Laws of Logarithms

$$\log_b M + \log_b N = \log_b(MN)$$

$$\log_b M - \log_b N = \log_b\left(\frac{M}{N}\right)$$

$$\log_b(N^p) = p\log_b(N)$$

Warm Up

Review of laws of logarithms...

Given that $\log_x M = -3$, $\log_x N = 5$ and $\log_x P = 4$, evaluate the following logarithmic expression:

$$\log_x \left[\frac{(M^3 N)^2 \sqrt{P}}{MP} \right]$$

$$\log_x \left[\frac{M^6 N^2 P^{1/2}}{MP} \right]$$

$$\log_x \left[M^5 N^2 P^{-1/2} \right]$$

$$\log_x \left[\frac{M^5 N^2}{P^{1/2}} \right]$$

$$\log_x M^5 + \log_x N^2 - \log_x P^{1/2}$$

$$5\log_x M + 2\log_x N - \frac{1}{2}\log_x P$$

$$5(-3) + 2(5) - \frac{1}{2}(4)$$

$$-15 + 10 - 2$$

$$\boxed{-7}$$

Solve the following equation: $\dfrac{3^{x-1}}{5 * 2^{3x}} = 6^{1-2x}$

take log of both sides

$$\log\left(\frac{3^{x-1}}{5 \cdot 2^{3x}}\right) = \log 6^{1-2x}$$

$$\log 3^{x-1} - \log 5 - \log 2^{3x} = \log 6^{1-2x}$$

$$(x-1)\log 3 - \log 5 - 3x\log 2 = (1-2x)\log 6$$

$$x\log 3 - \log 3 - \log 5 - 3x\log 2 = \log 6 - 2x\log 6$$

$$x\log 3 - 3x\log 2 + 2x\log 6 = \log 6 + \log 3 + \log 5$$

$$x\left(\log 3 - 3\log 2 + 2\log 6\right) = \log 6 + \log 3 + \log 5$$

$$x = \frac{\log 6 + \log 3 + \log 5}{\log 3 - 3\log 2 + 2\log 6}$$

$$x = \frac{\log 6 + \log 3 + \log 5}{\log 3 - \log 2^3 + \log 6^2}$$

$$x = \frac{\log(6 \cdot 3 \cdot 5)}{\log\left(\frac{3 \cdot 36}{8}\right)}$$

$$x = \frac{\log 90}{\log 13.5} \approx \boxed{1.73}$$

Questions from Homework

Rule: $d(\ln u) = \dfrac{1}{u} du = \dfrac{du}{u}$

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$u = \left(\dfrac{x}{2x+3}\right)^{1/2} = \dfrac{(x)^{1/2}}{(2x+3)^{1/2}}$

① k) $\quad y = \ln\sqrt{\dfrac{x}{2x+3}} = \ln\left(\dfrac{x}{2x+3}\right)^{1/2}$

$y' = \dfrac{(2x+3)^{1/2}}{x^{1/2}}\left[\dfrac{1}{2}\left(\dfrac{x}{2x+3}\right)^{-1/2} \cdot \dfrac{1(2x+3) - 2x}{(2x+3)^2}\right]$

$y' = \dfrac{(2x+3)^{1/2}}{x^{1/2}}\left[\dfrac{1}{2} \cdot \dfrac{(2x+3)^{1/2}}{x^{1/2}} \cdot \dfrac{3}{(2x+3)^2}\right]$

$y' = \dfrac{3(2x+3)}{2x(2x+3)^2} = \dfrac{3}{2x(2x+3)} = \dfrac{3}{4x^2+6x}$

m) $\quad y = \ln(\sec x + \tan x)$

$u = \sec x + \tan x$
$du = \sec x \tan x \cdot 1 + \sec^2 x \cdot$
$ = \sec x \tan x + \sec^2 x$

$y' = \dfrac{\sec x \tan x + \sec^2 x}{\sec x + \tan x}$

$y' = \dfrac{\sec x(\tan x + \sec x)}{(\sec x + \tan x)} = \sec x$

<p style="text-align:center;color:blue;">Questions from Homework</p>

① b) $f(x) = \sqrt{\ln x} = (\ln x)^{1/2}$

$$f'(x) = \frac{1}{2}(\ln x)^{-1/2}\left(\frac{1}{x}\right)$$

$$f'(x) = \frac{1}{2x\sqrt{\ln x}}$$

e) $y = \sin(\ln x)$

$$y' = \cos(\ln x)\left(\frac{1}{x}\right)$$

$$y' = \frac{\cos(\ln x)}{x}$$

g) $y = \frac{\ln x}{x^3}$

$$\frac{dy}{dx} = \frac{\left(\frac{1}{x}\right)(x^3) - 3x^2(\ln x)}{(x^3)^2}$$

$$\frac{dy}{dx} = \frac{x^2 - 3x^2 \ln x}{x^6}$$

$$\frac{dy}{dx} = \frac{x^2(1 - 3\ln x)}{x^{6\,4}} = \frac{1 - 3\ln x}{x^4}$$

Questions from Homework

① b) $y = \ln\left(\dfrac{x}{\sqrt{x^2+1}}\right)$

$u = \dfrac{x}{\sqrt{x^2+1}}$

$\dfrac{dy}{dx} = \dfrac{1}{\frac{x}{\sqrt{x^2+1}}} \cdot \dfrac{1}{(x^2+1)^{3/2}}$

$du = \dfrac{1(\sqrt{x^2+1}) - x\left(\frac{1}{2}\right)(x^2+1)^{-\frac{1}{2}}(2x)}{\left(\sqrt{x^2+1}\right)^2}$

$\dfrac{dy}{dx} = \dfrac{\sqrt{x^2+1}}{x} \cdot \dfrac{1}{(x^2+1)^{3/2}}$

$du = \dfrac{(x^2+1)^{1/2} - x^2(x^2+1)^{-1/2}}{(x^2+1)}$

$\boxed{\dfrac{dy}{dx} = \dfrac{1}{x(x^2+1)}}$

$du = \dfrac{(x^2+1)^{-1/2}\left[(x^2+1) - x^2\right]}{(x^2+1)}$

$\overset{x^2+1-x^2}{}$

$du = \dfrac{1}{(x^2+1)^{3/2}}$

We have now covered base "e"...both as an
exponential and logarithmic function...

What about other bases??

Will need to know the change of base formula for logarithms:

$$\log_N M = \frac{\log_b M}{\log_b N}$$

Whatever new
base you choose

$$\log_3 x = \frac{\ln x}{\ln 3}$$

$$= \frac{\log M}{\log N} = \frac{\log_e M}{\log_e N} = \boxed{\frac{\ln M}{\ln N}}$$

Rule: $d(\log_b u) = \dfrac{1}{u \ln b} du = \dfrac{du}{u \ln b}$

Differentiate:

$$y = \log_6 x^3 \qquad \begin{array}{l} b=6 \\ u=x^3 \\ du = 3x^2 \end{array}$$

$$y' = \frac{1}{x^3 \ln 6} \cdot 3x^2$$

$$y' = \frac{3x^2}{x^3 \ln 6} = \frac{3}{x \ln 6}$$

$$y = \log(5x^4) \qquad \begin{array}{l} b=10 \\ u=5x^4 \\ du = 20x^3 \end{array}$$

$$y' = \frac{20x^3}{5x^4 \ln 10} = \frac{4}{x \ln 10}$$

This leaves one form of exponential function remaining...

• What about a function such as $y = 3^{9x}$

$b = 3$
$u = 9x$
$du = 9$

Rule:

$$d(b^u) = b^u (\ln b) du \, , where \, b \in R$$

$y = 3^{9x}$

$y' = 3^{9x} (\ln 3) 9$

Try this one... $y = \pi^{x^5}$

$b = \pi$
$u = x^5$
$du = 5x^4$

$y' = \pi^{x^5} (\ln \pi) 5x^4$

Practice Problems:

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#1   #2 a   #3   #4

#5   #6   #7   #8

③ d) $g(x) = \dfrac{1 + \log_3 x}{x}$

$g'(x) = \dfrac{\left(\dfrac{1}{x\ln 3} \cdot 1\right) x - 1(1 + \log_3 x)}{x^2}$

$g'(x) = \dfrac{\dfrac{1}{\ln 3} - 1 - \log_3 x}{x^2 \ln 3}$ CD: $\ln 3$

$g'(x) = \dfrac{1 - \ln 3 - \ln 3(\log_3 x)}{x^2 \ln 3}$ $\dfrac{\ln x}{\ln 3}$

$g'(x) = \dfrac{1 - \ln 3 - \ln 3\left(\dfrac{\ln x}{\ln 3}\right)}{x^2 \ln 3} = \boxed{\dfrac{1 - \ln 3 - \ln x}{x^2 \ln 3}}$

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④ a) $\quad y = x^3 + 3^x \qquad$ $b = 3$
$u = x$
$du = 1$

$y' = 3x^2 + 3^x (\ln 3)(1)$

$y' = 3x^2 + 3^x \ln 3$

$\dfrac{x'}{x^{1/2}} = x^{1/2}$

④ c) $\quad y = (x)(5^{\sqrt{x}})$ $\qquad$ $b = 5$
$u = \sqrt{x}$
$du = \dfrac{1}{2\sqrt{x}}$

$y' = 1(5^{\sqrt{x}}) + x(5^{\sqrt{x}}) \ln 5 \left(\dfrac{1}{2} x^{-1/2}\right)$

$y' = \dfrac{5^{\sqrt{x}}}{1} + \dfrac{\sqrt{x}\, 5^{\sqrt{x}} \ln 5}{2}$

$y' = \dfrac{2(5^{\sqrt{x}})}{2} + \dfrac{\sqrt{x}\, 5^{\sqrt{x}} \ln 5}{2}$

$y' = \dfrac{2(5^{\sqrt{x}}) + \sqrt{x}\, 5^{\sqrt{x}} \ln 5}{2}$

$y' = \dfrac{5^{\sqrt{x}}\left(2 + \sqrt{x} \ln 5\right)}{2}$

⑤ a) find $f'(x)$

 ② Sub in x-value to find m

 ③ $y - y_1 = m(x - x_1)$

⑤ c) $y = 10^x$ @ $(1, 10)$

(i) $y = 10^x$

$y' = 10^x \ln 10 \cdot 1$

$y' = 10^x \ln 10$

(ii) $y' = 10^1 \ln 10$

$y' = 10 \ln 10$

$m = 10 \ln 10$

(iii) $y - y_1 = m(x - x_1)$

$y - 10 = 10 \ln 10 (x - 1)$

$y - 10 = 10x \ln 10 - 10 \ln 10$

$0 = 10x \ln 10 - y - 10 \ln 10 + 10$

$0 = 10x \ln 10 - y - 10(\ln 10 - 1)$

⑦ $y = x \ln x$

$y' = 1 \ln x + x \left(\frac{1}{x} \cdot 1 \right)$

$y' = \ln x + 1$

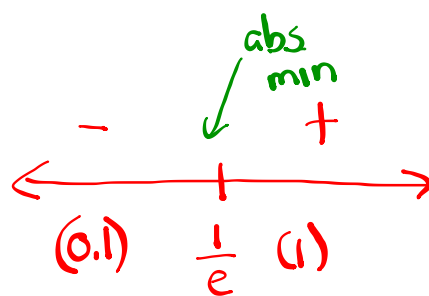CV: $0 = \ln x + 1$

$-1 = \ln x$ (log)

$e^{-1} = x$ (exp)

$\boxed{\frac{1}{e} = x}$

abs min

$(0,1) \quad \frac{1}{e} \quad (1)$

Find min

$y = x \ln x$

$y = \left(\frac{1}{e} \right) \ln \left(\frac{1}{e} \right)$

$y = \frac{1}{e} \boxed{\ln(e^{-1})}$

$y = \frac{1}{e}(-1) = \frac{-1}{e}$

$\boxed{\left(\frac{1}{e}, \frac{-1}{e} \right)}$