10.4

5b) $t_5 = 8$, $t_{10} = \frac{1}{4}$ $t_3 = ?$ Find "a" + "r"

$t_n = ar^{n-1}$

$t_5 = ar^{5-1}$

$t_5 = ar^4$

$8 = ar^4$

$8 = a\left(\frac{1}{2}\right)^4$

$8 = a\left(\frac{1}{16}\right)$

$16 \cdot 8 = \frac{a}{16} \cdot 16$

$\boxed{128 = a}$

$t_n = ar^{n-1}$

$t_{10} = ar^{10-1}$

$t_{10} = ar^9$

$\frac{1}{4} = ar^9$

Elimination

$$\frac{\frac{1}{4} = ar^9}{8 = ar^4}$$

$\frac{1}{4} \div 8 = 1r^{9-4}$

$\frac{1}{4} \times \frac{1}{8} = r^5$

$\frac{1}{32} = r^5$

$\frac{(1)^{1/5}}{(32)^{1/5}} = (r^5)^{1/5}$ or $\frac{\sqrt[5]{1}}{\sqrt[5]{32}} = \sqrt[5]{r^5}$

$\boxed{\frac{1}{2} = r}$ $\frac{1}{2} = r$

$t_3 = ar^{3-1}$

$t_3 = (128)\left(\frac{1}{2}\right)^2$

$t_3 = (128)\left(\frac{1}{4}\right)$

$t_3 = \frac{128}{4} = 32$

128, 64, 32

10.9

② Find S_7 for $30 - 5 + \frac{5}{6} - \ldots$

$-\frac{1}{6}$ $-\frac{1}{6}$

Given:

$a = 30$

$r = \frac{-5}{30} = \frac{-1}{6}$

$n = 7$

$$S_n = \frac{a(r^n - 1)}{r - 1}$$

$$S_7 = \frac{30\left[\left(-\frac{1}{6}\right)^7 - 1\right]}{-\frac{1}{6} - 1}$$

$$S_7 = \frac{30\left[\frac{-1}{279\,936} - \frac{1}{1}\right]}{-\frac{1}{6} - \frac{1}{1}}$$

$$S_7 = \frac{30\left[\frac{-1}{279\,936} - \frac{279\,936}{279\,936}\right]}{-\frac{1}{6} - \frac{6}{6}}$$

$$S_7 = 30\left(\frac{-279\,937}{279\,936}\right) \div \frac{-7}{6}$$

$$S_7 = \overset{5}{\cancel{30}}\left(\frac{-\overset{39\,991}{\cancel{279\,937}}}{\underset{\underset{7776}{\cancel{46\,656}}}{\cancel{279\,936}}}\right)\left(\frac{-\overset{1}{\cancel{6}}}{\underset{1}{\cancel{7}}}\right)$$

$$S_7 = 5\left(\frac{-39\,991}{7776}\right)\left(\frac{-1}{1}\right) = \boxed{\frac{199\,955}{7776}}$$

or

$$S_7 = \frac{50\,388\,660}{1\,959\,552} = \boxed{\frac{199\,955}{7776}}$$

10.9

(2) $\underline{30} - 5 + \frac{5}{6}$

$-\frac{1}{6}$ $-\frac{1}{6}$

Given:

$a = 30$

$r = -\frac{1}{6}$

$$S_7 = \frac{(30)\left[\left(-\frac{1}{6}\right)^7 - 1\right]}{\left(-\frac{1}{6}\right) - 1}$$

$$S_7 = \frac{(30)\left[\frac{-1}{279936} - \frac{1}{1}\right]}{-\frac{1}{6} - \frac{1}{1}}$$

$$S_7 = \frac{(30)\left[\frac{-1}{279936} - \frac{279936}{279936}\right]}{\frac{-1}{6} - \frac{6}{6}}$$

$$S_7 = 30\left(-\frac{279937}{279936}\right) \div -\frac{7}{6}$$

$$S_7 = 30\left(\frac{-279937}{279936}\right)\left(-\frac{6}{7}\right)$$

$$S_7 = \frac{50388660 \div 6}{1959552 \div 6}$$

$$S_7 = \frac{8398110 \div 2}{326592 \div 2}$$

$$S_7 = \frac{4199055 \div 21}{163296 \div 21}$$

$$\boxed{S_7 = \frac{199955}{7776}} \text{ or } 25\frac{5555}{7776}$$

10.9

3 c) $81+27+9+\ldots$ $S_6 = \frac{(81)\left[\left(\frac{1}{3}\right)^6-1\right]}{\left(\frac{1}{3}\right)-1}$

$S_6=?$
$a=81$
$r=\frac{1}{3}$

$S_6 = 81\frac{\left[\frac{1}{729}-\frac{1}{1}\right]}{\frac{1}{3}-\frac{1}{1}}$

$S_6 = 81\frac{\left[\frac{1}{729}-\frac{729}{729}\right]}{\frac{1}{3}-\frac{3}{3}}$

$S_6 = 81\left(\frac{-728}{729}\right)\div\frac{-2}{3}$

$S_6 = 81\left(\frac{-728}{729}\right)\left(-\frac{3}{2}\right)$

$S_6 = \frac{176904}{1458} = \frac{364}{3}$

5 b) $1+\frac{5}{2}+\frac{25}{4}+\ldots+\frac{15625}{64}$

$a=1$
$r=\frac{5}{2}$
$t_n=\frac{15625}{64}$

① Find n:

$t_n = ar^{n-1}$

$\frac{\frac{15625}{64}}{1} = \frac{(\cancel{1})\left(\frac{5}{2}\right)^{n-1}}{\cancel{1}}$

$\frac{15625}{64} = \left(\frac{5}{2}\right)^{n-1}$ $\frac{\log\left(\frac{15625}{64}\right)}{\log\left(\frac{5}{2}\right)} = 6$

$\left(\cancel{\frac{5}{2}}\right)^6 = \left(\cancel{\frac{5}{2}}\right)^{n-1}$

$6 \overset{+1}{=} n-1^{+1}$

$\boxed{7=n}$

② Find S_7

$S_7 = \frac{(1)\left[\left(\frac{5}{2}\right)^7-1\right]}{\left(\frac{5}{2}\right)-1}$

$S_7 = 1\frac{\left[\frac{78125}{128}-\frac{1}{1}\right]}{\frac{5}{2}-\frac{1}{1}}$

$S_7 = 1\frac{\left[\frac{78125}{128}-\frac{128}{128}\right]}{\frac{5}{2}-\frac{2}{2}}$

$S_7 = 1\left(\frac{77997}{128}\right)\div\frac{3}{2}$

$S_7 = 1\left(\frac{77997}{128}\right)\left(\frac{2}{3}\right)$

$S_7 = \frac{155994}{384} = \boxed{\frac{25999}{64}}$

6 $S_7=1093$
$r=\frac{1}{3}$
$a=?$

$S_7 = \frac{a\left[\left(\frac{1}{3}\right)^7-1\right]}{\left(\frac{1}{3}\right)-1}$

$1093 = a\frac{\left[\frac{1}{2187}-\frac{2187}{2187}\right]}{\frac{1}{3}-\frac{3}{3}}$

$1093 = a\left[\frac{-2186}{2187}\right]\div\frac{-2}{3}$

$1093 = a\left(\frac{-2186}{2187}\right)\left(-\frac{3}{2}\right)$

$4374\cdot 1093 = \frac{6558a}{\cancel{4374}}\cdot 4374$

$\frac{4780782}{6558} = \frac{6558a}{6558}$

$\boxed{729=a}$

10.9

① b) $2 + \frac{2}{3} + \frac{2}{9} + \frac{2}{27} + \cdots$

$a = 2$

$r = \frac{2}{3} \div \frac{2}{1}$

$= \frac{\cancel{2}}{3} \times \frac{1}{\cancel{2}}$

$= \frac{1}{3}$

$n = 4$

$$S_4 = \frac{(2)\left[\left(\frac{1}{3}\right)^4 - 1\right]}{\left[\left(\frac{1}{3}\right) - 1\right]}$$

$$= \frac{2\left[\frac{1}{81} - \frac{81}{81}\right]}{\left[\frac{1}{3} - \frac{3}{3}\right]}$$

$$= \frac{2\left(-\frac{80}{81}\right)}{\left(\frac{-2}{3}\right)}$$

$$= \overset{1}{\cancel{2}}\left(-\frac{80}{\cancel{81}_{27}}\right)\left(-\frac{\cancel{3}^{1}}{\cancel{2}_{1}}\right)$$

$$= \frac{480}{162}$$

$$= \frac{80}{27}$$

10.9

(7) Given:

$t_7 = 192$

$a = 3$

$S_8 = ?$

(1) Find r:

$t_n = ar^{n-1}$

$192 = (3)r^{7-1}$

$\frac{192}{3} = \frac{3r^6}{3}$

$64 = r^6$

$\pm 2 = r$

if $r = 2$

$S_8 = \frac{(3)[(2)^8 - 1]}{(2-1)}$

$= \frac{3(256-1)}{1}$

$= 3(255)$

$= 765$

if $r = -2$

$S_8 = \frac{(3)[(-2)^8 - 1]}{(-2-1)}$

$= \frac{3(256-1)}{-3}$

$= \frac{\cancel{3}(255)}{-\cancel{3}}$

$= -255$

Review:

① a) $2, -4, 8, -16, \ldots, 512$

$a = 2$

$r = \frac{-4}{2} = -2$

$t_n = 512$

$t_n = ar^{n-1}$

$\frac{512}{2} = \frac{(2)(-2)^{n-1}}{2}$

$256 = (-2)^{n-1}$

$(\cancel{-2})^8 = (\cancel{-2})^{n-1}$

$8 = n-1$

$\boxed{9 = n}$

b) $\frac{2}{5}, \frac{9}{10}, \frac{7}{5}, \ldots, \frac{22}{5}$

$a = \frac{2}{5}$

$d = \frac{9}{10} - \frac{2}{5}$

$d = \frac{9}{10} - \frac{4}{10}$

$d = \frac{5}{10} = \frac{1}{2}$

$t_n = \frac{22}{5}$

$t_n = a + (n-1)d$

$\frac{22}{5} = \left(\frac{2}{5}\right) + (n-1)\left(\frac{1}{2}\right)$

$\frac{20}{5} = \frac{n}{2} - \frac{1}{2}$

$4 + \frac{1}{2} = \frac{n}{2}$

$\frac{8}{2} + \frac{1}{2} = \frac{n}{2}$

$\cancel{2} \cdot \frac{9}{\cancel{2}} = \frac{n}{\cancel{2}} \cdot \cancel{2}$

$\boxed{9 = n}$

Review:

② a) $-5 - 1 + 3 + 7 + \ldots + 51$

$a = -5$

$d = 4$

$t_n = 51$

(i) Find n:

$t_n = a + (n-1)d$

$51 = -5 + (n-1)4$

$\frac{56}{4} = \frac{(n-1)4}{4}$

$14 = n - 1$

$15 = n$

(ii) Find S_n or S_{15}

$S_n = \frac{n}{2}[a + t_n]$

$S_{15} = \frac{15}{2}[-5 + 51]$

$S_{15} = \frac{15}{\cancel{2}_1}(\cancel{46}^{23}) = \boxed{345}$

Review

② b) $16+8+4+2+\dots+\frac{1}{32}$

$\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$

$a=16$

$r=\frac{1}{2}$

$t_n=\frac{1}{32}$

(i) Find n:

$t_n=ar^{n-1}$

$\frac{1}{32}\div 16=(16)\left(\frac{1}{2}\right)^{n-1}\div 16$

$\frac{1}{512}=\left(\frac{1}{2}\right)^{n-1}$

$\left(\cancel{\frac{1}{2}}\right)^9=\left(\cancel{\frac{1}{2}}\right)^{n-1}$ $\frac{\log\frac{1}{512}}{\log\frac{1}{2}}=9$

$9=n-1$

$10=n$

(ii) Find S_n or S_{10}

$S_n=\frac{a[r^n-1]}{r-1}$

$S_{10}=\frac{16\left[\left(\frac{1}{2}\right)^{10}-1\right]}{\frac{1}{2}-1}$

$S_{10}=\frac{16\left[\frac{1}{1024}-\frac{1024}{1024}\right]}{\frac{1}{2}-\frac{2}{2}}$

$S_{10}=\overset{1}{\cancel{16}}\left[\frac{-1023}{\cancel{1024}_{\cancel{512}_{32}}}\right]\left(-\frac{\overset{1}{\cancel{2}}}{1}\right)$

$S_{10}=\frac{1023}{32}$

Review:

Elimination by Subtracting

③ $t_3 = 7$ | $t_9 = 55$

$t_n = a + (n-1)d$ | $t_n = a + (n-1)d$

$t_3 = a + (3-1)d$ | $t_9 = a + (9-1)d$

$t_3 = a + 2d$ | $t_9 = a + 8d$

$7 = a + 2d$ | $55 = a + 8d$

$a + 2d = 7$ | $a + 8d = 55$

$$\begin{array}{rl} & a + 8d = 55 \\ (-) & a + 2d = 7 \\ \hline & \frac{6d}{6} = \frac{48}{6} \end{array}$$

$$\boxed{d = 8}$$

$a + 2(8) = 7$

$a + 16 = 7$

$$\boxed{a = -9}$$

$t_n = a + (n-1)d$

$t_n = -9 + (n-1)(8)$

$t_n = -9 + 8n - 8$

$$\boxed{t_n = 8n - 17}$$

Review:

Elimination by Division

4

$t_4 = -36$

$t_n = ar^{n-1}$

$t_4 = ar^{4-1}$

$t_4 = ar^3$

$-36 = ar^3$

$t_7 = 972$

$t_n = ar^{n-1}$

$t_7 = ar^{7-1}$

$t_7 = ar^6$

$972 = ar^6$

$$\frac{972 = ar^6}{-36 = ar^3}$$

$-27 = r^3$

$\boxed{-3 = r}$

$-36 = ar^3$

$-36 = a(-3)^3$

$\frac{-36}{-27} = \frac{-27a}{-27}$

$\boxed{\frac{4}{3} = a}$

$t_n = ar^{n-1}$

$t_n = \left(\frac{4}{3}\right)(-3)^{n-1}$

Review

5 a) $S_6 = 1365$

$r = \frac{1}{4}$

$n = 6$

$a = ?$

$$S_n = \frac{a(r^n - 1)}{r - 1}$$

$$1365 = \frac{a\left(\frac{1}{4}^6 - 1\right)}{\frac{1}{4} - 1}$$

$$1365 = \frac{a\left(\frac{1}{4096} - \frac{4096}{4096}\right)}{\frac{1}{4} - \frac{4}{4}}$$

$$1365 = a\left(-\frac{\cancel{4095}^{1365}}{\cancel{4096}_{1024}}\right)\left(-\frac{\cancel{4}^{1}}{\cancel{3}_{1}}\right)$$

$$1365 = \frac{1365a}{1024}$$

$$\boxed{1024 = a}$$

b) $t_n = ar^{n-1}$

$$t_8 = (1024)\left(\frac{1}{4}\right)^{8-1}$$

$$t_8 = 1024\left(\frac{1}{16384}\right)$$

$$t_8 = \frac{1024}{16384} = \boxed{\frac{1}{16}}$$

4. If the sum of the first five terms of a geometric series is 1089 and the common ratio is $\frac{1}{3}$, find:

a) The first term

b) The 9th term

Given:

$S_5 = 1089$

$r = \frac{1}{3}$

a) $S_n = \dfrac{a[r^n - 1]}{r-1}$

$1089 = \dfrac{a\left[\left(\frac{1}{3}\right)^5 - 1\right]}{\frac{1}{3} - 1}$

$1089 = \dfrac{a\left[\frac{1}{243} - \frac{243}{243}\right]}{\frac{1}{3} - \frac{3}{3}}$

$1089 = a\left(-\dfrac{\cancel{242}^{121}}{\cancel{243}_{81}}\right)\left(-\dfrac{\cancel{3}^{1}}{\cancel{2}_{1}}\right)$

$81 \cdot 1089 = \dfrac{121a}{\cancel{81}} \cdot \cancel{81}$

$\dfrac{88209}{121} = \dfrac{121a}{121}$

$729 = a$

b) $t_n = ar^{n-1}$

$t_9 = (729)\left(\frac{1}{3}\right)^{9-1}$

$t_9 = 729\left(\frac{1}{3}\right)^8$

$t_9 = 729\left(\frac{1}{6561}\right)$

$t_9 = \dfrac{729}{6561} = \dfrac{1}{9}$

10.9

⑤ b) $1+\frac{5}{2}+\frac{25}{4}+\ldots+\frac{15625}{64}$ ← last term

$a=1$

$r=\frac{5}{2}$

$t_n=\frac{15625}{64}$

① Find n:

$t_n=ar^{n-1}$

$\frac{15625}{64}=(1)\left(\frac{5}{2}\right)^{n-1}$

$\frac{15625}{64}=\left(\frac{5}{2}\right)^{n-1}$

$\left(\frac{5}{2}\right)^{6}=\left(\frac{5}{2}\right)^{n-1}$

$6=n-1$

$7=n$

② Find S_7

$S_n=\frac{a(r^n-1)}{r-1}$

$S_7=\frac{(1)\left(\left(\frac{5}{2}\right)^7-1\right)}{\left(\frac{5}{2}\right)-1}$

$S_7=\frac{1\left(\frac{78125}{128}-\frac{128}{128}\right)}{\frac{5}{2}-\frac{2}{2}}$

$S_7=\left(\frac{77997}{128}\right)\div\left(\frac{3}{2}\right)$

$S_7=\left(\frac{77997}{128}\right)\left(\frac{2}{3}\right)$

$S_7=\frac{155994}{384}=\boxed{\frac{25999}{64}}$

10.9

6) $S_7 = 1093$

$r = \frac{1}{3}$

$n = 7$

$a = ?$

$$S_n = \frac{a(r^n - 1)}{r - 1}$$

$$1093 = \frac{a\left(\left(\frac{1}{3}\right)^7 - 1\right)}{\frac{1}{3} - 1}$$

$$1093 = \frac{a\left(\frac{1}{2187} - \frac{2187}{2187}\right)}{\frac{1}{3} - \frac{3}{3}}$$

$$1093 = a\left(\frac{-2186}{2187}\right) \times -\frac{3}{2}$$

$$1093 = \frac{6558a}{4374}$$

$$6558a = 4780782$$

$$\boxed{a = 729}$$

b) $t_4 = ?$

$a = 729$

$r = \frac{1}{3}$

$n = 4$

$$t_4 = (729)\left(\frac{1}{3}\right)^{4-1}$$

$$t_4 = 729\left(\frac{1}{27}\right)$$

$$t_4 = \frac{729}{27}$$

$$\boxed{t_4 = 27}$$

10.9

1 b) $2 + \frac{2}{3} + \frac{2}{9} + \frac{2}{27} + \dots$

$a = 2$

$r = \frac{1}{3}$

$$S_n = \frac{2\left(\frac{1}{3}^n - 1\right)}{\frac{1}{3} - 1}$$

$$= \frac{2\left(\frac{1}{3}^n - 1\right)}{-\frac{2}{3}}$$

$$= \cancel{2}\left(\frac{1}{3}^n - 1\right) \times \frac{3}{-\cancel{2}}$$

$$= -3\left(\frac{1}{3}^n - 1\right)$$

10.9

3 c) $81+27+9\ldots$

$a=81$

$r=\frac{1}{3}$

$n=6$

$$S_6 = \frac{81\left(\left(\frac{1}{3}\right)^6-1\right)}{\frac{1}{3}-1}$$

$$= \frac{81\left(\frac{1}{729}-\frac{729}{729}\right)}{\frac{1}{3}-\frac{3}{3}}$$

$$= \frac{\cancel{81}^{1}\left(\frac{-728}{\cancel{729}_{9}}\right)}{-\frac{2}{3}}$$

$$= \frac{-\cancel{728}^{364}}{\cancel{9}_{3}} \times \frac{\cancel{3}^{1}}{-\cancel{2}_{1}}$$

$\frac{-2184}{-18}$

$$\boxed{=\frac{364}{3}} = \boxed{121\tfrac{1}{3}}$$

Ex 10.9

⑤ b) $1 + \frac{5}{2} + \frac{25}{4} + \ldots + \frac{15625}{64}$

$S_n = ?$

$a = 1$

$r = \frac{5}{2}$

$t_n = \frac{15625}{64}$

Solve for n:

$t_n = ar^{n-1}$

$\frac{15625}{64} = (\cancel{1})\left(\frac{5}{2}\right)^{n-1}$

$\frac{15625}{64} = \left(\frac{5}{2}\right)^{n-1}$

$\left(\frac{5}{2}\right)^6 = \left(\frac{5}{2}\right)^{n-1}$

$6 = n-1$

$\boxed{7 = n}$

Find S_7:

$$S_7 = \frac{1\left(\left(\frac{5}{2}\right)^7 - 1\right)}{\frac{5}{2} - 1}$$

$$= \frac{1\left(\frac{78125}{128} - \frac{128}{128}\right)}{\frac{5}{2} - \frac{2}{2}}$$

$$= \left(\frac{77997}{128}\right) \div \left(\frac{3}{2}\right)$$

$$= \frac{77997}{128} \times \frac{2}{3}$$

$$= \frac{155994}{384} = \boxed{\frac{25999}{64}}$$

Review

② a) $n=?$ $\quad t_n = a+(n-1)d$

$a=3 \quad 39 = 3+(n-1)4$

$d=4 \quad 36 = 4n-4$

$t_n=39 \quad 40 = 4n$

$10 = n$

3, 7, 11, 15, 19, 23, 27, 31, 35, 39

① b) $t_9=-6 \quad t_{12}=-12$

$t_9 = a+8d \quad t_{12} = a+11d$

$$\boxed{a+8d=-6 \qquad a+11d=-12}$$

$$\begin{array}{r} a+11d=-12 \\ (-)\ a+8d=-6 \\ \hline 3d=-6 \\ \boxed{d=-2} \end{array}$$

$a+8(-2)=-6$

$a-16=-6$

$\boxed{a=10}$

$t_n = a+(n-1)d$

$t_n = 10+(n-1)(-2)$

$t_n = 10-2n+2$

$t_n = 12-2n$

⑥ b) $t_5 = 8 \quad t_{10} = \frac{1}{4} \quad t_3 = ?$

$t_5 = ar^4 \quad t_{10} = ar^9$

$$\boxed{ar^4 = 8 \qquad ar^9 = \frac{1}{4}}$$

$$\frac{ar^9 = \frac{1}{4}}{ar^4 = 8}$$

$r^5 = \frac{1}{32}$

$\boxed{r = \frac{1}{2}}$

$a\left(\frac{1}{2}\right)^4 = 8$

$a\left(\frac{1}{16}\right) = 8$

$\frac{a}{16} = 8$

$\boxed{a = 128}$

$t_3 = (128)\left(\frac{1}{2}\right)^2$

$t_3 = 128\left(\frac{1}{4}\right)$

$t_3 = 32$

$$512 = 2(-2)^{n-1}$$

$$256 = (-2)^{n-1}$$

$$\cancel{(-2)}^{8} = \cancel{(-2)}^{n-1}$$

$$8 = n-1$$

$$9 = n$$

Review

⑩ $t_7 = 192$ $\boxed{a = t_1 = 3}$ $S_8 = ?$

$t_7 = ar^{7-1}$

$t_7 = ar^6$

$ar^6 = 192$

$3r^6 = 192$

$r^6 = 64$

$\boxed{r = \pm 2}$

$$S_8 = \frac{3(2^8 - 1)}{2 - 1} = \frac{3(256-1)}{1} = 3(255) \quad \boxed{= 765}$$

$$S_8 = \frac{3((-2)^8 - 1)}{(-2) - 1} = \frac{3(256-1)}{-3} = \frac{3(255)}{-3} \quad \boxed{= -255}$$