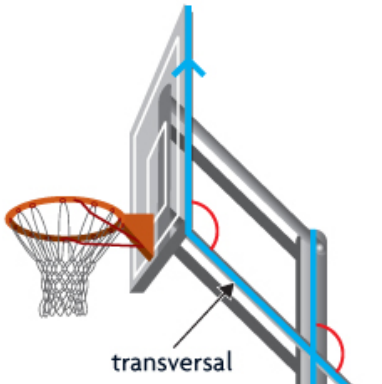
2.1 Exploring Parallel Lines

GOAL

Identify relationships among the measures of angles formed by intersecting lines.

EXPLORE the Math

A sports equipment manufacturer builds portable basketball systems, like those shown here. These systems can be adjusted to different heights.



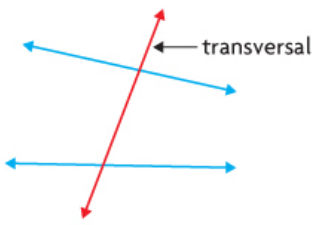
When the adjusting arm is moved, the measures of the angles formed with the backboard and the supporting post change. The adjusting arm forms a **transversal**.

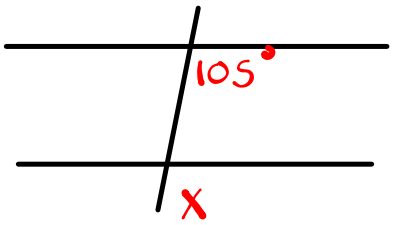
When a system is adjusted, the backboard stays perpendicular to the ground and parallel to the supporting post.

? When a transversal intersects two parallel lines, how are the angle measures related?

transversal

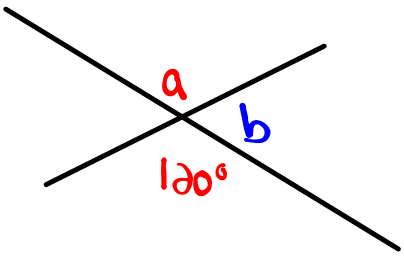
A line that intersects two or more other lines at distinct points.





"F pattern"

$x = 105$ → corresponding angles are equal



$a = 120°$ → vertically opposite angles are equal.

$b + 120° = 180°$ → supplementary angles add to 180°

$b = 180° - 120°$

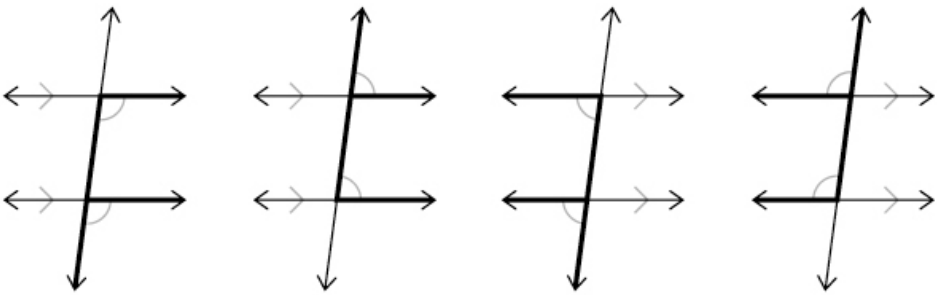
$b = 60°$

? When a transversal intersects two parallel lines, how are the angle measures related?

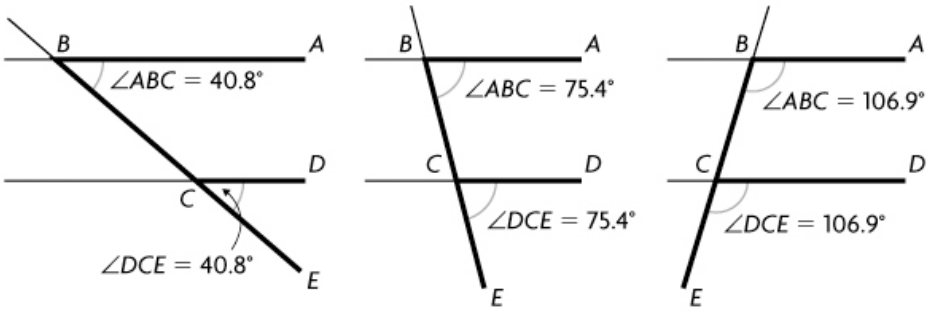
Sample solution

I measured the angle formed by the backboard and the lower adjusting arm. I also measured the angle facing the same direction, formed by the lower adjusting arm and the post. In the first diagram, these angles both measured 132°. In the second diagram, they both measured 70°.

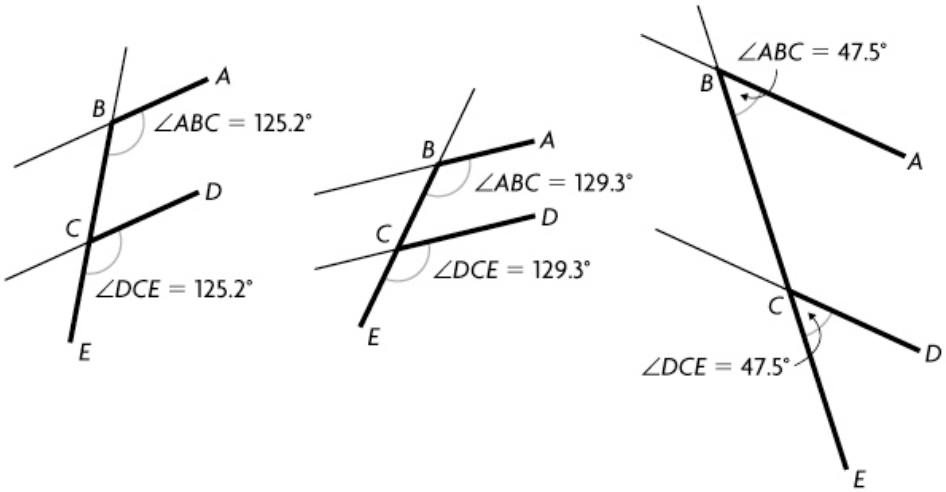
I imagined these angles as forming an F. I conjectured that any angles determined by two parallel lines and a transversal that formed an F would be equal. I drew a sketch of what these angles would look like.



I constructed parallel line segments AB and CD and a transversal using dynamic geometry software. I measured one set of angles that formed an F. Then I moved the transversal to form several different sets of angles and measured the angles. I noticed that no matter how I angled the transversal, the F angles were always equal.

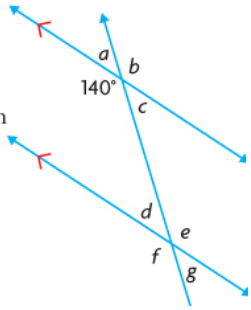


Next, I changed the angle of the parallel line segments and the distance between the parallel line segments. The F angles were still equal.



Reflecting

A. Use the relationships you observed to predict the measures of as many of the angles a to g in this diagram as you can. Explain each of your predictions.



B. Jonathan made the following conjecture: "When a transversal intersects two parallel lines, the **corresponding angles** are always equal." Do you agree or disagree? Explain, using examples.

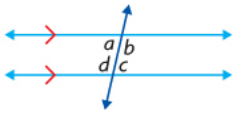
C. Did you discover any counterexamples for Jonathan's conjecture? What does this imply?

D. Sarah says that the **converse** of Jonathan's conjecture is also true: "When a transversal intersects two lines and creates corresponding angles that are equal, the two lines are parallel." Do you agree or disagree? Explain.

E. Do your conjectures about angle measures hold when a transversal intersects a pair of non-parallel lines? Use diagrams to justify your decision.

interior angles

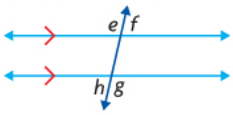
Any angles formed by a transversal and two parallel lines that lie inside the parallel lines.



a, *b*, *c*, and *d* are interior angles.

exterior angles

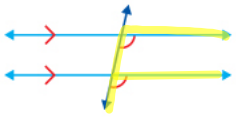
Any angles formed by a transversal and two parallel lines that lie outside the parallel lines.



e, *f*, *g*, and *h* are exterior angles.

corresponding angles

One interior angle and one exterior angle that are non-adjacent and on the same side of a transversal.



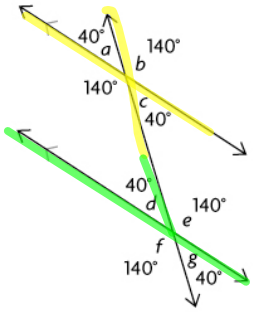
converse

A statement that is formed by switching the premise and the conclusion of another statement.

Answers

A. Explanation:

$\angle a = 40°$	Supplementary angles: $180° - 140°$
$\angle b = 140°$	Vertically opposite angles
$\angle c = 40°$	Vertically opposite angles
$\angle d = \angle a = 40°$	Angles on the same side of the transversal, facing the same direction, are equal.
$\angle e = \angle b = 140°$	Angles on the same side of the transversal, facing the same direction, are equal.
$\angle f = 140°$	Vertically opposite angles
$\angle g = 40°$	Vertically opposite angles

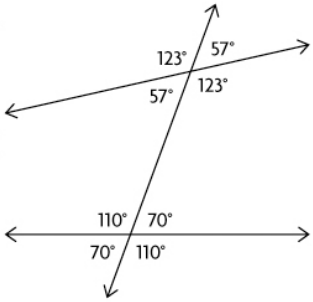


B. Yes, I agree. No matter what angle I made the transversal to the parallel lines I drew, the corresponding angles were equal. The corresponding angles in the diagram above are equal.

C. No, I didn't. This implies that Jonathan's conjecture about corresponding angles being equal is probably valid for parallel lines, no matter where the transversal is.

D. I agree. If I draw a first line, which will be the transversal, and then draw a second line intersecting the first, and then draw a third line also intersecting the transversal at the same angle as the second line, the second and third lines will be parallel.

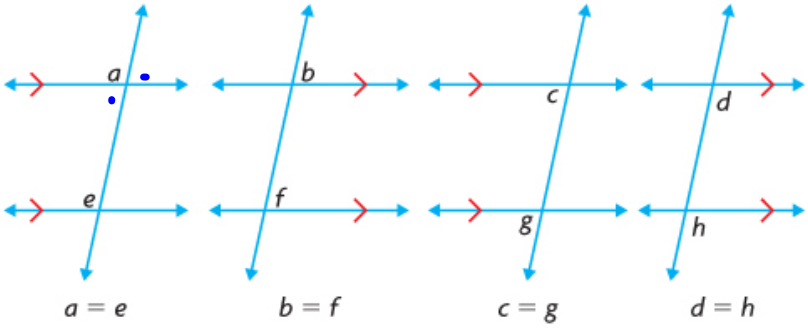
E. No, they don't. I drew a set of non-parallel lines and a transversal, and I measured the angles. The angles formed by the first line and the transversal were different from the angles formed by the second line and the transversal.



In Summary

Key Ideas

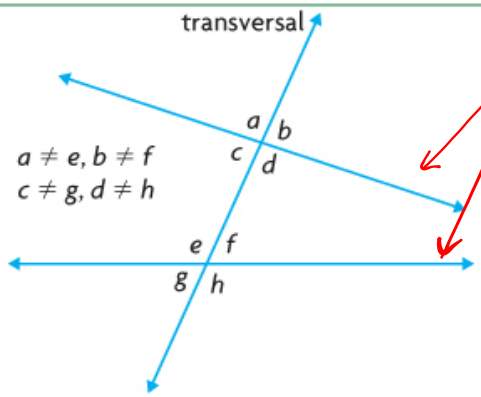
- When a transversal intersects a pair of parallel lines, the corresponding angles that are formed by each parallel line and the transversal are equal.



- When a transversal intersects a pair of lines creating equal corresponding angles, the pair of lines is parallel.

Need to Know

- When a transversal intersects a pair of non-parallel lines, the corresponding angles are not equal.
- There are also other relationships among the measures of the eight angles formed when a transversal intersects two parallel lines.



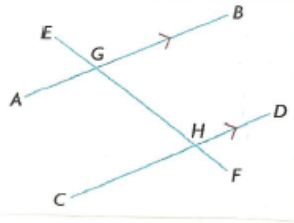
not parallel

Assignment: pgs. 72 Ques. 2, 3, 5, 6

***For #2-list only corresponing and vertically opposite angles**

SOLUTIONS => 2.1 Exploring Parallel Lines

2. Which pairs of angles are equal in this diagram? Is there a relationship between the measures of the pairs of angles that are not equal?



SOLUTION

Corresponding Angles	Vertically Opposite Angles
$\angle EGB = \angle GHD$	$\angle EGA = \angle BGH$
$\angle EGA = \angle GHC$	$\angle AGH = \angle EGB$
$\angle BGH = \angle DHF$	$\angle GHC = \angle DHF$
$\angle AGH = \angle CHF$	$\angle CHF = \angle GHD$

* These are all of the equal pairs of angles that we can pick out so far => there are more!

** There definitely is a relationship between the measures of the pairs of angles that are not equal => they are supplementary.

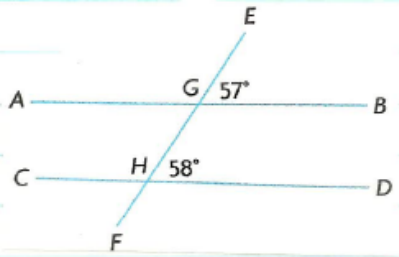
3. Explain how you could construct parallel lines using only a protractor and a ruler.

SOLUTION

First, you could draw a line and then draw a transversal passing through that line.
Next, you could use your practor to create an equal corresponding angle on the same side of the transversal.
Lastly, you could use that angle to draw a line parallel to the first one.

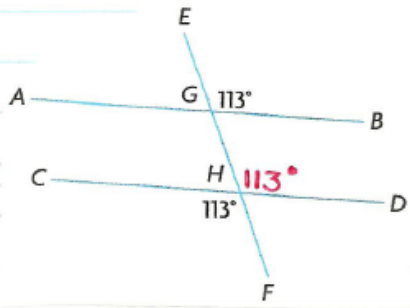
5. In each diagram, is AB parallel to CD? Explain how you know.

a)



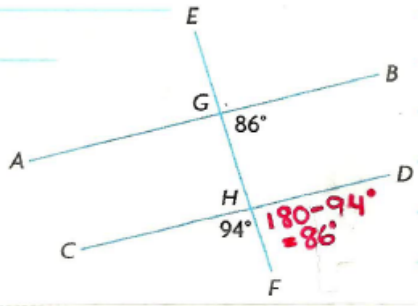
No, AB is not parallel to CD in this diagram since corresponding angles are not equal.
$\angle EGB \neq \angle GHD$

b)



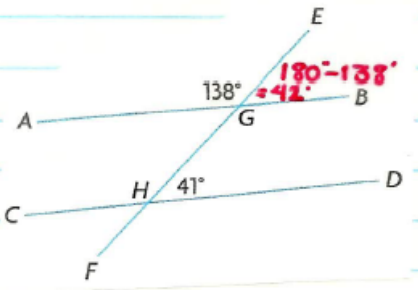
Yes, AB is parallel to CD in this diagram since corresponding angles are equal.
$\angle EGB = \angle GHD$

c)



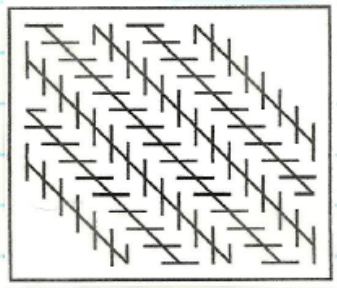
Yes, AB is parallel to CD in this diagram since corresponding angles are equal.
$\angle BGH = \angle DHF$

d)



No, AB is not parallel to CD in this diagram since corresponding angles are not equal.
$\angle GHD \neq \angle EGB$

6. Nancy claims that the diagonal lines in the diagram to the left are not parallel. Do you agree or disagree? Justify your decision.



SOLUTION

I disagree with Nancy's claims since the lines are all 5mm from each other. It is an optical illusion.

Attachments

PM11-2s1.gsp