Correct Homework Sheet

② $\dfrac{1-\boxed{\cos 2\theta}}{\sin^2\theta} = 2$

$\dfrac{1-(\cos^2\theta-\sin^2\theta)}{\sin^2\theta}$

$\dfrac{\boxed{1-\cos^2\theta}+\sin^2\theta}{\sin^2\theta}$

$\dfrac{\sin^2\theta+\sin^2\theta}{\sin^2\theta}$

$\dfrac{2\cancel{\sin^2\theta}}{\cancel{\sin^2\theta}}$

2

③ $\boxed{\sin(x+y)}\boxed{\sin(x-y)} = \cos^2 y-\cos^2 x$

$(\sin x\cos y+\cos x\sin y)(\sin x\cos y-\cos x\sin y)$

$\sin^2 x\cos^2 y-\cos^2 x\sin^2 y$

$(1-\cos^2 x)\cos^2 y-\cos^2 x(1-\cos^2 y)$

$\cos^2 y-\cancel{\cos^2 x\cos^2 y}-\cos^2 x+\cancel{\cos^2 x\cos^2 y}$

$\cos^2 y-\cos^2 x$

⑤ $\boxed{\tan^4\theta} = \sec^4\theta\,(1-2\cos^2\theta+\cos^4\theta)$

$\dfrac{\sin^4\theta}{\cos^4\theta}$

$\boxed{\sec^4\theta}\,(1-\cos^2\theta)(1-\cos^2\theta)$

$\dfrac{1}{\cos^4\theta}$

9) $\dfrac{\cos\theta}{1+\sin\theta} + \dfrac{1+\sin\theta}{\cos\theta} = 2\boxed{\sec\theta}$

$\dfrac{\cos^2\theta + (1+\sin\theta)(1+\sin\theta)}{\cos\theta(1+\sin\theta)}$ $\qquad 2\left(\dfrac{1}{\cos\theta}\right)$

$\dfrac{\cos^2\theta + 1 + 2\sin\theta + \sin^2\theta}{\cos\theta(1+\sin\theta)}$ $\qquad \dfrac{2}{\cos\theta}$

$\dfrac{2+2\sin\theta}{\cos\theta(1+\sin\theta)}$

$\dfrac{2\cancel{(1+\sin\theta)}}{\cos\theta\cancel{(1+\sin\theta)}}$

$\dfrac{2}{\cos\theta}$

10) $\dfrac{\tan^2\theta}{\boxed{\tan^2\theta+1}} = \sin^2\theta$

$\boxed{\tan^2\theta} \div \sec^2\theta$

$\dfrac{\sin^2\theta}{\cos^2\theta} \div \dfrac{1}{\cos^2\theta}$

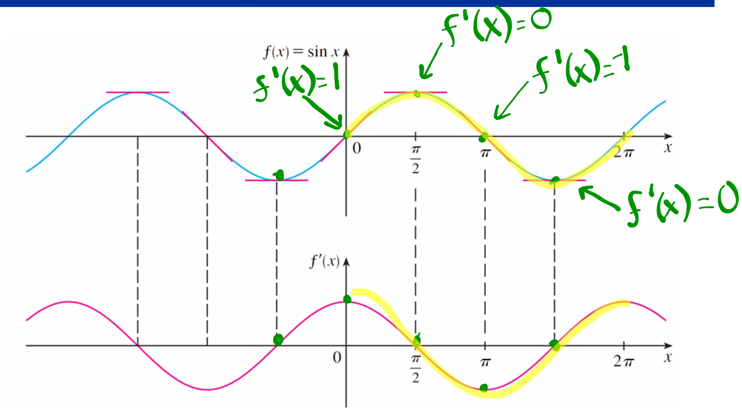
$\dfrac{\sin^2\theta}{\cancel{\cos^2\theta}} \times \cancel{\cos^2\theta}$

$\sin^2\theta$

Derivatives of Trigonometric Functions

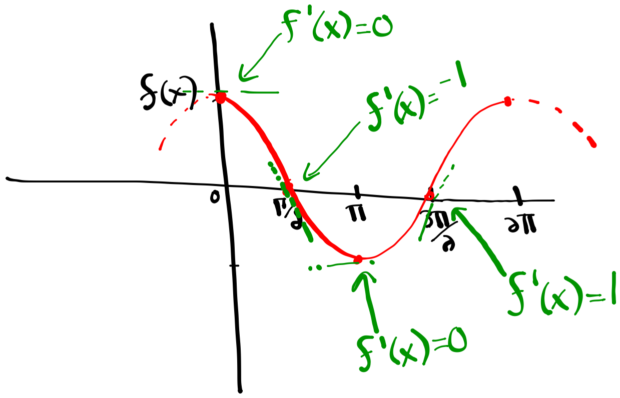
The Sine Function

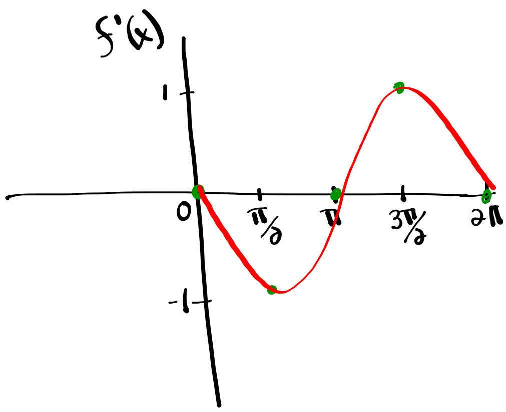
- We recall that the derivative $f'(x)$ of a function $f(x)$ gives the slope of the tangent.
- On the next slide we graph $f(x) = \sin x$ together with $f'(x)$, as determined by the slope of the tangent to the sine curve.
  - Note that x is measured in radians.
- The derivative graph resembles the graph of the cosine!



if $f(x) = \sin x$

$f'(x) = \cos x$





Let's check this using the definition of a derivative...

$$f'(x) = \lim_{h\to 0} \frac{f(x+h) - f(x)}{h} = \lim_{h\to 0} \frac{\sin(x+h) - \sin x}{h}$$

$$= \lim_{h\to 0} \frac{\sin x \cos h + \cos x \sin h - \sin x}{h}$$

$$= \lim_{h\to 0} \left[\frac{\sin x \cos h - \sin x}{h} + \frac{\cos x \sin h}{h} \right]$$

$$= \lim_{h\to 0} \left[\frac{\sin x \cos h - \sin x}{h} + \frac{\cos x \sin h}{h} \right]$$

$$= \lim_{h\to 0} \left[\sin x \left(\frac{\cos h - 1}{h} \right) + \cos x \left(\frac{\sin h}{h} \right) \right]$$

$$= \lim_{h\to 0} \sin x \cdot \lim_{h\to 0} \frac{\cos h - 1}{h} + \lim_{h\to 0} \cos x \cdot \lim_{h\to 0} \frac{\sin h}{h}$$

- Our calculations have brought us to four limits, two of which are easy:
  - Since x is constant while $h \to 0$,

$$\lim_{h\to 0} \sin x = \sin x \quad \text{and} \quad \lim_{h\to 0} \cos x = \cos x$$

- With some work we can also show that

$$\lim_{h\to 0} \frac{\sin h}{h} = 1 \quad \text{and} \quad \lim_{h\to 0} \frac{\cos h - 1}{h} = 0$$

$f(x) = \sin x$

$f'(x) = \cos x$

- Thus our guess is confirmed:

$$f'(x) = \lim_{h\to 0} \sin x \cdot \lim_{h\to 0} \frac{\cos h - 1}{h} + \lim_{h\to 0} \cos x \cdot \lim_{h\to 0} \frac{\sin h}{h}$$

$$= (\sin x) \cdot 0 + (\cos x) \cdot 1 = \cos x$$

Rules to differentiate trigonometric functions:

Given that "u" represents some differentiable function...

$$\frac{d}{du}(\sin u) = \cos u \bullet du$$

$$\frac{d}{du}(\csc u) = -\csc u \cot u \bullet du$$

$$\frac{d}{du}(\cos u) = -\sin u \bullet du$$

$$\frac{d}{du}(\sec u) = \sec u \tan u \bullet du$$

$$\frac{d}{du}(\tan u) = \sec^2 u \bullet du$$

$$\frac{d}{du}(\cot u) = -\csc^2 u \bullet du$$

Ex: $f(x) = \tan(5x^2)$

$u = 5x^2$

$du = 10x$

$f'(x) = \sec^2(5x^2) \bullet 10x$

$f'(x) = 10x \sec^2(5x^2)$

Let's Practice...

Differentiate the following:

$y = \sin 3x$ $u = 3x$ $du = 3$

$y' = \cos(3x) \cdot 3$

$y' = 3\cos(3x)$

$y = \sin(x+2)$ $u = x+2$ $du = 1$

$y' = \cos(x+2) \cdot 1$

$y' = \cos(x+2)$

$y = \sin(kx+d)$ $u = kx+d$ $du = k$

$y' = \cos(kx+d) \cdot k$

$y' = k\cos(kx+d)$

Ex #2.

Differentiate:

$u = x^3$
$du = 3x^2$

$y = (\sin x)^3$ $u = x$ $du = 1$

$y = [\sin(x^2-1)]^3$ $u = x^2 - 1$ $du = 2x$

a) $y = \sin(x^3)$

b) $y = \sin^3 x$

c) $y = \sin^3(x^2 - 1)$

$y' = \cos(x^3) \cdot 3x^2$
$y' = 3x^2 \cos(x^3)$

$y' = 3(\sin x)^2 (\cos x) \cdot 1$
$y' = 3\sin^2 x \cos x$

$y' = 3[\sin(x^2-1)]^2 \cos(x^2-1) 2x$

$y' = 6x \sin^2(x^2-1) \cos(x^2-1)$

Ex #3.

Differentiate:

Product Rule:
$f'(x)g(x) + f(x)g'(x)$

$$y = (x^2)(\cos x)$$

$$y' = 2x\cos x + x^2(-\sin x \cdot 1)$$

$$y' = 2x\cos x - x^2\sin x$$

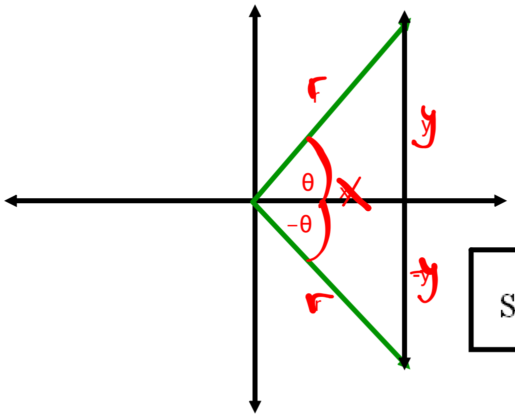
$$y' = x(2\cos x - x\sin x)$$

Homework

Do Questions 1 and 3 from Exercise 7.2 Page 313

Worksheet on derivatives of trigonometric functions

Negative Angles



$$\sin\theta = \frac{y}{r} \qquad \cos(\theta) = \frac{x}{r}$$

$$\sin(-\theta) = -\frac{y}{r} \qquad \cos(-\theta) = \frac{x}{r}$$

$$\boxed{\sin(-\theta) = -\sin\theta} \qquad \boxed{\cos(-\theta) = \cos\theta}$$

Exercise 7.2

1 a) $y = \cos(-4x)$ $\quad u = -4x$, $du = -4$

$y' = -\sin u \cdot du$

$y' = -\sin(-4x) \cdot -4$

$y' = 4\sin(-4x)$

$y' = -4\sin(4x)$

$y = \cos(-4x)$

$y = \cos(4x)$ $\quad u = 4x$, $du = 4$

$y' = -\sin u \cdot du$

$y' = -\sin(4x) \cdot 4$

$y' = -4\sin(4x)$

7.2

1 c) $y = 4\sin(-2x^2-3)$

$y' = \cancel{0(\sin(-2x^2-3))} + 4(\cos(-2x^2-3))(-4x)$

$y' = -16x\cos(-2x^2-3)$

g) $y = \sin^{-2}(x^3)$

$y = [\sin(x^3)]^{-2}$

$y' = -2[\sin(x^3)]^{-3}(\cos(x^3))(3x^2)$

$y' = -6x^2\sin^{-3}(x^3)\cos(x^3)$

$y' = -\dfrac{6x^2\cos(x^3)}{\sin^3(x^3)}$

i) $y = 3\sin^4(2-x)^{-1}$

$y = 3[\sin(2-x)^{-1}]^4$

$u = (2-x)^{-1}$
$du = -(2-x)^{-2}(-1)$
$= (2-x)^{-2}$

$y' = 12[\sin(2-x)^{-1}]^3\cos(2-x)^{-1}\cdot(2-x)^{-2}$

$y' = \dfrac{12\sin^3(2-x)^{-1}\cos(2-x)^{-1}}{(2-x)^2}$

l) $y = \dfrac{\sin x}{1+\cos x}$

$\dfrac{dy}{dx} = \dfrac{\cos x(1+\cos x) - \sin x(-\sin x)}{(1+\cos x)^2}$

$\dfrac{dy}{dx} = \dfrac{\cos x + \cos^2 x + \sin^2 x}{(1+\cos x)^2}$ Pythagorean Identity

$\dfrac{dy}{dx} = \dfrac{\cancel{1+\cos x}}{(1+\cos x)^{\cancel{2}}} = \dfrac{1}{1+\cos x}$

Quiz:

$(x^2)^5$

$x^{2 \cdot 5}$

x^{10}

② c) $f(x) = \sqrt{\left(\frac{x^2+1}{x^2-1}\right)^3}$

$f(x) = \left[\left(\frac{x^2+1}{x^2-1}\right)^3\right]^{1/2}$

$f(x) = \left(\frac{x^2+1}{x^2-1}\right)^{3/2}$

$\cancel{2x^3} - 2x \quad -\cancel{2x^3} - 2x$

$f'(x) = \frac{3}{2}\left(\frac{x^2+1}{x^2-1}\right)^{1/2}\left[\frac{2x(x^2-1) - 2x(x^2+1)}{(x^2-1)^2}\right]$

$f'(x) = \frac{3}{2} \cdot \frac{(x^2+1)^{1/2}}{(x^2-1)^{1/2}} \cdot \frac{-4x}{(x^2-1)^2}$

$f'(x) = \frac{-\overset{-6}{\cancel{12}}x(x^2+1)^{1/2}}{\underset{1}{\cancel{2}}(x^2-1)^{5/2}}$

$f'(x) = \frac{-6x(x^2+1)^{1/2}}{(x^2-1)^{5/2}}$

Quiz:

② d)

$$y = \frac{(6x-10)^3}{\sqrt{4x-7}}$$

$$y = \frac{(6x-10)^3}{(4x-7)^{1/2}}$$

$$\frac{dy}{dx} = \frac{3(6x-10)^2(6)(4x-7)^{1/2} - (6x-10)^3\left(\frac{1}{2}\right)(4x-7)^{-1/2}(4)}{\left((4x-7)^{1/2}\right)^2}$$

$$\frac{dy}{dx} = \frac{18(6x-10)^2(4x-7)^{1/2} - 2(6x-10)^3(4x-7)^{-1/2}}{(4x-7)}$$

$$\frac{dy}{dx} = \frac{2(6x-10)^2(4x-7)^{-1/2}\left[9\overset{36x-63}{(4x-7)} - \overset{-6x+10}{(6x-10)}\right]}{(4x-7)}$$

$$\frac{dy}{dx} = \frac{2(6x-10)^2(30x-53)}{(4x-7)^{1/2}(4x-7)}$$

$$\frac{dy}{dx} = \frac{2(6x-10)^2(30x-53)}{(4x-7)^{3/2}}$$

$f(x) = \frac{3}{2x}$

$f(x) = \frac{3}{2}x^{-1}$

$f'(x) = -\frac{3}{2}x^{-2}$

$f'(x) = -\frac{3}{2x^2}$

$f(x) = \frac{3}{2x}$

$f(x) = 3(2x)^{-1}$

$f'(x) = -3(2x)^{-2}(2)$

$f'(x) = \frac{-6}{(2x)^2}$

$f'(x) = \frac{-6}{4x^2}$

$f'(x) = -\frac{3}{2x^2}$

Attachments

Derivatives Worksheet.doc