Questions from homework

① $x^2 + xy + y^2 = 1$

$$2x + y + x\frac{dy}{dx} + 2y\frac{dy}{dx} = 0$$

$$x\frac{dy}{dx} + 2y\frac{dy}{dx} = -2x - y$$

$$\frac{\frac{dy}{dx}\cancel{(x+2y)}}{\cancel{(x+2y)}} = \frac{-2x-y}{x+2y}$$

$$\frac{dy}{dx} = \frac{-2x-y}{x+2y}$$

$$\frac{dy}{dx} = -\frac{2x+y}{x+2y}$$

Questions from homework

2e) $\sqrt{x+y} + \sqrt{xy} = 4 \quad @ (2,2)$

$$(x+y)^{1/2} + (xy)^{1/2} = 4$$

$$\frac{1}{2}(x+y)^{-1/2}\left(1+\frac{dy}{dx}\right) + \frac{1}{2}(xy)^{-1/2}\left(y + x\frac{dy}{dx}\right) = 0$$

$\cancel{2\sqrt{x+y}}\,\sqrt{xy}$ $\quad \cancel{2\sqrt{x+y}}\,\cancel{\sqrt{xy}}$ $\quad 2\sqrt{x+y}\,\sqrt{xy}$

$$\frac{1}{\cancel{2\sqrt{x+y}}}\left(1+\frac{dy}{dx}\right) + \frac{1}{\cancel{2\sqrt{xy}}}\left(y + x\frac{dy}{dx}\right) = 0$$

$$\sqrt{xy}\left(1+\frac{dy}{dx}\right) + \sqrt{x+y}\left(y + x\frac{dy}{dx}\right) = 0$$

$$\underline{\sqrt{xy}} + \frac{dy}{dx}\sqrt{xy} + \underline{y\sqrt{x+y}} + x\frac{dy}{dx}\sqrt{x+y} = 0$$

$$\frac{dy}{dx}\sqrt{xy} + x\frac{dy}{dx}\sqrt{x+y} = -\sqrt{xy} - y\sqrt{x+y}$$

$$\frac{\frac{dy}{dx}\left(\cancel{\sqrt{xy} + x\sqrt{x+y}}\right)}{\cancel{\sqrt{xy} + x\sqrt{x+y}}} = \frac{-\sqrt{xy} - y\sqrt{x+y}}{\sqrt{xy} + x\sqrt{x+y}}$$

$$\frac{dy}{dx} = -\frac{\left(\sqrt{xy} + y\sqrt{x+y}\right)}{\sqrt{xy} + x\sqrt{x+y}}$$

$$\frac{dy}{dx} = -\frac{\sqrt{(2)(2)} + (2)\sqrt{2+2}}{\sqrt{(2)(2)} + (2)\sqrt{2+2}}$$

$$\frac{dy}{dx} = -\frac{2+4}{2+4}$$

$$\frac{dy}{dx} = -\frac{6}{6}$$

$$\frac{dy}{dx} = -1$$

Questions from homework

Higher Derivatives

Since the derivative of a function *f* is itself a function *f '*, we can take its derivative *(f ')'* . The result is a function called the second derivative of *f* and is denoted by *f ''*

In Leibniz notation we write:

$$f''(x) = \frac{d^2y}{dx^2}$$

Find $y'' = f''(x) = \dfrac{d^2y}{dx^2}$

Examples

$y = x^6$

$y' = 6x^5$

$y'' = 30x^4$

$f(x) = 5x^2 + \sqrt{x}$

$f(x) = 5x^2 + x^{1/2}$

$f'(x) = 10x + \frac{1}{2}x^{-1/2}$

$f''(x) = 10 - \frac{1}{4}x^{-3/2}$

$f(x) = (2 - x^2)^{10}$

$f'(x) = 10(2-x^2)^9(-2x)$

$f'(x) = \underbrace{-20x}_{f}\underbrace{(2-x^2)^9}_{g}$

$f''(x) = \underbrace{-20}_{f'}\underbrace{(2-x^2)^9}_{g} + \underbrace{-20x}_{f}\underbrace{(9)(2-x^2)^8(-2x)}_{g'}$

$f''(x) = -20(2-x^2)^9 + 360x^2(2-x^2)^8$

common factor

$f''(x) = -20(2-x^2)^8\left[(2-x^2) - 18x^2\right]$

$2 - x^2 - 18x^2$

$f''(x) = -20(2-x^2)^8(2-19x^2)$

Since the first derivative of a function can be interpreted either as the slope of a tangent line or as a rate of change, the second derivative can be interpreted as *the rate of change of the slope of the tangent line.*
This idea will be pursued later where the second derivative gives valuable information about the shape of a graph, or when dealing with acceleration.

Higher derivatives can also be defined. The third derivative is the derivative of the second derivative.

$$y''' = f'''(x) = \frac{d^3 y}{dx^3}$$

Find the first five derivatives of

$$y = x^4 + 2x^3 - 5x^2 + 3x - 6$$

$y' = 4x^3 + 6x^2 - 10x + 3$

$y'' = 12x^2 + 12x - 10$

$y''' = 24x + 12$

$y^{iv} = 24$

$y^{v} = 0$

Homework

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① c) $f(t) = 2t - \frac{1}{t+1} = 2t - 1(t+1)^{-1}$

$f'(t) = 2 + (t+1)^{-2}(1) = 2 + \frac{1}{(t+1)^2}$

$f''(t) = -2(t+1)^{-3}(1) = \frac{-2}{(t+1)^3}$

g) $y = \sqrt{x^2+1} = (x^2+1)^{1/2}$

$y' = \frac{1}{2}(x^2+1)^{-1/2}(2x) = x(x^2+1)^{-1/2} = \frac{x}{\sqrt{x^2+1}}$

$y'' = 1(x^2+1)^{-1/2} + x\left(-\frac{1}{2}(x^2+1)^{-3/2}(2x)\right)$

$y'' = (x^2+1)^{-1/2} - x^2(x^2+1)^{-3/2}$

$y'' = (x^2+1)^{-3/2}\left[(x^2+1)^1 - x^2\right]$

$y'' = \frac{1}{(x^2+1)^{3/2}} = \frac{1}{\sqrt{(x^2+1)^3}}$

$-\frac{1}{2} - \left(-\frac{3}{2}\right)$

$-\frac{1}{2} + \frac{3}{2}$

$\frac{2}{2}$

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