

Warm-Up

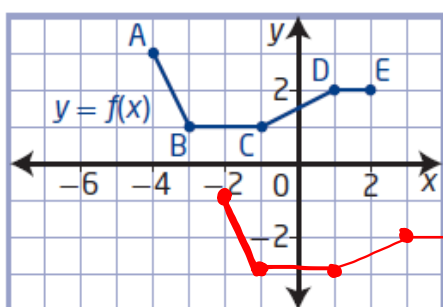
horizontal
↓ (change sign)
 $y = f(x-h) + k$
↑ vertical

8. Copy and complete the table.

Translation	Transformed Function	Transformation of Points	
vertical	$y = f(x) + \underline{5}$	$(x, y) \rightarrow (x, y + 5)$	$k = 5$ (Up)
horizontal	$y = f(x + \underline{7})$	$(x, y) \rightarrow (x - 7, y)$	$h = -7$ (Left)
horizontal	$y = f(x - \underline{3})$	$(x, y) \rightarrow (x + 3, y)$	$h = 3$ (Right)
vertical	$y = f(x) - \underline{6}$	$(x, y) \rightarrow (x, y - 6)$	$k = -6$ (Down)
horizontal and vertical	$y = f(x + \underline{4}) - \underline{9}$ $y + 9 = f(x + 4)$	$(x, y) \rightarrow (x - 4, y - 9)$	$h = -4$ Left $k = -9$ Down
horizontal and vertical	$y = f(x - \underline{4}) - \underline{6}$	$(x, y) \rightarrow (x + 4, y - 6)$	$h = 4$ Right $k = -6$ Down
$h + v$	$y = f(x + \underline{2}) + \underline{3}$	$(x, y) \rightarrow (x - 2, y + 3)$	$h = -2$ $k = 3$
horizontal and vertical	$y = f(x - \underline{h}) + \underline{k}$	$(x, y) \rightarrow (x + h, y + k)$	

Questions from Homework

4.



$$b) y = f(x - \underline{2}) - \underline{4}$$

$$h = 2 \quad k = -4$$

$$(x, y) \rightarrow (x + 2, y - 4)$$

$$A(-4, 3) \rightarrow (-2, -1)$$

$$B(-3, 1) \rightarrow (-1, -3)$$

$$C(-1, 1) \rightarrow (1, -3)$$

$$D(1, 2) \rightarrow (3, -2)$$

$$E(2, 2) \rightarrow (4, -2)$$

Transformations:

New Functions From Old Functions

✓ ~~Translations~~

✓ ~~Stretches~~

✓ ~~Reflections~~

Reflections and Stretches

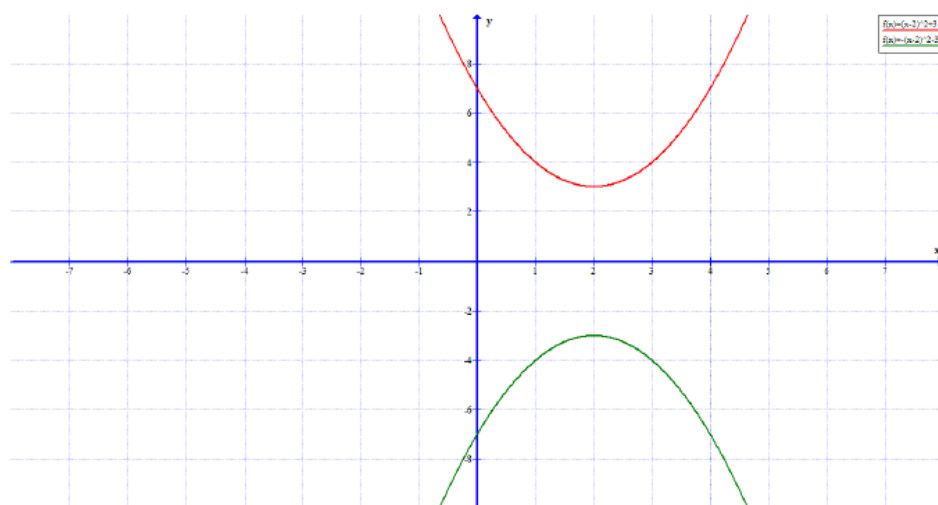
Focus on...

- developing an understanding of the effects of reflections on the graphs of functions and their related equations
- developing an understanding of the effects of vertical and horizontal stretches on the graphs of functions and their related equations

A **reflection** of a graph creates a mirror image in a line called the line of reflection. Reflections, like translations, do not change the shape of the graph. However, unlike translations, reflections may change the orientation of the graph.

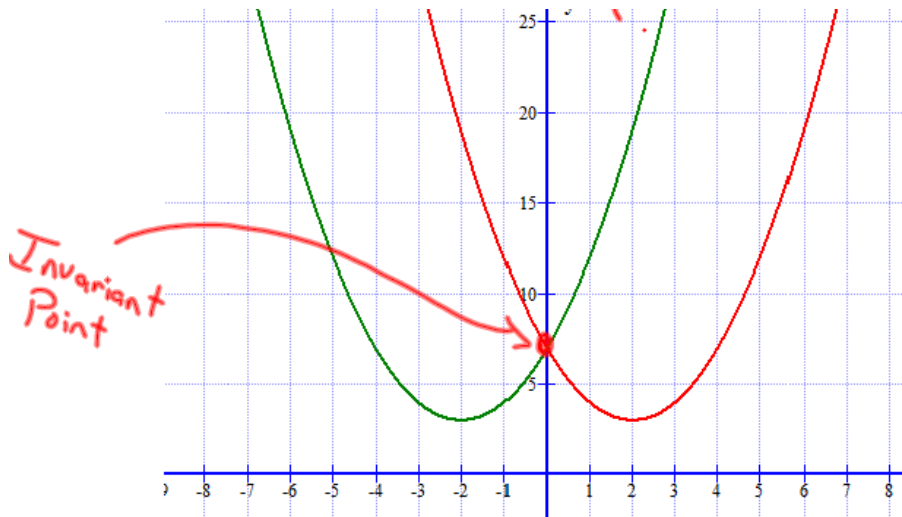
Vertical reflection $(x, y) \rightarrow (x, -y)$

- When the output of a function $y = f(x)$ is multiplied by -1 , the result, $y = -f(x)$, is a reflection of the graph in the x -axis.



Horizontal Reflection $(x, y) \rightarrow (-x, y)$

- When the input of a function $y = f(x)$ is multiplied by -1 , the result, $y = f(-x)$, is a reflection of the graph in the y -axis.

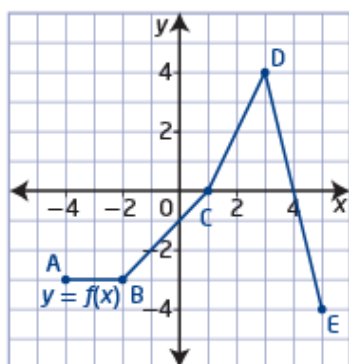


invariant point

- a point on a graph that remains unchanged after a transformation is applied to it
- any point on a curve that lies on the line of reflection is an invariant point

Example 1**Compare the Graphs of $y = f(x)$, $y = -f(x)$, and $y = f(-x)$**

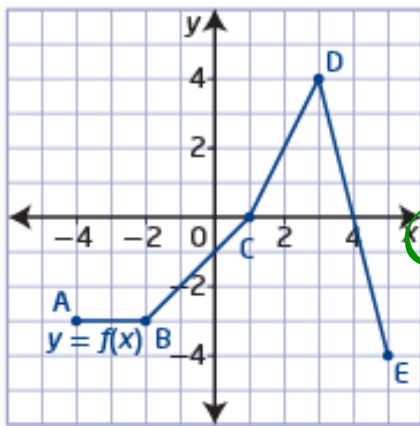
- a) Given the graph of $y = f(x)$, graph the functions $y = -f(x)$ and $y = f(-x)$.
- b) How are the graphs of $y = -f(x)$ and $y = f(-x)$ related to the graph of $y = f(x)$?



Remember...

- When the output of a function $y = f(x)$ is multiplied by -1 , the result, $y = -f(x)$, is a reflection of the graph in the x -axis.

- Sketch $y = -f(x)$ on the axis below (Vertical Reflection)



$$(x, y) \rightarrow (x, -y)$$

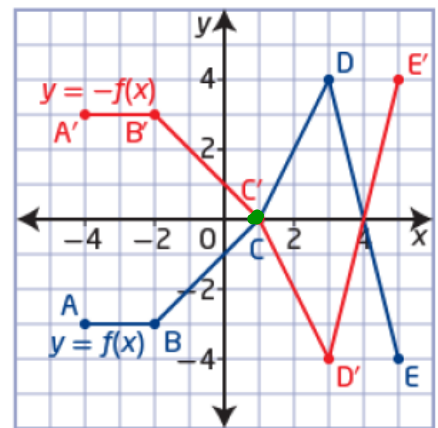
$$(-4, -3) \rightarrow (-4, 3)$$

$$(-2, -3) \rightarrow (-2, 3)$$

$$(1, 0) \rightarrow (1, 0)$$

$$(3, 4) \rightarrow (3, -4)$$

$$(5, -4) \rightarrow (5, 4)$$

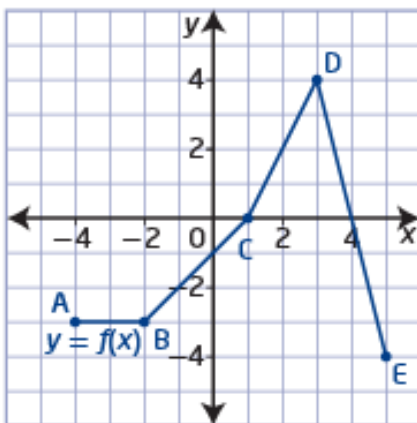


Invariant Point

Remember...

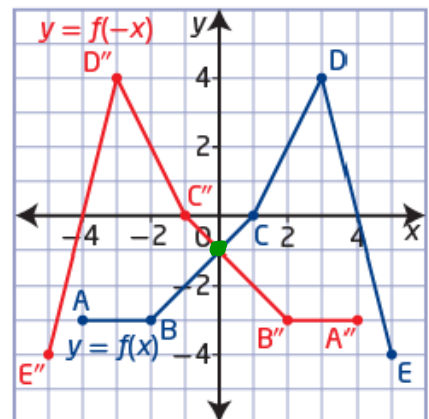
- When the input of a function $y = f(x)$ is multiplied by -1 , the result, $y = f(-x)$, is a reflection of the graph in the y -axis.

- Sketch $y = f(-x)$ on the axis below Horizontal reflection



$(x, y) \rightarrow (-x, y)$

$(-4, -3)$	$(4, -3)$
$(-2, -3)$	$(2, -3)$
$(1, 0)$	$(-1, 0)$
$(3, 4)$	$(-3, 4)$
$(5, -4)$	$(-5, -4)$



Homework

$$\begin{aligned} *f(-4) &= 2(-4)+1 && \text{Page 28 \#1, 3, 4} \\ &= -8+1 \\ &= -7 \end{aligned}$$

$$f(x) = 2x+1$$

x	y
-4	-7
-2	-3
0	1
2	5
4	9

Vertical

$$g(x) = -f(x)$$

x	y
-4	7
-2	3
0	-1
2	-5
4	-9

Horizontal

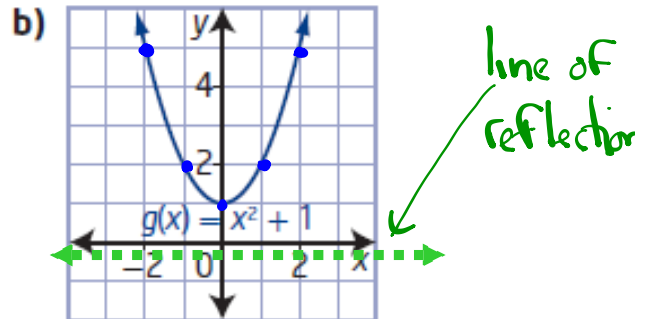
$$h(x) = f(-x)$$

x	y
4	-7
2	-3
0	1
-2	5
-4	9

Questions from Homework

3. Consider each graph of a function.

- Copy the graph of the function and sketch its reflection in the x-axis on the same set of axes. *(Vertical)*
- State the equation of the reflected function in simplified form.
- State the domain and range of each function.



(i) $g(x) = x^2 + 1$ original $(x, y) \rightarrow (x, -y)$

$h(x) = -(x^2 + 1)$

$h(x) = -x^2 - 1$

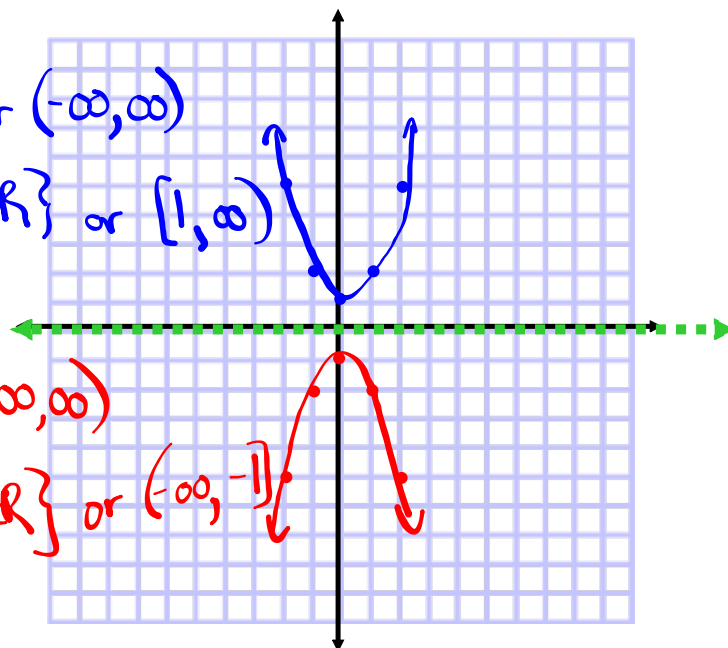
$(-2, 5)$	$(-2, -5)$
$(-1, 2)$	$(-1, -2)$
$(0, 1)$	$(0, -1)$
$(1, 2)$	$(1, -2)$
$(2, 5)$	$(2, -5)$

D: $\{x | x \in \mathbb{R}\}$ or $(-\infty, \infty)$

R: $\{y | y \geq 1, y \in \mathbb{R}\}$ or $[1, \infty)$

D: $\{x | x \in \mathbb{R}\}$ or $(-\infty, \infty)$

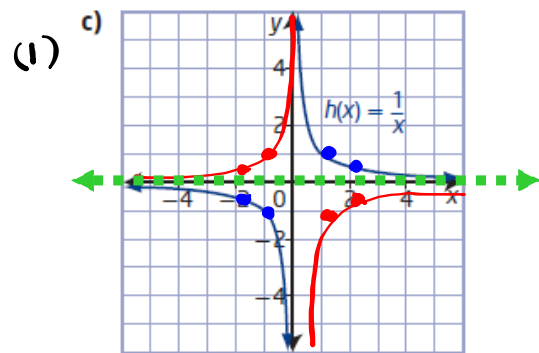
R: $\{y | y \leq -1, y \in \mathbb{R}\}$ or $(-\infty, -1]$



Questions from Homework

3. Consider each graph of a function.

- Copy the graph of the function and sketch its reflection in the x-axis on the same set of axes. *vertical*
- State the equation of the reflected function in simplified form.
- State the domain and range of each function.



(a)

$$h(x) = \frac{1}{x} \text{ original}$$

$$k(x) = -\frac{1}{x} \text{ transformed}$$

$$D: \{x \mid x \neq 0, x \in \mathbb{R}\}$$

$$R: \{y \mid y \neq 0, y \in \mathbb{R}\}$$

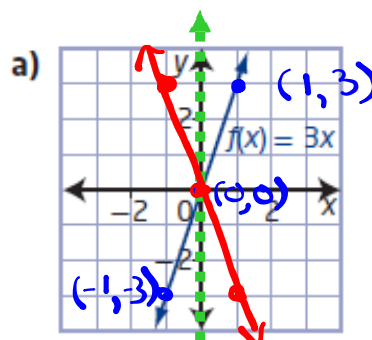
$$h(x) = \frac{1}{x} \rightarrow (x, y) \rightarrow (x, -y)$$

$(-2, -\frac{1}{2})$	$(-2, \frac{1}{2})$
$(-1, -1)$	$(-1, 1)$
$(0, \text{undefined})$	$(0, \text{undefined})$
$(1, 1)$	$(1, -1)$
$(2, \frac{1}{2})$	$(2, -\frac{1}{2})$

Questions from Homework

4. Consider each function in #3.

- (i) ✓ Copy the graph of the function and sketch its reflection in the y-axis on the same set of axes. *horizontal*
- (ii) • State the equation of the reflected function.
- (iii) • State the domain and range for each function.



$$(x, y) \rightarrow (-x, y)$$

(ii) $f(x) = 3x$

$$f(-x) = 3(-x)$$

$$f(-x) = -3x$$

$$g(x) = -3x$$

$$f(x) = 3x$$

x	y
-2	-6
-1	-3
0	0
1	3
2	6

$$g(x) = -3x$$

x	y
2	-6
1	-3
0	0
-1	3
-2	6

(iii) $f(x) = 3x$

$$D: \{x \mid x \in \mathbb{R}\}$$

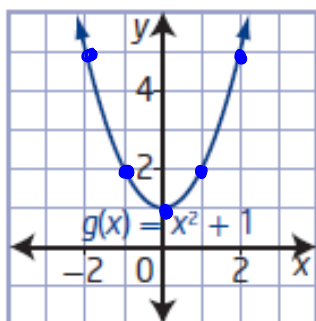
$$R: \{y \mid y \in \mathbb{R}\}$$

$$g(x) = -3x$$

$$D: \{x \mid x \in \mathbb{R}\}$$

$$R: \{y \mid y \in \mathbb{R}\}$$

④ b)



$$(x, y) \rightarrow (-x, y)$$

$(-2, 5)$	$(2, 5)$
$(-1, 2)$	$(1, 2)$
$(0, 1)$	$(0, 1)$
$(1, 2)$	$(-1, 2)$
$(2, 5)$	$(-2, 5)$

$$g(x) = x^2 + 1$$

$$g(-x) = (-x)^2 + 1$$

$$g(-x) = x^2 + 1$$

$$h(x) = x^2 + 1 \quad (\text{transformed})$$

Vertical and Horizontal Stretches

A **stretch**, unlike a translation or a reflection, changes the shape of the graph. However, like translations, stretches do not change the orientation of the graph.

- When the output of a function $y = f(x)$ is multiplied by a non-zero constant a , the result, $y = af(x)$ or $\frac{y}{a} = f(x)$, is a vertical stretch of the graph about the x -axis by a factor of $|a|$. If $a < 0$, then the graph is also reflected in the x -axis.
- When the input of a function $y = f(x)$ is multiplied by a non-zero constant b , the result, $y = f(bx)$, is a horizontal stretch of the graph about the y -axis by a factor of $\frac{1}{|b|}$. If $b < 0$, then the graph is also reflected in the y -axis.

stretch

- a transformation in which the distance of each x -coordinate or y -coordinate from the line of reflection is multiplied by some scale factor

- *Compression* scale factors between 0 and 1 result in the point moving closer to the line of reflection; scale factors greater than 1 result in the point moving farther away from the line of reflection
Ex: 0.5, $\frac{1}{4}$

* If you can't see a value in place of "a" or "b" then we let them equal 1

If you can't see a value in place of "h" or "k" then we let them equal 0

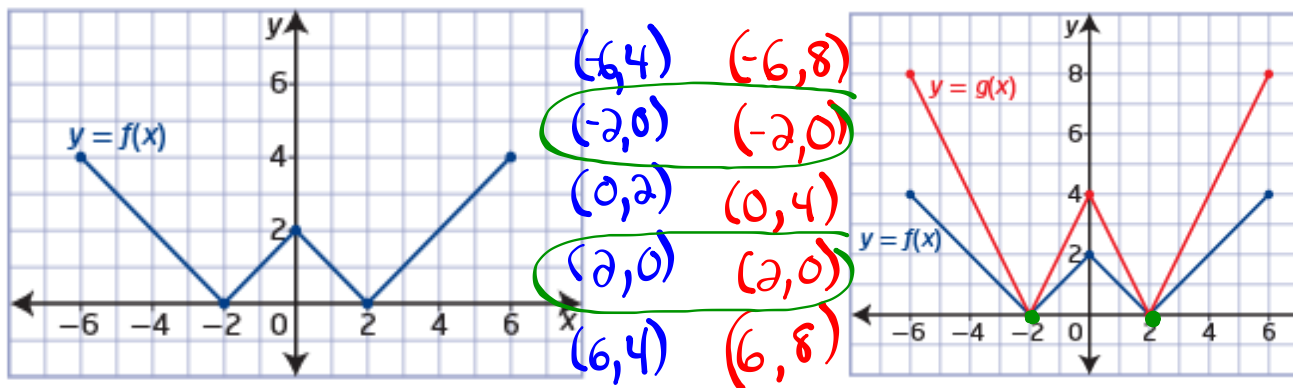
Vertical Stretch or Compression...

- When the output of a function $y = f(x)$ is multiplied by a non-zero constant a , the result, $y = \underline{af(x)}$ or $\frac{y}{a} = f(x)$, is a vertical stretch of the graph about the x-axis by a factor of $|a|$. If $a < 0$, then the graph is also reflected in the x-axis. (negative)

$a=2 \rightarrow$ Vertical Stretch by a factor of 2

a) $g(x) = \underline{2f(x)}$

$(x, y) \rightarrow (x, 2y)$



The invariant points are $(-2, 0)$ and $(2, 0)$.

For $f(x)$, the domain is $\{x \mid -6 \leq x \leq 6, x \in \mathbb{R}\}$, or $[-6, 6]$, and the range is $\{y \mid 0 \leq y \leq 4, y \in \mathbb{R}\}$, or $[0, 4]$.

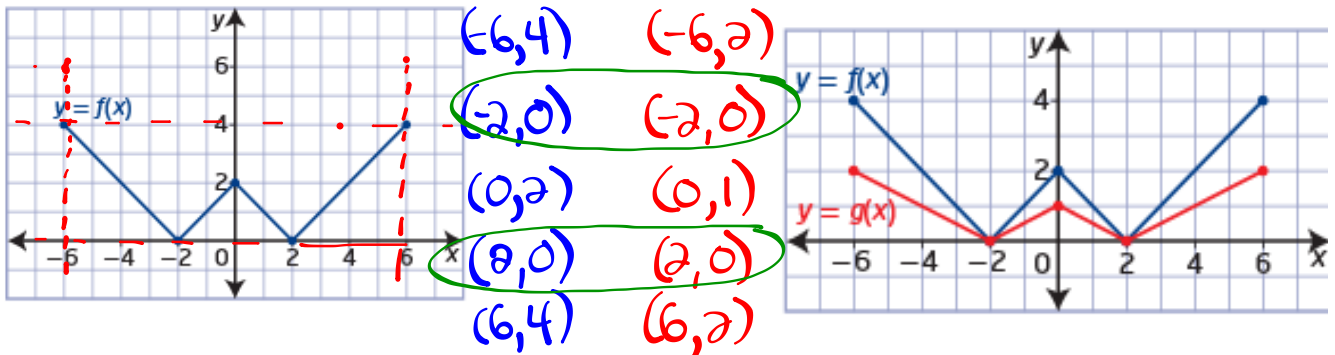
← interval notation

For $g(x)$, the domain is $\{x \mid -6 \leq x \leq 6, x \in \mathbb{R}\}$, or $[-6, 6]$, and the range is $\{y \mid 0 \leq y \leq 8, y \in \mathbb{R}\}$, or $[0, 8]$.

$$b) \underline{g(x) = \frac{1}{2}f(x)}$$

$a = \frac{1}{2} \rightarrow$ vertical stretch by a factor of $\frac{1}{2}$

$$(x, y) \rightarrow (x, \frac{1}{2}y)$$



The invariant points are $(-2, 0)$ and $(2, 0)$.

For $f(x)$, the domain is

$\{x \mid -6 \leq x \leq 6, x \in \mathbb{R}\}$, or $[-6, 6]$,

and the range is

$\{y \mid 0 \leq y \leq 4, y \in \mathbb{R}\}$, or $[0, 4]$.

For $g(x)$, the domain is $\{x \mid -6 \leq x \leq 6, x \in \mathbb{R}\}$, or $[-6, 6]$,

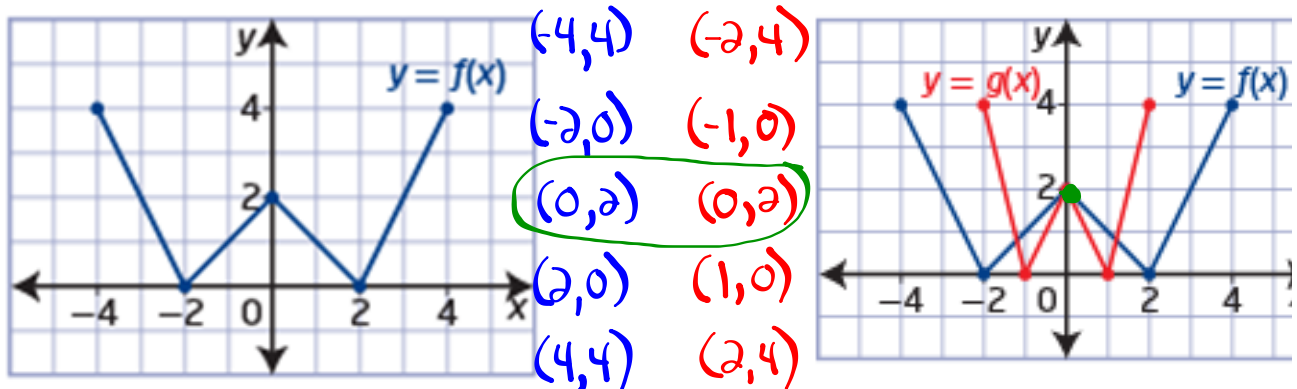
and the range is $\{y \mid 0 \leq y \leq 2, y \in \mathbb{R}\}$, or $[0, 2]$.

Horizontal Stretch or Compression... (Reciprocal)

- When the input of a function $y = f(x)$ is multiplied by a non-zero constant b , the result, $y = f(bx)$, is a horizontal stretch of the graph about the y-axis by a factor of $\frac{1}{|b|}$. If $b < 0$, then the graph is also reflected in the y-axis.

$b=2 \rightarrow$ Horizontal Stretch by a factor $\frac{1}{2}$

a) $g(x) = f(2x)$ $(x, y) \rightarrow (\frac{1}{2}x, y)$



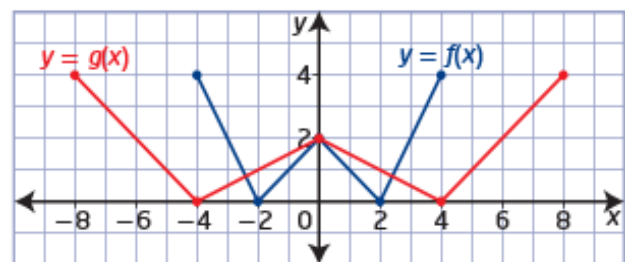
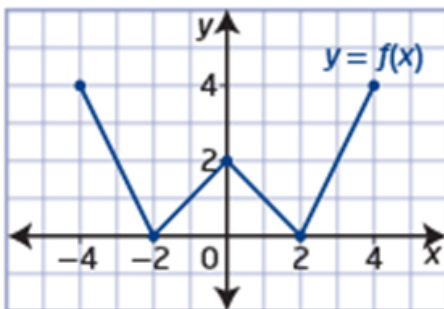
The invariant point is $(0, 2)$.

For $f(x)$, the domain is $\{x \mid -4 \leq x \leq 4, x \in \mathbb{R}\}$, or $[-4, 4]$, and the range is $\{y \mid 0 \leq y \leq 4, y \in \mathbb{R}\}$, or $[0, 4]$.

For $g(x)$, the domain is $\{x \mid -2 \leq x \leq 2, x \in \mathbb{R}\}$, or $[-2, 2]$, and the range is $\{y \mid 0 \leq y \leq 4, y \in \mathbb{R}\}$, or $[0, 4]$.

$$\text{b) } g(x) = f\left(\frac{1}{2}x\right)$$

$b = \frac{1}{2} \rightarrow$ Horizontal stretch by a factor of 2

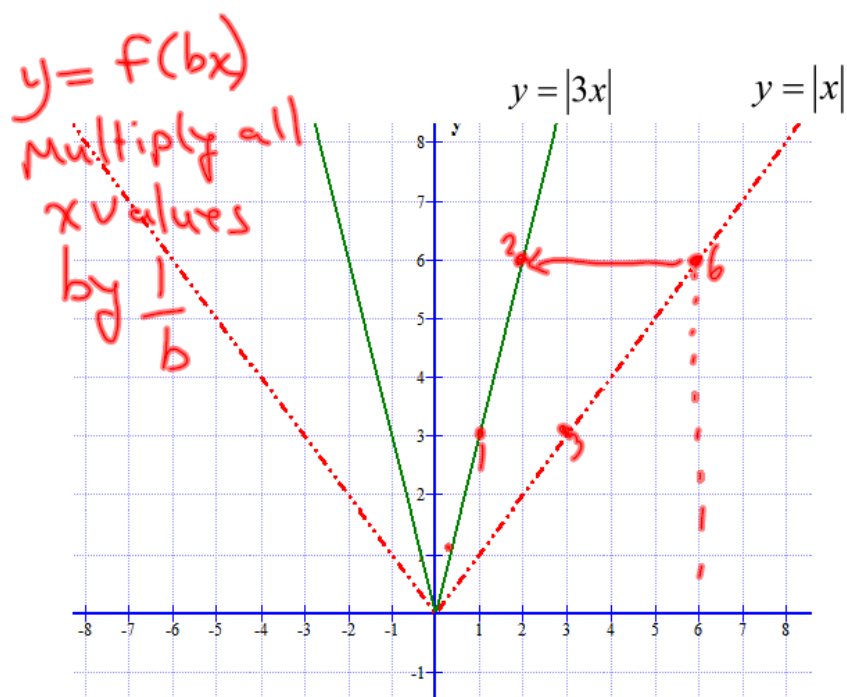


The invariant point is $(0, 2)$.

For $f(x)$, the domain is $\{x \mid -4 \leq x \leq 4, x \in \mathbb{R}\}$, or $[-4, 4]$, and the range is $\{y \mid 0 \leq y \leq 4, y \in \mathbb{R}\}$, or $[0, 4]$.

For $g(x)$, the domain is $\{x \mid -8 \leq x \leq 8, x \in \mathbb{R}\}$, or $[-8, 8]$, and the range is $\{y \mid 0 \leq y \leq 4, y \in \mathbb{R}\}$, or $[0, 4]$.

Horizontal Stretch or Compression...



Horizontal Stretch or Compression...

- When the input of a function $y = f(x)$ is multiplied by a non-zero constant b , the result, $y = f(bx)$, is a horizontal stretch of the graph about the y -axis by a factor of $\frac{1}{|b|}$. If $b < 0$, then the graph is also reflected in the y -axis.

$$y = -3f(-2x) + 7$$

Homework

Page 28 # 2, 5, 6, 7

Determine the Equation of a Translated Function:

