

1.6

Reasoning to Solve Problems

GOAL

Solve problems using inductive or deductive reasoning.

EXPLORE...

- Suppose that you are lost in the woods for hours and come upon a cabin. In the cabin, you find a lantern, a candle, a wood stove with wood in it, and a match. What do you light first?

**SAMPLE ANSWER**

I would light the match first. If I didn't, I couldn't light any of the other items. I would light the candle next, since it would stay lit for longer than the match and would allow me to light the other two items. Also, it's less likely that I would make an error or fail when lighting the candle. The lantern and the stove would be more difficult to light.

INVESTIGATE the Math

Emma was given this math trick:

- Choose a number.
- Multiply by 6.
- Add 4.
- Divide by 2.
- Subtract 2.

$$\begin{array}{l} 6n \\ 6n+4 \\ 3n+2 \\ 3n \end{array}$$

Emma was asked to use inductive reasoning to make a conjecture about the relationship between the starting and ending numbers, and then use deductive reasoning to prove that her conjecture is always true. Here is her response to the problem:

Inductive reasoning:

#	×6	+4	÷2	-2
<u>5</u>	30	34	17	<u>15</u>
<u>-3</u>	-18	-14	-7	<u>-9</u>
<u>0</u>	0	4	2	<u>0</u>
<u>24</u>	144	148	74	<u>72</u>

I followed the steps to work through four examples.

Conjecture: It is 3 times. *the original #*

$$\begin{array}{l} 3(5) = 15 \\ 3(-3) = -9 \\ 3(0) = 0 \\ 3(24) = 72 \end{array}$$

Deductive reasoning: (Algebra)

I chose d .

Then I multiplied, added, divided, and subtracted to get an expression.

$$\left(\frac{6d + 4}{2}\right) - 2$$

It worked.

$$\begin{array}{l} \left(\frac{6d+4}{2}\right) - 2 \\ 3d + 2 - 2 \\ 3d \end{array}$$

It simplified to $3d$.

? How can Emma's communication about her reasoning be improved?

- With a partner, explain why Emma might have chosen the values she did.
- What details are missing from the deductive reasoning Emma used to arrive at the expression $3d$?
- Improve Emma's conjecture, justifications, and explanations.

Answers

- Emma might have chosen the four values because each value represents a different attribute. One value is positive, another is negative, another is zero, and the last is a larger number. With this variety, Emma might have thought that she had sampled sufficiently from the range of possible values.
- The explanation does not include reasons for each step, nor does it show what each step looks like. It provides only a summary.
- Conjecture: The resulting value will always be three times the starting value. Justification and explanation:

Let d represent any number.	d
Multiply by 6.	$6d$
Add 4.	$6d + 4$
Divide by 2.	$\frac{6d + 4}{2} = 3d + 2$
Subtract 2.	$3d + 2 - 2 = 3d$
The resulting value is three times the starting value.	$3d$

Reflecting

- D. How does it help to understand the mathematics when both symbols and words are used in an explanation?
- E. Why is it important to explain your reasoning clearly?

Answers

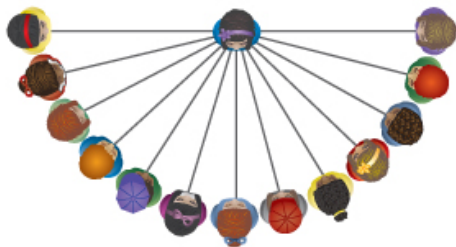
- D. Understanding the mathematics represented by both symbols and words makes it easy to explain. For example, because I know that doubling a number means multiplying by 2, I can represent the words as $2x$.
- E. A clear explanation ensures that the person who is reading it will follow your reasoning all the way through. If you miss steps, then the reader won't understand or may reject your argument as invalid. If you don't use precise language, diagrams, or algebra, then the reader may not understand.

APPLY the Math

EXAMPLE 1 Using reasoning to solve a problem

The members of a recently selected varsity basketball team met each other at their first team meeting. Each person shook the hand of every other person. The team had 12 players and 2 coaches. How many handshakes were exchanged?

Kim's Solution



I decided to think about how many times each person shook hands. There were 14 people in total, so person 1 shook hands with each of the other 13 people.

13 handshakes



Person 2 had already shaken hands with person 1. Person 2 shook hands with each of the remaining 12 people.

13 + 12 handshakes

$$\begin{aligned}
 &13 + 12 + 11 + 10 + 9 + 8 + 7 \\
 &+ 6 + 5 + 4 + 3 + 2 + 1 \\
 &= 91 \text{ handshakes}
 \end{aligned}$$

This pattern of handshakes continued until there were two people left when the last handshake happened.

Inductive Reasoning

EXAMPLE 1 Using reasoning to solve a problem

The members of a recently selected varsity basketball team met each other at their first team meeting. Each person shook the hand of every other person. The team had 12 players and 2 coaches. How many handshakes were exchanged?

Your Turn

Discuss, with a partner, whether Kim used inductive or deductive thinking in her solution. How do you know?

**Answer**

Kim used inductive reasoning. To solve the problem, Kim determined the new number of handshakes based on the pattern identified in the first two cases. I know that Kim used inductive reasoning because the result was specific to this number of people, not a generalization that would be true for any number of people.

$$14 \text{ people} = 91 \text{ shaks}$$

EXAMPLE 2 Using reasoning to solve a problem

Sue signed up for games at her school's fun night. Seven other people were assigned to her group, making up four pairs of partners. The other members of her group were Dave, Angie, Josh, Tanya, Joy, Stu, and Linus. When the games started, Dave and his partner were to the left of Stu. Across from Dave was Sue, who was to the right of Josh. Dave's brother's partner, Tanya, was across from Stu. Joy was not on Stu's right.

Name the four pairs of partners.

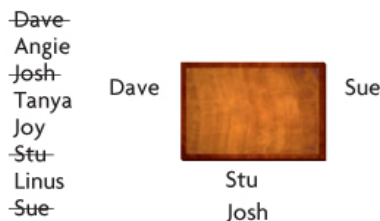
Vicky's Solution



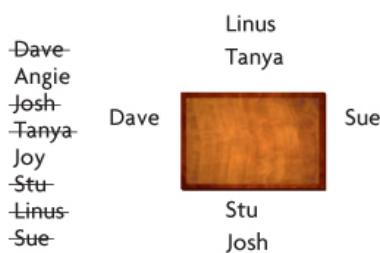
I drew a rectangle to represent a table. I made a list of the students' names so I could cross them off as I put them in place.



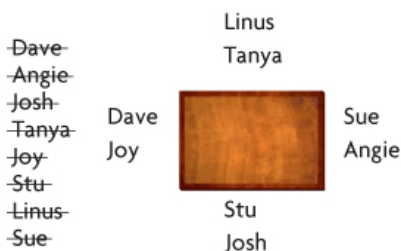
The first names I wrote in were Dave and Stu, since they were the first two mentioned. It didn't matter where I started, as long as I kept the relationships of left, right, and across the table. I crossed Dave and Stu off my list.



I knew that Sue was across from Dave and to the right of Josh. I crossed Sue and Josh off my list.



The next clue mentioned that Dave's brother and his partner Tanya were across from Stu. The only male name left was Linus, so Linus and Tanya were partners. I crossed their names off my list.



If Joy was not on Stu's right, then she must have been on his left. Therefore, she must have been Dave's partner. So, the last person to match was Angie with Sue.

The four pairs of partners were Linus and Tanya, Dave and Joy, Sue and Angie, and Stu and Josh.

The partners sat together, on the same side of the table.

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Name the four pairs of partners.

Your Turn

Discuss with a partner whether inductive or deductive reasoning was used for this solution. How do you know?

**Answer**

Vicky used deductive reasoning. She used the given information to deduce the seating arrangements. The language in her explanation followed the pattern of if . . . then statements, which may be present in deductive reasoning.

In Summary**Key Idea**

- Inductive and deductive reasoning are useful in problem solving.

Need to Know

- Inductive reasoning involves solving a simpler problem, observing patterns, and drawing a logical conclusion from your observations to solve the original problem.
- Deductive reasoning involves using known facts or assumptions to develop an argument, which is then used to draw a logical conclusion and solve the problem.

Assignment: pages 48-51

Questions: 1, 2, 3, 5a,7, 8, 9, 15

SOLUTIONS ⇒ 1.6 Reasoning to Solve Problems

1. Explain which type of reasoning is demonstrated by each statement.

a) Over the past 12 years, a tree has produced plums every other year. Last year, the tree did not produce plums. Therefore, the tree will produce plums this year.

↳ Inductive Reasoning

b) Mammals have hair. Dogs are mammals. Therefore, dogs have hair.

↳ Deductive Reasoning

c) Every Thursday, a train arrives at 2:30 pm. Today is Thursday, so the train will arrive at 2:30 pm.

↳ Inductive Reasoning

d) Every even number has a factor of 2. 24 is an even number. Therefore, 24 has a factor of 2.

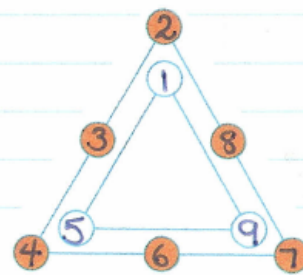
↳ Deductive Reasoning

e) For the pattern 3, 12, 21, 30, 39, the next term is 48.

↳ Inductive Reasoning

2. Place the digits 1 through 9 in the circles so that the sum of the numbers on the outside triangle is double the sum of the numbers on the inside triangle. Explain whether more than one solution is possible.

For Example:



Several Solutions are possible.
Since $1+2+3+4+5+6+7+8+9=45$, any combination of numbers in the middle that add up to 15 and any combination of numbers on the outside that add up to 30 will work.

3. Draw the next figure in this sequence.

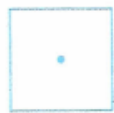


Figure 1

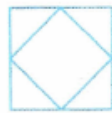


Figure 2

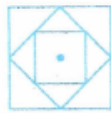
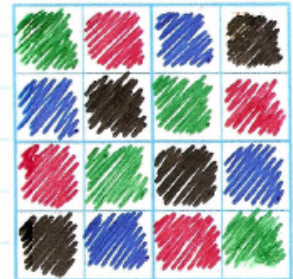


Figure 3

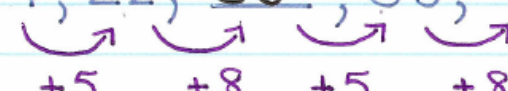


Figure 4

5a) Choose four different colours. Fill in the cells in a copy of this chart, so that each row and column has four different colours and each quadrant also has four different colours.

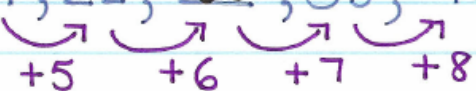


7. Determine the unknown term in this pattern: 17, 22, —, 35, 43.
Explain your reasoning.

$$\Rightarrow 17, 22, \underline{30}, 35, 43$$


+5 +8 +5 +8

* More than 1 possibility

$$\Rightarrow 17, 22, \underline{28}, 35, 43$$


+5 +6 +7 +8

8. Suppose that you are marooned on an island where there are only liars and truth-tellers. Liars always tell lies, and truth-tellers always tell the truth. You meet two siblings. The brother says, "My sister told me that she is a liar."
Is he a liar or a truth-teller?
Explain how you know.

The brother is the liar. WHY?

9. Bob, Kurt, and Morty are football players. One is quarterback, one is a receiver, and one is a kicker. The kicker, who is the shortest of the three is not married. Bob who is Kurt's father-in-law, is taller than the receiver. Who plays which position?

The kicker, who is the shortest of the three is not married.

↳ Kicker is either Bob or Morty.

Bob who is Kurt's father-in-law, is taller than the receiver.

↳ Receiver is either Kurt or Morty.

↳ Kicker is not Bob. (not the shortest)

Therefore, the kicker is Morty, the receiver is Kurt and the quarterback is Bob.

15. Max, Karl, Terri, and Suganthy live on the first floor of an apartment building. One is a manager, one is a computer programmer, one is a singer, and one is a teacher.

a) Use the statements below to determine which person is the manager.

- Suganthy and Terri each lunch with the singer
 - ↳ Singer is either Max or Karl
- Karl and Max carpool with the manager.
 - ↳ Manager is either Suganthy or Terri.
- Terri watches football on television with the manager and the singer.
 - ↳ The manager must be Suganthy.

b) Describe the reasoning you used to solve this problem.

I used deductive reasoning to solve this problem.

Attachments

1s6e1 finalt.mp4

1s6e2 final.mp4