Review Binomial Expansion

① a) $\left(\frac{a}{b}+2\right)^3$ $x=\frac{a}{b}$ $y=2$ $n=3$

$${}_3C_0\left(\frac{a}{b}\right)^3(2)^0+{}_3C_1\left(\frac{a}{b}\right)^2(2)^1+{}_3C_2\left(\frac{a}{b}\right)^1(2)^2+{}_3C_3\left(\frac{a}{b}\right)^0(2)^3$$

$$(1)\left(\frac{a^3}{b^3}\right)(1)+(3)\left(\frac{a^2}{b^2}\right)(2)+(3)\left(\frac{a}{b}\right)(4)+(1)(1)(8)$$

$$\boxed{\frac{a^3}{b^3}+\frac{6a^2}{b^2}+\frac{12a}{b}+8}$$

$\therefore$

① b) $(3x-2y)^5$ $x = 3x$ $y = -2y$ $n = 5$

$$\binom{5}{0}(3x)^5(-2y)^0 + \binom{5}{1}(3x)^4(-2y)^1 + \binom{5}{2}(3x)^3(-2y)^2 + \binom{5}{3}(3x)^2(-2y)^3 + \binom{5}{4}(3x)^1(-2y)^4 + \binom{5}{5}(3x)^0(-2y)^5$$

$$(1)(243x^5)(1) + (5)(81x^4)(-2y) + (10)(27x^3)(4y^2) + (10)(9x^2)(-8y^3) + (5)(3x)(16y^4) + (1)(1)(-32y^5)$$

$$243x^5 - 810x^4y + 1080x^3y^2 - 720x^2y^3 + 240xy^4 - 32y^5$$

Review Composite Functions

② Suppose $f(x) = x^2 - 3x + 5$ and $g(x) = 2x - 3$

composed with

a) find $(f \circ g)(x)$

$f(g(x)) = (g(x))^2 - 3(g(x)) + 5$

$f(2x-3) = (2x-3)^2 - 3(2x-3) + 5$

$f(2x-3) = 4x^2 - 12x + 9 - 6x + 9 + 5$

$f(2x-3) = 4x^2 - 18x + 23$

b) find $g(f(x))$

$g(f(x)) = 2(f(x)) - 3$

$g(x^2-3x+5) = 2(x^2-3x+5) - 3$

$g(x^2-3x+5) = 2x^2 - 6x + 10 - 3$

$g(x^2-3x+5) = 2x^2 - 6x + 7$

c) find $f(g(3))$

(i) $g(3) = 2(3) - 3$

$g(3) = 6 - 3$

$g(3) = 3$

(ii) $f(3) = (3)^2 - 3(3) + 5$

$f(3) = 9 - 9 + 5$

$\boxed{f(3) = 5}$

d) find $g(f(-1))$

(i) $f(-1) = (-1)^2 - 3(-1) + 5$

$f(-1) = 1 + 3 + 5$

$f(-1) = 9$

(ii) $g(9) = 2(9) - 3$

$g(9) = 18 - 3$

$\boxed{g(9) = 15}$

Review Combining Functions

③ Suppose: $f(x) = (x+2)^3 - 3$, $g(x) = 3x+1$, $h(x) = \sqrt{x+5}$, $i(x) = \log(x-3)$

a) find $(f \cdot g)(x)$ and state its domain. (multiply)

b) find $(h-g)(x)$ and state its domain.

c) find $\left(\frac{f}{h}\right)(x)$ and state its domain.

d) find $(f+i)(x)$ and state its domain.

$f(x) = (x+2)^3 - 3$
$f(x) = x^3+4x+4-3$
$f(x) = x^3+4x+1$
✓ (degree 3)
D: $\{x \mid x \in \mathbb{R}\}$
or $(-\infty, \infty)$

$g(x) = 3x+1$
(degree 1)
D: $\{x \mid x \in \mathbb{R}\}$
or $(-\infty, \infty)$

$h(x) = \sqrt{x+5}$
(radical)
$x+5 \geq 0$
$x \geq -5$
D: $\{x \mid x \geq -5, x \in \mathbb{R}\}$
or $[-5, \infty)$

$i(x) = \log(x-3)$
(logarithm)
$x-3 > 0$
$x > 3$
D: $\{x \mid x > 3, x \in \mathbb{R}\}$
or $(3, \infty)$

a) $(f \cdot g)(x) = f(x)g(x)$

$(f \cdot g)(x) = (x^3+4x+1)(3x+1)$

$(f \cdot g)(x) = 3x^4 + x^3 + 12x^2 + 4x + 3x + 1$

$(f \cdot g)(x) = 3x^4 + 13x^2 + 7x + 1$ (degree 4)

D: $\{x \mid x \in \mathbb{R}\}$ or $(-\infty, \infty)$

b) $(h-g)(x) = h(x) - g(x)$

$(h-g)(x) = \sqrt{x+5} - (3x+1)$

$(h-g)(x) = \sqrt{x+5} - 3x - 1$

D: $\{x \mid x \geq -5, x \in \mathbb{R}\}$ or $[-5, \infty)$

c) $\left(\frac{f}{h}\right)(x) = \frac{f(x)}{h(x)}$

$\left(\frac{f}{h}\right)(x) = \frac{x^3+4x+1}{\sqrt{x+5}}$ → $\sqrt{x+5} \neq 0$, $x+5 \neq 0$, $x \neq -5$

D: $\{x \mid x > -5, x \in \mathbb{R}\}$
or $(-5, \infty)$

d) $(f+i)(x) = f(x) + i(x)$

$(f+i)(x) = (x+2)^3 - 3 + \log(x-3)$ → $x-3 > 0$, $x > 3$

D: $\{x \mid x > 3, x \in \mathbb{R}\}$
or $(3, \infty)$