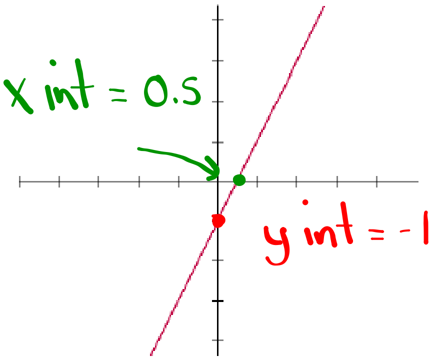
Catalog of Essential Functions

1. Linear



highest exponent

Straight Line

Equation will be degree one

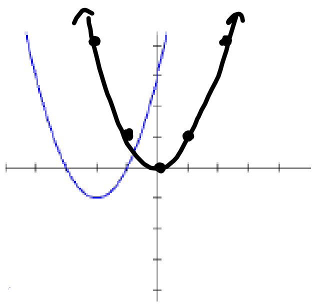
Should be able to identify the ***slope, intercepts, and equation*** from the graph

$m = \frac{rise}{run} = \frac{2}{1} = 2$

$y = mx + b$ (equation)

$y = 2x - 1$

2. Quadratic



degree of 2

Parabola (U-Shaped)

Either *y or x* will be squared (not both!)

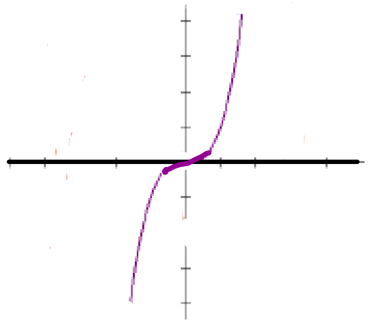
Should know the 4 basic quadratic functions

Should be able to apply transformations to the basic quadratic functions

$y = x^2$

x	y
-2	4
-1	1
0	0
1	1
2	4

3. Cubic



S-Shaped

We will work with functions that are raised to the third power

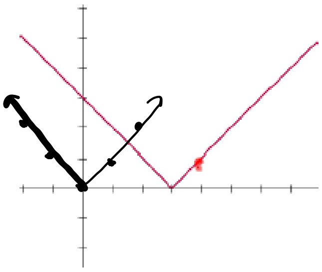
degree of 3

$y = x^3$

x	y
-2	-8
-1	-1
0	0
1	1
2	8

Catalog of Essential Functions

4. Absolute Value



V-Shaped

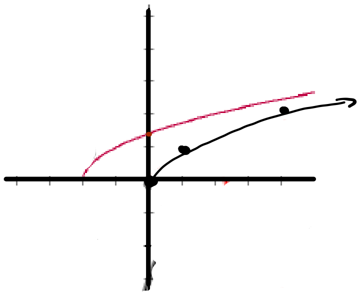
Equation will have a variable within the absolute value bars

Should be able to apply transformations to the basic absolute value function

$y=|x|$

x	y
-2	2
-1	1
0	0
1	1
2	2

5. Square Root /Radical



Half Parabola

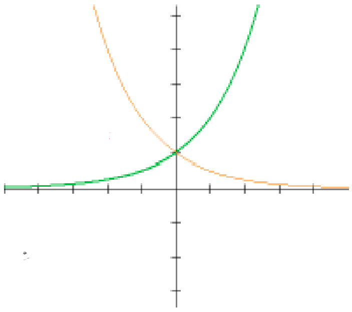
Equation will have a variable under the square root sign

Should be able to apply transformations to the basic square root function

$y=\sqrt{x}$

x	y
0	0
1	1
4	2
9	3

6. Exponential



Steadily increasing or decreasing

Base will be a number and variable will appear in the exponent

$y=2^x$ | $y=5^x$ | $y=(\frac{1}{2})^x$

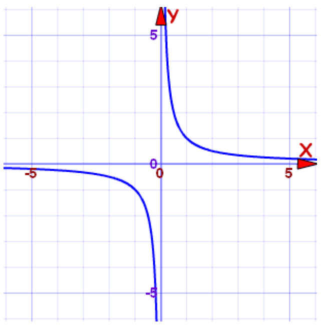
Should be able to identify the ***horizontal asymptote***

$y=2^x$

x	y
-2	1/4
-1	1/2
0	1
1	2
2	4

Catalog of Essential Functions

7. Reciprocal



Will have two branches

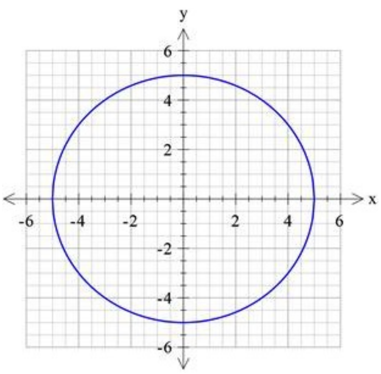
Equation will have a variable within the denominator of a rational expression

Should be able to identify the ***vertical and horizontal asymptotes***

$y = \frac{1}{x}$

x	y
-2	$-\frac{1}{2}$ or -0.5
-1	-1
0	undefined
1	1
2	$\frac{1}{2}$ or 0.5

8. Circle

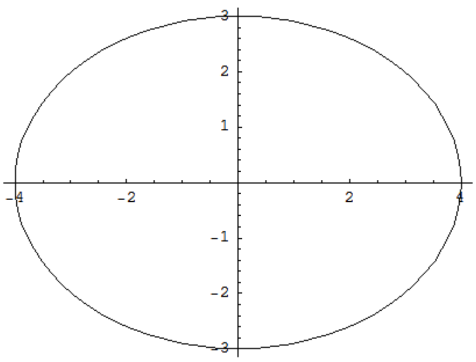


- General form: $(x-h)^2 + (y-k)^2 = r^2$

  * center: (h, k)
  * radius $= r$

- Be able to identify the function that would describe either just the top or bottom of the circle.

9. Ellipse



- General form: $\frac{(x-h)^2}{a^2} + \frac{(y-k)^2}{b^2} = 1$

Where...
- Center: (h, k)
- a > b
- If a is the denominator of the "y" term the ellipse will have a vertical major axis.

Transformations:

New Functions From Old Functions

1. Translations
2. Stretches
3. Reflections

Translations

h = horizontal translation (Shift Left/Right) add to x-values

K = vertical translation (Shift Up/Down) add to y-values

Focus on...

- determining the effects of h and k in $\boxed{y - k = f(x - h)}$ or $y = f(x-h)+k$ on the graph of $y = f(x)$
- sketching the graph of $y - k = f(x - h)$ for given values of h and k, given the graph of $y = f(x)$
- writing the equation of a function whose graph is a vertical and/or horizontal translation of the graph of $y = f(x)$

Ex: ① $y = (x-3)^2 + 2$

$k = 2 \rightarrow$ Up 2

$h = 3 \rightarrow$ Right 3

② $y - 4 = |x+3|$

$y = |x+3| + 4$

$k = 4 \rightarrow$ Up 4

$h = -3 \rightarrow$ Left 3

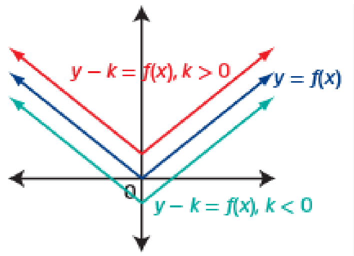
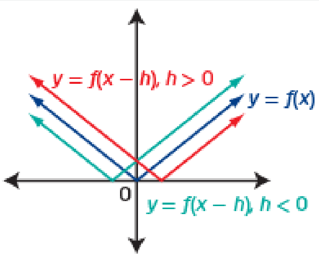
Function notation

③ $g(x) = f(x+2) - 1$

$k = -1 \rightarrow$ down 1

$h = -2 \rightarrow$ Left 2

- Translations are transformations that shift all points on the graph of a function up, down, left, and right without changing the shape or orientation of the graph.
- The table summarizes translations of the function $y = f(x)$.

<table>
<tr><th>Function</th><th>Transformation from $y = f(x)$</th><th>Mapping</th><th>Example</th></tr>
<tr><td>$y - k = f(x)$ or $y = f(x) + k$</td><td>A vertical translation
If $k > 0$, the translation is up.
If $k < 0$, the translation is down.</td><td>$(x, y) \rightarrow (x, y + k)$</td><td>
</td></tr>
<tr><td>$y = f(x - h)$</td><td>A horizontal translation
If $h > 0$, the translation is to the right.
If $h < 0$, the translation is to the left.</td><td>$(x, y) \rightarrow (x + h, y)$</td><td>
</td></tr>
</table>

* change the sign when you remove from brackets

- A sketch of the graph of $y - k = f(x - h)$, or $y = f(x - h) + k$, can be created by translating key points on the graph of the base function $y = f(x)$.

Translation

- To *translate* or *shift* a graph is to move it up, down, left, or right without changing its shape.
- Translation is summarized by the following table and illustration:

Vertical and Horizontal Shifts Suppose $c > 0$. To obtain the graph of

$y = f(x) + c$, shift the graph of $y = f(x)$ a distance c units upward

$y = f(x) - c$, shift the graph of $y = f(x)$ a distance c units downward

$y = f(x - c)$, shift the graph of $y = f(x)$ a distance c units to the right

$y = f(x + c)$, shift the graph of $y = f(x)$ a distance c units to the left

Translations illustrated...

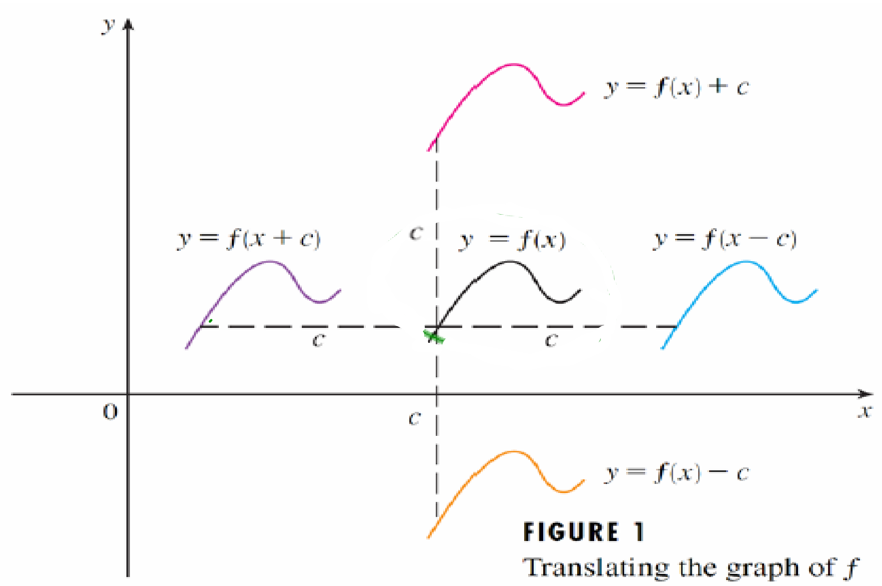


FIGURE 1
Translating the graph of f

Using Mapping Notation to Describe Transformations:

*Think of this as a set of instructions to follow to transform a graph.

$y=x^2$

x	$y = x^2$
−3	9
−2	4
−1	1
0	0
1	1
2	4
3	9

$k=2 \rightarrow$ up 2

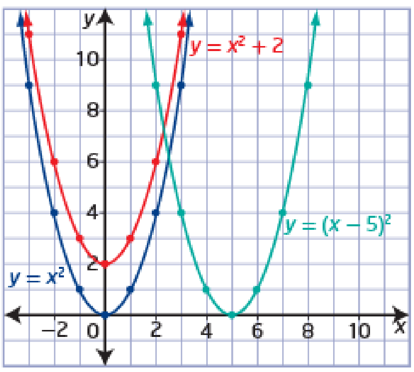
x	$y = x^2 + 2$
−3	11
−2	6
−1	3
0	2
1	3
2	6
3	11

$(x,y) \rightarrow (x, y+2)$

$h=5$ Right 5

x	$y = (x - 5)^2$
2	9
3	4
4	1
5	0
6	1
7	4
8	9

$(x,y) \rightarrow (x+5, y)$



Make table showing $y = \sqrt{x+1} - 3$

$h = -1$

$k = -3$

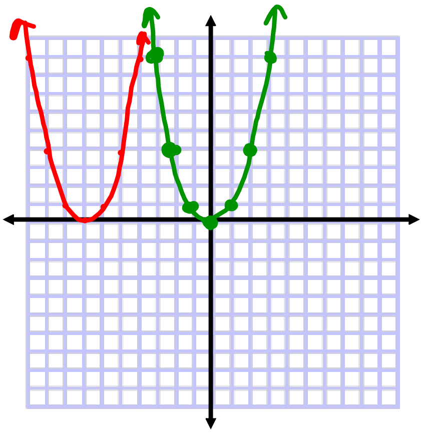
$(x,y) \rightarrow (x-1, y-3)$

base $y = \sqrt{x}$

x	y
0	0
1	1
4	2
9	3

x	y
−1	−3
0	−2
3	−1
8	0

Identify the translations for each of the following and sketch the transformation



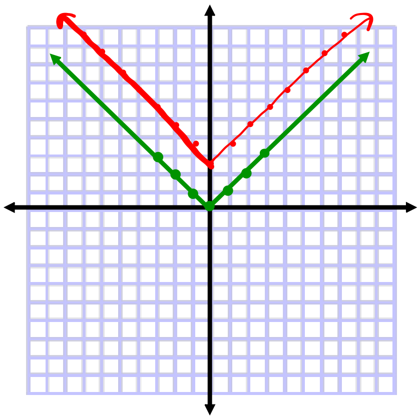
base:
$f(x)=x^2$

x	$f(x)$
-2	4
-1	1
0	0
1	1
2	4

$h=-7$

$f(x)=(x+7)^2$

x	$f(x)$
-9	4
-8	1
-7	0
-6	1
-5	4



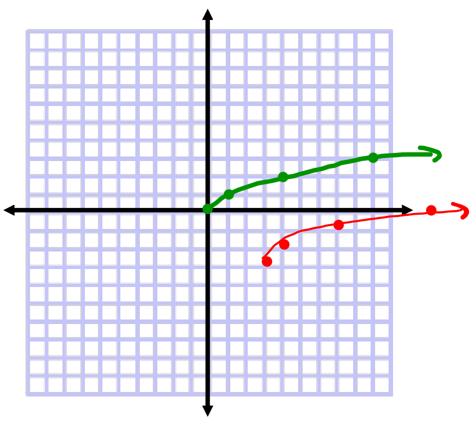
base:
$f(x)=|x|$

x	$f(x)$
-2	2
-1	1
0	0
1	1
2	2

$k=3$

$f(x)=|x|+3$

x	$f(x)$
-2	5
-1	4
0	3
1	4
2	5



base:
$f(x)=\sqrt{x}$

x	$f(x)$
0	0
1	1
4	2
9	3

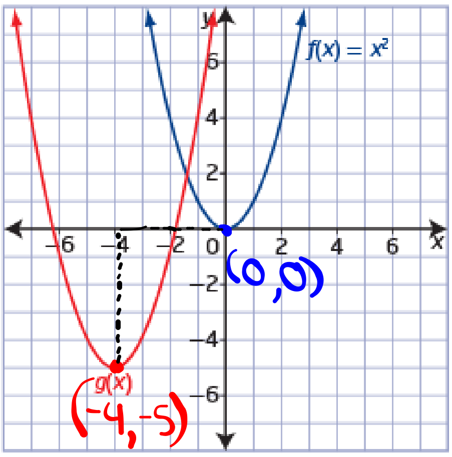
$h=3$ $k=-2$

$f(x)=\sqrt{x-3}-2$

x	$f(x)$
3	-2
4	-1
7	0
12	1

Determine the Equation of a Translated Function:

a)



Translated:

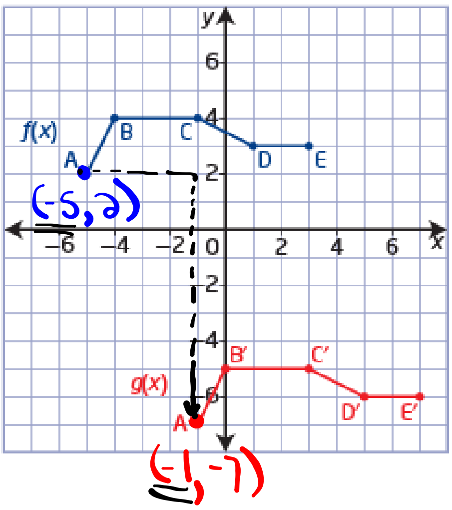
left 4 units $h = -4$

down 5 units $k = -5$

$g(x) = (x+4)^2 - 5$

or $g(x) = f(x+4) - 5$

b)



Translated:

Right 4 $h = 4$

down 9 $k = -9$

$g(x) = f(x-4) - 9$

Homework

Page 12 #1, 2, 4, and 8

Ex:

$y = |x+\underline{\underline{3}}| - \underline{\underline{2}}$

$h = -3 \quad k = -2$

Mapping:

$(x,y) \rightarrow (x-3, y-2)$