodic behavior



Sinusoidal Function: A periodic function that looks like waves, where any portion of the curve can be translated onto another portion of the curve.

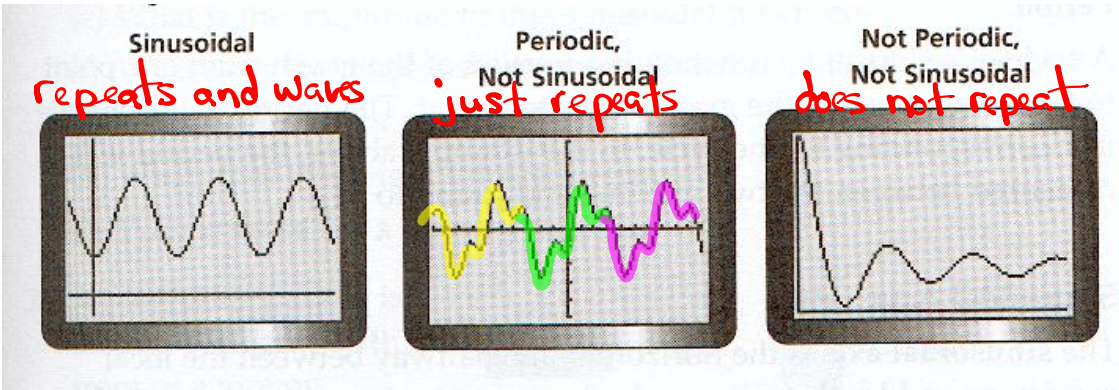
(Repeats and looks like a smooth wave).

Example of sinusoidal behavior



$P = 7.5 - 2.5 = 5$

These illustrations should summarize periodic and sinusoidal...

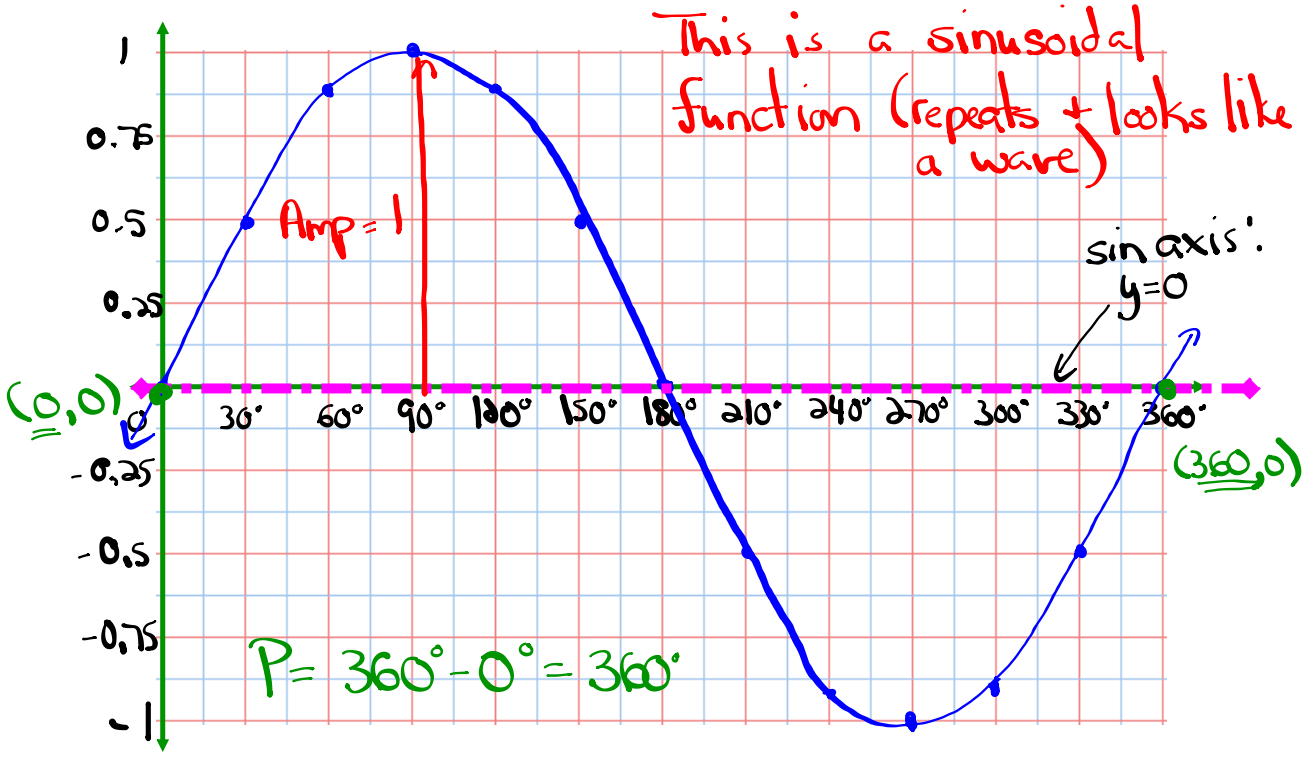


Let's examine the graph of $y = \sin\theta$

$y = \sin x$

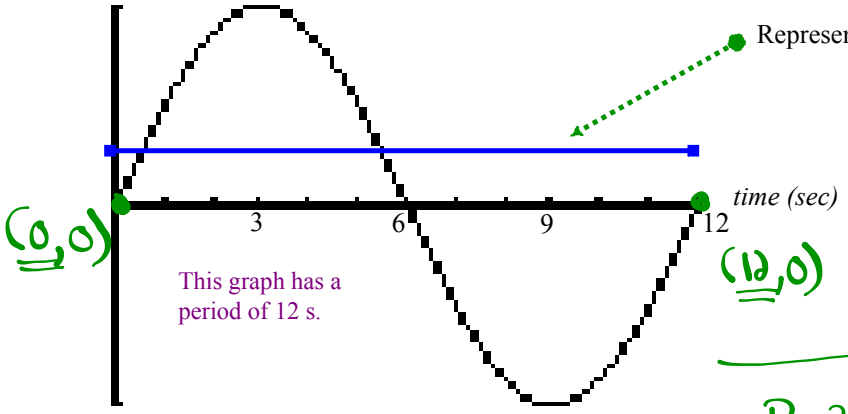
θ	0	30	60	90	120	150	180	210	240	270	300	330	360
y	0	0.5	0.87	1	0.87	0.5	0	-0.5	-0.87	-1	-0.87	-0.5	0

Now plot the above points...



Vocabulary of Sinusoidal Functions

I. Period: The change in x corresponding to one cycle. (one repetition)



$P = 12 - 0 = 12$

$P = \text{max} - \text{max}$

$P = \text{min} - \text{min}$

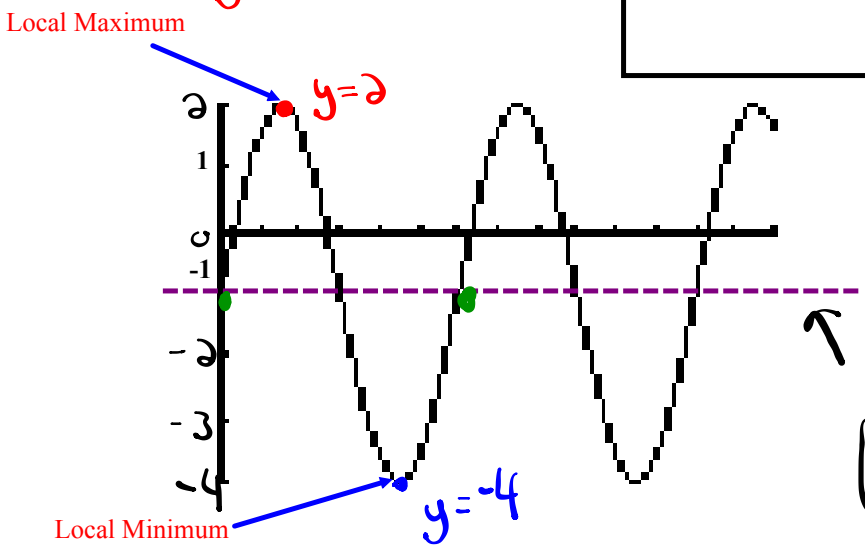
$P = \frac{360}{b}$ or $P = \frac{2\pi}{b}$

II. Sinusoidal Axis: The horizontal line halfway between the local maximum and local minimum.

highest (y) lowest (y)

$$\text{Sinusoidal Axis} = \frac{\text{Max} + \text{Min}}{2}$$

$$= \frac{2 + (-4)}{2} = \frac{-2}{2} = -1$$



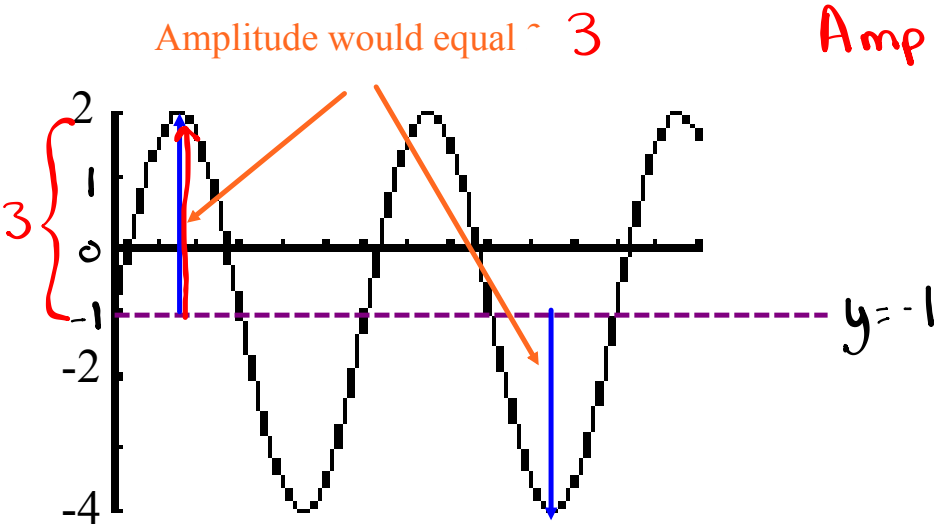
Equation of sinusoidal axis is $y = -1$

$k = -1$

III. Amplitude: The vertical distance from the sinusoidal axis to a local maximum or local minimum.

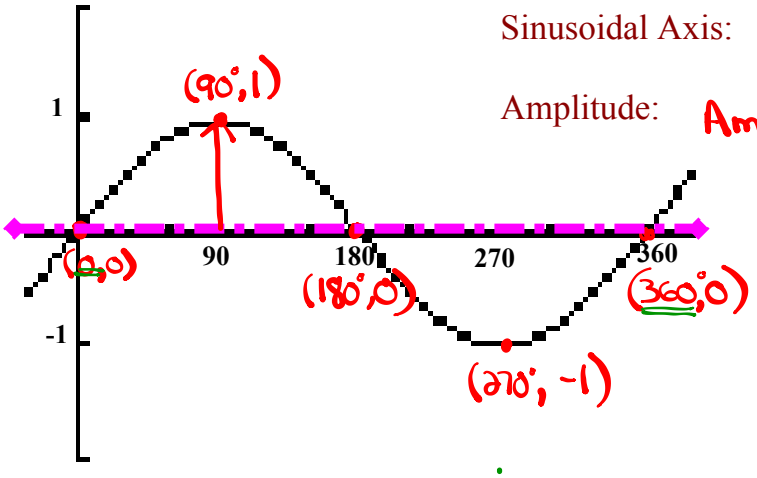
Amplitude = |a|

Amp is always positive



Summarize...

Here is the graph of $y = \sin\theta$



Period : $P = 360° - 0° = 360°$

Sinusoidal Axis: $\frac{1+(-1)}{2} = \frac{0}{2} = 0$ $(y=0)$

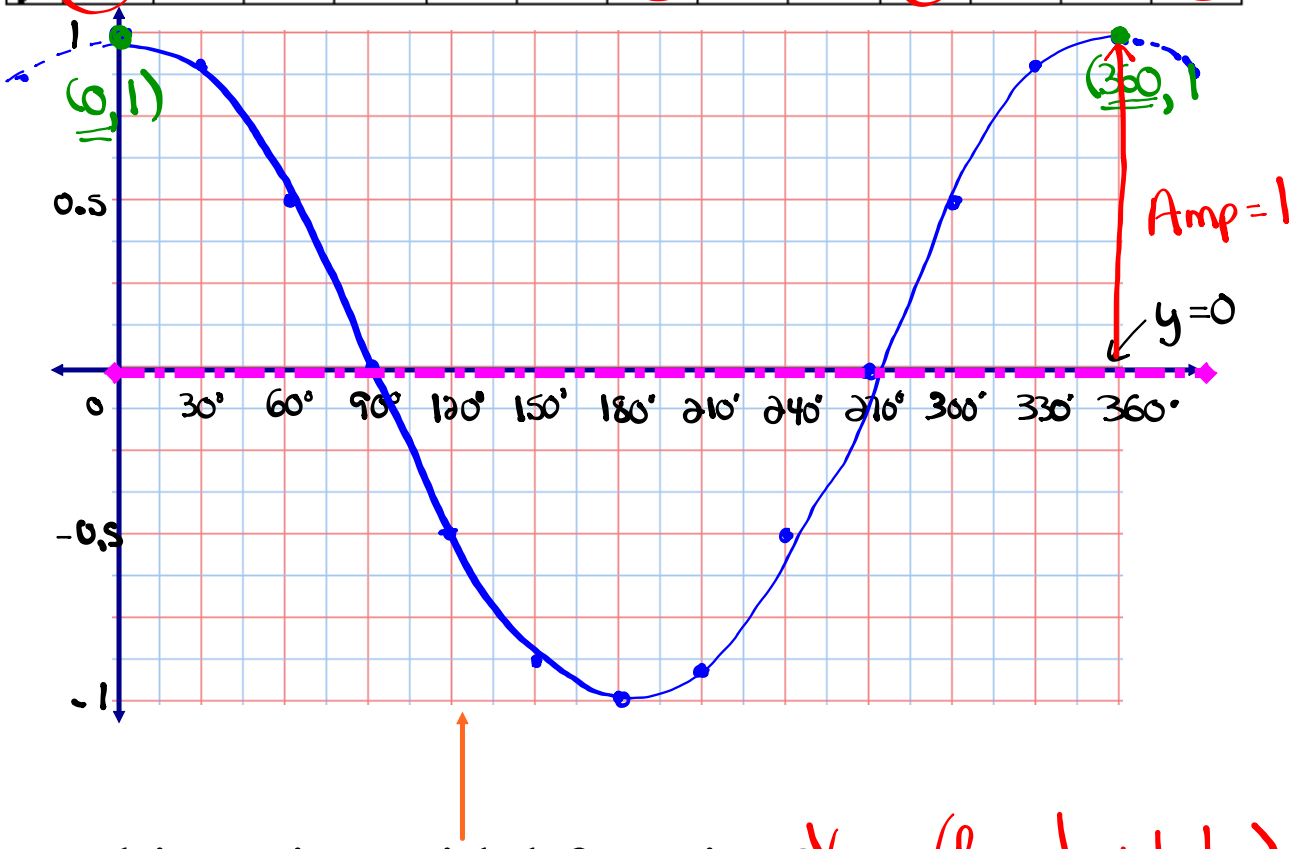
Amplitude: Amp = 1

What about $y = \cos\theta$?

$y = \cos x$

Complete the table of values and sketch below

θ	0	30	60	90	120	150	180	210	240	270	300	330	360
y	1	0.87	0.5	0	-0.5	-0.87	-1	-0.87	-0.5	0	0.5	0.87	1



Is this a sinusoidal function? Yes (repeats + looks like waves)

What about the period, sinusoidal axis, and amplitude?

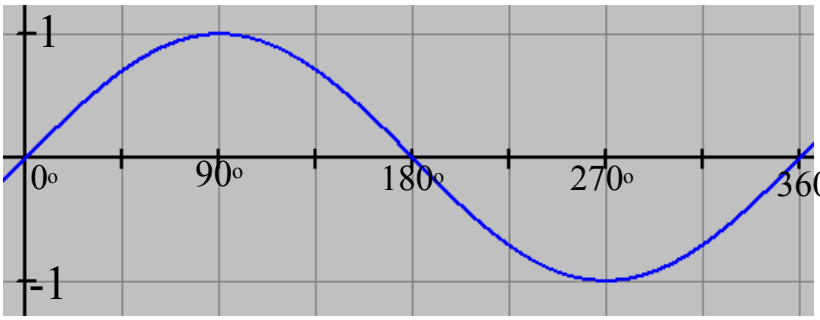
Period = $360° - 0° = 360°$

sinusoidal axis = $\frac{\text{Max} + \text{min}}{2} = \frac{1 + (-1)}{2} = \frac{0}{2} = 0$ $(y = 0)$

Amplitude = 1

Basic Trig Graphs

y = sin θ



Period = 360°
Amplitude = 1
Eq'n of Sinusoidal Axis: y = 0
Domain: {θ ∈ R}
Range: {-1 ≤ y ≤ 1}

θ	y
0°	0
90°	1
180°	0
270°	−1
360°	0

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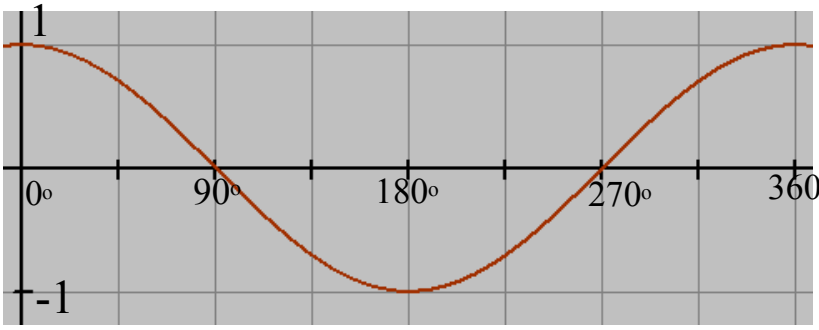
y = cos θ



Period = 360°
Amplitude = 1
Eq'n of Sinusoidal Axis: y = 0
Domain: {θ ∈ R}
Range: {-1 ≤ y ≤ 1}

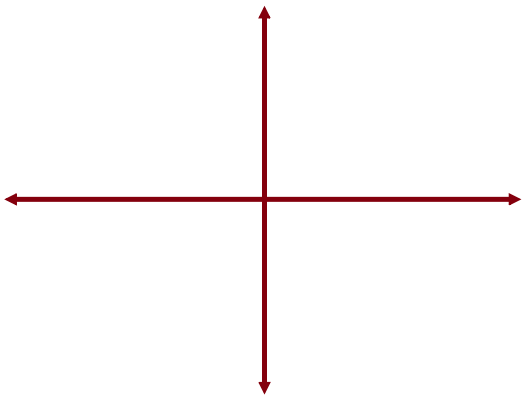
θ	y
0°	1
90°	0
180°	−1
270°	0
360°	1

Transformations of the Sinusoidal Function

Recall... $y = -2(x-3)^2 + 4$

Vertex $\Rightarrow$

Sketch $\Rightarrow$



Now, let's look at a sinusoidal function...

$$y = -2\sin[3(\theta - 60°)] - 1$$

Equations in Standard Form

$$y = a\sin[b(x-c)]+d \quad \text{or} \quad y = a\cos[b(x-h)]+k$$

a = ***Amplitude*** → influences how tall the sine curve is. (or cosine) Amp = |a|

$b = \frac{360^\circ}{P}$ → influences how often the pattern repeats. $b = \frac{2\pi}{P}$

h = ***Horizontal Translation*** → Influences how far to the left or the right that the graph will shift. h = Phase Shift

- If h is positive → Shift Left
- If h is negative → Shift Right

} Inside Brackets

k = ***Vertical Translation*** → influences how far up and down the graph will shift.

- If k is positive → Shift Up
- If k is negative → Shift Down
- equal to the sinusoidal axis: $y = k$

Example:

$2y+5 = -6\sin\left(\frac{1}{3}x-30^\circ\right)-3$ (Subtract 5 from both sides)

$\frac{2y}{2} = \frac{-6\sin\left(\frac{1}{3}x-30^\circ\right)}{2} - \frac{8}{2}$ (Divide by 2 a+k)

$y = -3\sin\left(\frac{1}{3}x-30^\circ\right)-4$ (Factor out a $\frac{1}{3}$)

$y = -3\sin\left[\frac{1}{3}(x-90^\circ)\right]-4$

$y = -3\sin\left[\left(\frac{1}{3}\right)(x-90^\circ)\right]-4$

$a = -3$ $b = \frac{1}{3}$ $h = 90^\circ$ $k = -4$

Amp = 3 $P = \frac{360^\circ}{\frac{1}{3}}$ equation of sinusoidal axis: $y = -4$

$P = 360^\circ \times 3$

$P = 1080^\circ$

Homework

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ex: $\frac{y}{2} - 5 = -4\cos[3x - 90°] - 7$

2. $\frac{y}{2} = -4\cos[3x - 90°] - 2$ ·2

$y = -8\cos[3x - 90°] - 4$

$y = -8\cos[3(x - 30°)] - 4$

$y = -8\cos[3(x - 30°)] - 4$

$a = -8 \rightarrow$ vertically stretched by a factor of 8. vertically reflected in the x-axis.

$b = 3 \rightarrow$ horizontally stretched by a factor of $\frac{1}{3}$.

$h = 30° \rightarrow$ horizontally translated 30° right.

$K = -4 \rightarrow$ vertically translated 4 units down.

Worksheet

h) 2. $\frac{1}{2}(y+2) \overset{2.}{=} 3\cos(x-90^\circ)$

$(y+2) = 6\cos(x-90^\circ)$

$y+2 = 6\cos(x-90^\circ)$

$y = 6\cos(x-90^\circ) - 2$

$a = 6$ $b = 1$ $h = 90^\circ$ $k = -2$

Amp = 6 $P = \frac{360^\circ}{1} = 360^\circ$ sin axis: $y = -2$

How to Sketch Sinusoidal Functions . . .

3. $\frac{y+4}{3} = -2\cos\left[3\theta - \frac{\pi}{2}\right] + 1 \quad \cdot 3$

(Multiply/Divide only a + K)

$y + 4 = -6\cos\left[3\theta - \frac{\pi}{2}\right] + 3 \quad -4 \quad -4$

$y = -6\cos\left[3\theta - \frac{\pi}{2}\right] - 1$

$y = -6\cos\left[3\left(\theta - \frac{\pi}{6}\right)\right] - 1$

$-\frac{\pi}{2} \div 3$

$-\frac{\pi}{2} \times \frac{1}{3} = -\frac{\pi}{6}$

$a = -6 \quad b = 3 \quad h = \frac{\pi}{6} \quad K = -1$

$Amp = 6 \quad P = \frac{2\pi}{b}$

$P = \frac{2\pi}{3}$

equation of sin axis: $y = -1$

$(\theta, y) \rightarrow \left[\frac{1}{3}\theta + \frac{\pi}{6}, -6y - 1\right]$

$y = \cos\theta \longrightarrow$

θ	y
0	1
$\pi/2$	0
π	-1
$3\pi/2$	0
2π	1

θ	y
$\pi/6$	-7
$2\pi/6$	-1
$3\pi/6$	5
$4\pi/6$	-1
$5\pi/6$	-7

Attachments

worksheet-sketching in radian measure.doc

Worksheet - Finding the Equation.doc

Worksheet - Sketching Trigonometric Functions.doc

Worksheet Solns - Sketching Sinusoidal Relations.doc

Worksheet - Sketching Sinusoidal relations (sept06).pdf

Bonus Soln - Fox Population.doc

Worksheet Solns - Applications of Sinusoidal Relations.doc

Review - Practice Test for Sinusoidal Functions.doc

Review - Trigonometric Functions(3)(4).doc