Questions from Homework

4. Given $f(x) = 3x^2 + 2$, $g(x) = \sqrt{x + 4}$, and $h(x) = 4x - 2$, determine each combined function and state its domain.

a) $y = (f + g)(x)$ **b)** $y = (h - g)(x)$
c) $y = (g - h)(x)$ **d)** $y = (f + h)(x)$

$f(x) = 3x^2 + 2$ (V)
D: $\{x \mid x \in R\}$ or $(-\infty, \infty)$

$g(x) = \sqrt{x+4}$
$x + 4 \geq 0$
$x \geq -4$
D: $\{x \mid x \geq -4, x \in R\}$
or $[-4, \infty)$

$h(x) = 4x - 2$
D: $\{x \mid x \in R\}$ or $(-\infty, \infty)$

a) $y = f(x) + g(x)$
$y = 3x^2 + 2 + \sqrt{x+4}$
$y = 3x^2 + 2 + \sqrt{x+4}$
D: $\{x \mid x \geq -4, x \in R\}$
$[-4, \infty)$

d) $y = f(x) + h(x)$
$y = 3x^2 + 2 + (4x - 2)$
$y = 3x^2 + 2 + 4x - 2$
$y = 3x^2 + 4x$
D: $\{x \mid x \in R\}$ or $(-\infty, \infty)$

11. If $h(x) = (f - g)(x)$ and $f(x) = 5x + 2$, determine $g(x)$.

a) $h(x) = -x^2 + 5x + 3$
b) $h(x) = \sqrt{x - 4} + 5x + 2$
c) $h(x) = -3x + 11$
d) $h(x) = -2x^2 + 16x + 8$

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Function Operations

To combine two functions, $f(x)$ and $g(x)$, multiply or divide as follows:

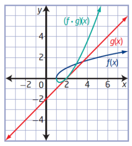
Product of Functions	*Quotient of Functions*
$h(x) = f(x)g(x)$	$h(x) = \frac{f(x)}{g(x)}$
$h(x) = (f \cdot g)(x)$	$h(x) = \left(\frac{f}{g}\right)(x)$

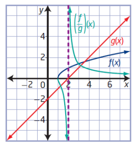
The domain of a product of functions is the domain common to the original functions. However, the domain of a quotient of functions must take into consideration that division by zero is undefined. The domain of a quotient, $h(x) = \frac{f(x)}{g(x)}$, is further restricted for values of x where $g(x) = 0$.

Example

Consider $f(x) = \sqrt{x - 1}$ and $g(x) = x - 2$.

The domain of $f(x)$ is $\{x \mid x \geq 1, x \in R\}$, and the domain of $g(x)$ is $\{x \mid x \in R\}$. So, the domain of $(f \cdot g)(x)$ is $\{x \mid x \geq 1, x \in R\}$, while the domain of $\left(\frac{f}{g}\right)(x)$ is $\{x \mid x \geq 1, x \neq 2, x \in R\}$





$f(x) = \sqrt{x-1}$
$g(x) = x - 2$
$(f \cdot g)(x) = f(x) \cdot g(x)$
$(f \cdot g)(x) = (\sqrt{x-1})(x-2)$

$f(x) = \sqrt{x-1}$
$g(x) = x - 2$
$\left(\frac{f}{g}\right)(x) = \frac{\sqrt{x-1}}{x-2}$

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Key Ideas

- The combined function $h(x) = (f \cdot g)(x)$ represents the product of two functions, $f(x)$ and $g(x)$.
- The combined function $h(x) = \left(\frac{f}{g}\right)(x)$ represents the quotient of two functions, $f(x)$ and $g(x)$, where $g(x) \neq 0$.
- The domain of a product or quotient of functions is the domain common to both $f(x)$ and $g(x)$. The domain of the quotient $\left(\frac{f}{g}\right)(x)$ is further restricted by excluding values where $g(x) = 0$.
- The range of a combined function can be determined using its graph.

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Example 1

Determine the Product of Functions

Given $f(x) = (x + 2)^2 - 5$ and $g(x) = 3x - 4$, determine $h(x) = (f \cdot g)(x)$. State the domain and range of $h(x)$.

Solution

$f(x) = (x+2)(x+2) - 5$
$f(x) = x^2 + 2x + 2x + 4 - 5 = x^2 + 4x - 1$

To determine $h(x) = (f \cdot g)(x)$, multiply the two functions.

$h(x) = (f \cdot g)(x)$
$h(x) = f(x)g(x)$
$h(x) = ((x + 2)^2 - 5)(3x - 4)$
$h(x) = (x^2 + 4x - 1)(3x - 4)$
$h(x) = 3x^3 - 4x^2 + 12x^2 - 16x - 3x + 4$
$h(x) = 3x^3 + 8x^2 - 19x + 4$

How can you tell from the original functions that the product is a cubic function?

$f(x) = x^2 + 4x - 1$ (V)
D: $\{x \mid x \in R\}$
or $(-\infty, \infty)$

$g(x) = 3x - 4$
D: $\{x \mid x \in R\}$
or $(-\infty, \infty)$

$h(x) = 3x^3 + 8x^2 - 19x + 4$
D: $\{x \mid x \in R\}$
or $(-\infty, \infty)$
R: $\{y \mid y \in R\}$
or $(-\infty, \infty)$

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Example 2

Determine the Quotient of Functions

Consider the functions $f(x) = x^2 + x - 6$ and $g(x) = 2x + 6$.

a) Determine the equation of the function $h(x) = \left(\frac{g}{f}\right)(x)$.
b) Sketch the graphs of $f(x)$, $g(x)$, and $h(x)$ on the same set of coordinate axes.
c) State the domain and range of $h(x)$.

$f(x) = x^2 + x - 6$
D: $\{x \mid x \in R\}$

$g(x) = 2x + 6$
D: $\{x \mid x \in R\}$

$h(x) = \frac{2}{x-2}$
D: $\{x \mid x \neq -3, 2, x \in R\}$

Solution

a) To determine $h(x) = \left(\frac{g}{f}\right)(x)$, divide the two functions.

$h(x) = \left(\frac{g}{f}\right)(x)$
$h(x) = \frac{g(x)}{f(x)}$
$h(x) = \frac{2x + 6}{x^2 + x - 6}$ Common; Simple Trinomial $-2 \times 3 = -6$, $-2 + 3 = 1$
$h(x) = \frac{2(x + 3)}{(x + 3)(x - 2)}$ Factor.
$h(x) = \frac{2\cancel{(x + 3)}}{\cancel{(x + 3)}(x - 2)}$
$h(x) = \frac{2}{x - 2}, x \neq -3, 2$ Identify any non-permissible values.

VA: $x - 2 = 0$
$x = 2$
There is an infinite discontinuity @ $x = 2$

Hole: $x + 3 = 0$
$x = -3$
$h(-3) = \frac{2}{-3-2}$
$h(-3) = -\frac{2}{5}$
$(-3, -\frac{2}{5})$
There is a removable discontinuity @ $x = -3$

c) $f(x) = x^2 + x - 6$
D: $\{x \mid x \in R\}$
$(-\infty, \infty)$

$g(x) = 2x + 6$
D: $\{x \mid x \in R\}$
$(-\infty, \infty)$

$h(x) = \frac{2}{x-2}$
D: $\{x \mid x \neq -3, x \neq 2, x \in R\}$
$(-\infty, -3) \cup (-3, 2) \cup (2, \infty)$

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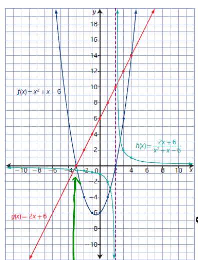
b) Method 1: Use Paper and Pencil

x	$f(x) = x^2 + x - 6$	$g(x) = 2x + 6$	$h(x) = \frac{2}{x - 2}, x \neq -3, 2$	
–3	0	0	does not exist	hole @ $(-3, -2/5)$
–2	–4	2	$-\frac{1}{2}$	
–1	–6	4	$-\frac{2}{3}$	
0	–6	6	–1	
1	–4	8	–2	
2	0	10	undefined	VA @ $x = 2$
3	6	12	2	
4	14	14	1	



How are the y-coordinates of points on the graph of $h(x)$ related to those on the graphs of $f(x)$ and $g(x)$?

$h(x) = \frac{2}{x-2}$

R: $\{y \mid y \neq -\frac{2}{5}, y \neq 0, y \in R\}$
or $(-\infty, -\frac{2}{5}) \cup (-\frac{2}{5}, 0) \cup (0, \infty)$

hole @ $(-3, -\frac{2}{5})$

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c) The function $f(x) = x^2 + x - 6$ is quadratic with domain $\{x \mid x \in R\}$.
The function $g(x) = 2x + 6$ is linear with domain $\{x \mid x \in R\}$.
The domain of $h(x) = \left(\frac{g}{f}\right)(x)$ consists of all values that are in both the domain of $f(x)$ and the domain of $g(x)$, excluding values of x where $f(x) = 0$.

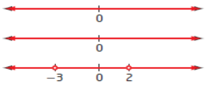
Since the function $h(x)$ does not exist at $\left(-3, -\frac{2}{5}\right)$ and is undefined at $x = 2$, the domain is $\{x \mid x \neq -3, x \neq 2, x \in R\}$. This is shown in the graph by the point of discontinuity at $\left(-3, -\frac{2}{5}\right)$ and the vertical asymptote that appears at $x = 2$.

How do you know there is a point of discontinuity and an asymptote?

Domain of $f(x) = x^2 + x - 6$

Domain of $g(x) = 2x + 6$

Domain of $h(x) = \left(\frac{g}{f}\right)(x)$



The range of $h(x)$ is $\left\{y \mid y \neq 0, -\frac{2}{5}, y \in R\right\}$.

horizontal asymptote

hole @ x = −3

$h(x) = \frac{2}{x-2}$

$h(-3) = \frac{2}{-3-2} = \frac{2}{-5} = -\frac{2}{5}$ $\quad (-3, -\frac{2}{5})$

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Homework

finish #1-9 on page 496-497

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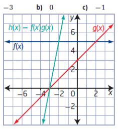
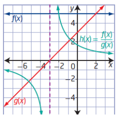
10.2 Products and Quotients of Functions, pages 496 to 498

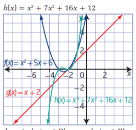
1. a) $h(x) = x^2 - 49$, $k(x) = \frac{x+7}{x-7}$, $x \neq 7$
   b) $h(x) = 6x^2 + 5x - 4$, $k(x) = \frac{2x-1}{3x+4}$, $x \neq -\frac{4}{3}$
   c) $h(x) = (x+2)\sqrt{x+5}$, $k(x) = \frac{\sqrt{x+5}}{x+2}$, $x \geq -5$, $x \neq -2$
   d) $h(x) = \sqrt{-x^2 + 7x - 6}$, $k(x) = \frac{\sqrt{x-1}}{\sqrt{6-x}}$, $1 \leq x < 6$
2. a) −3 b) 0 c) −1 d) 0
3. a)



b)

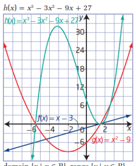


4. a) $h(x) = x^3 + 7x^2 + 16x + 12$



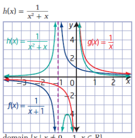
domain $\{x \mid x \in R\}$, range $\{y \mid y \in R\}$

b) $h(x) = x^3 - 3x^2 - 9x + 27$



domain $\{x \mid x \in R\}$, range $\{y \mid y \in R\}$

c) $h(x) = \frac{1}{x^2 + x}$



domain $\{x \mid x \neq 0, -1, x \in R\}$, range $\{y \mid y \leq -4 \text{ or } y > 0, y \in R\}$

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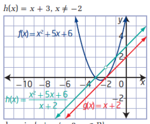
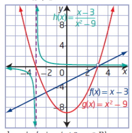
5. a) $h(x) = x + 3$, $x \neq -2$



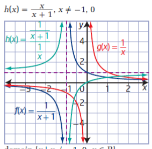
domain $\{x \mid x \neq -2, x \in R\}$, range $\{y \mid y \neq 1, y \in R\}$

b) $h(x) = \frac{1}{x+3}$, $x \neq \pm 3$



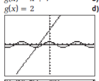
domain $\{x \mid x \neq \pm 3, x \in R\}$, range $\left\{y \mid y \neq 0, \frac{1}{6}, y \in R\right\}$

c) $h(x) = \frac{x}{x+1}$, $x \neq -1, 0$



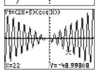
domain $\{x \mid x \neq -1, 0, x \in R\}$, range $\{y \mid y \neq 0, 1, y \in R\}$

6. a) $y = x^3 + 3x^2 - 10x - 24$
   b) $y = \frac{x^2 - x - 6}{x + 4}$, $x \neq -4$ c) $y = \frac{2x-1}{x+4}$, $x \neq -4$
   d) $y = \frac{x^2 - x - 6}{x^2 + 8x + 16}$, $x \neq -4$
7. a) $g(x) = 3$ b) $g(x) = -x$
   c) $g(x) = \sqrt{x}$ d) $g(x) = 5x - 6$
8. a) $g(x) = x + 7$ b) $g(x) = \sqrt{x+6}$
   c) $g(x) = 2$ d) $g(x) = 3x^2 + 26x - 9$
9. a)



$f(x)$: domain $\{x \mid x \in R\}$, range $\{y \mid y \in R\}$
$g(x)$: domain $\{x \mid x \in R\}$, range $\{y \mid -1 \leq y \leq 1, y \in R\}$

b)



domain $\{x \mid x \in R\}$, range $\{y \mid y \in R\}$

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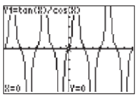
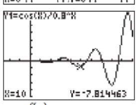
10. a)



domain $\{x \mid x \neq (2n-1)\frac{\pi}{2}, n \in I, x \in R\}$, range $\{y \mid y \in R\}$

b)



domain $\{x \mid x \in R\}$, range $\{y \mid y \in R\}$

11. a) $y = \frac{f(x)}{g(x)}$ b) $y = f(x)f(x)$
   c) The graphs of $y = \frac{\sin x}{\cos x}$ and $y = \tan x$ appear to be the same. The graphs of $y = 1 - \cos^2 x$ and $y = \sin^2 x$ appear to be the same.

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