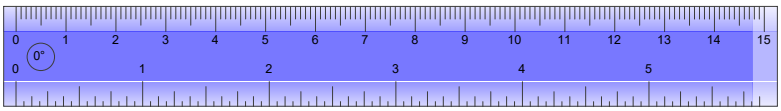
1.2 Exploring the Validity of Conjectures

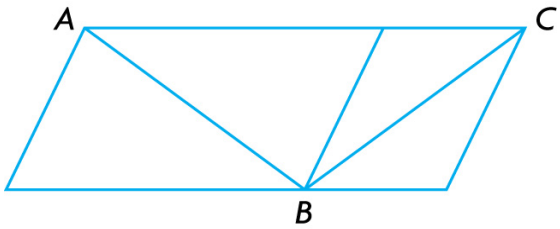
GOAL

Determine whether a conjecture is valid.

EXPLORE the Math

Your brain can be deceived.





Make a conjecture about diagonal *AB* and diagonal *BC*.

? How can you check the validity of your conjecture?

Conjecture: AB is longer than BC

AB = 3.7cm
BC = 3.7cm
} Using a ruler

Revision: AB is the same as BC

EXPLORE the Math

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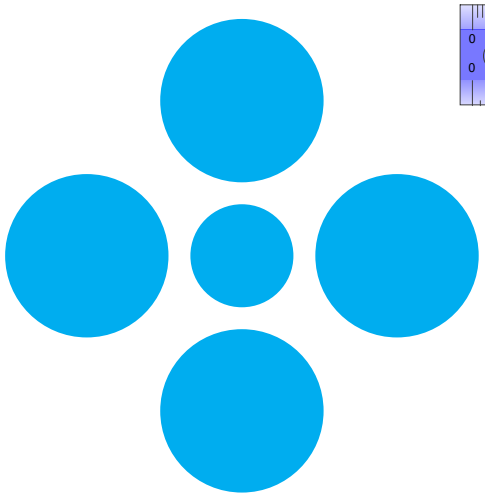

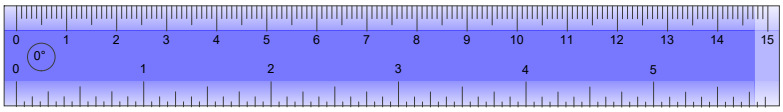

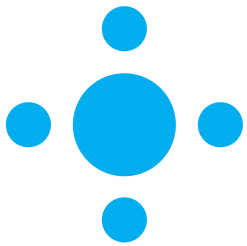

Make a conjecture about the circles in the centre.

How can you check the validity of your conjecture?

Conjecture: The center circle on the right is larger

Both circles have a diameter of 2cm (Using a ruler)

Revision: Both circles are the same size

EXPLORE the Math

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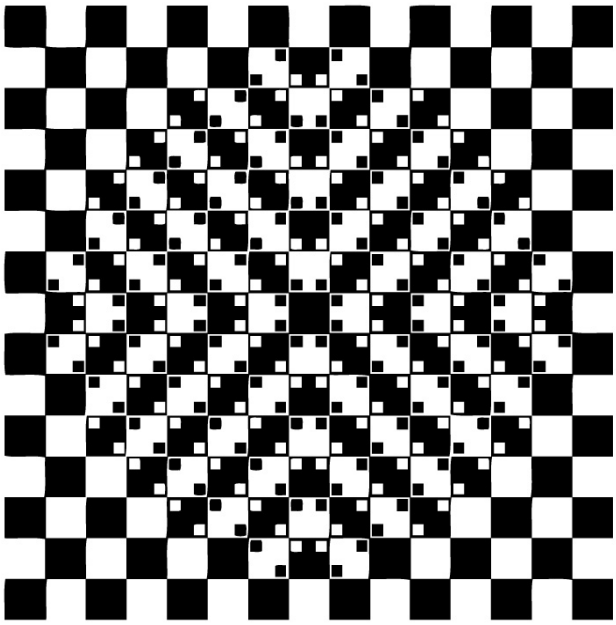
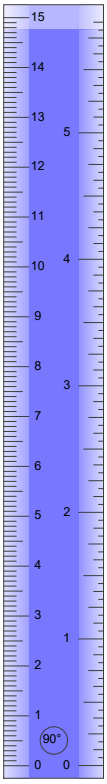




Make a conjecture about the lines.

How can you check the validity of your conjecture?

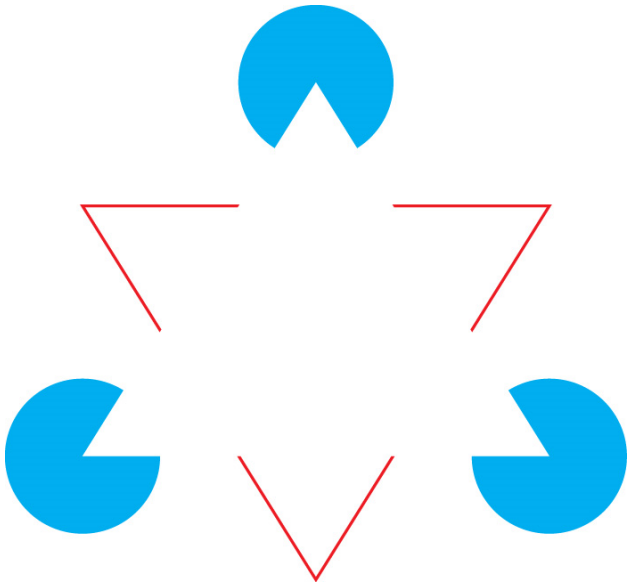
Conjecture: The lines are curved

We used the ruler again and found...

Revision: The lines are actually straight

EXPLORE the Math

Your brain can be deceived.



Make a conjecture about the number of triangles.

How can you check the validity of your conjecture?

Conjecture: There are no triangles ✓

Reflecting

A. Describe the steps you took to verify your conjectures.

B. After collecting evidence, did you decide to revise either of your conjectures? Explain.

C. Can you be certain that the evidence you collect leads to a correct conjecture? Explain.

Answers

A. Both measurement and visual inspection helped to verify or discredit the conjectures.

B. My conjectures changed as follows after collecting more evidence:

- First image: Both diagonals are the same length.
- Second image: The centre circles of the figures are the same size.
- Third image: The rows and columns of white and black shapes are placed in straight lines.
- Fourth image: There are no triangles in the figure.

C. For these images, the revised conjectures hold true for the accuracy of the tools I used. I cannot be absolutely sure that my new conjectures are valid until the precision of the tools is considered.

In Summary

Key Idea

- Some conjectures initially seem to be valid, but are shown not to be valid after more evidence is gathered.

Need to Know

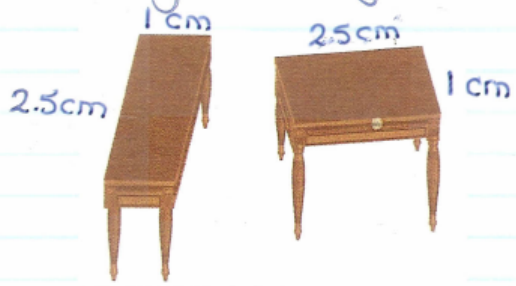
- The best we can say about a conjecture reached through inductive reasoning is that there is evidence either to support or deny it.
- A conjecture may be revised, based on new evidence.

Assignment: page 17

Questions: 1, 2, 3

SOLUTIONS => 1.2 Exploring the Validity of Conjectures.

1. Make a conjecture about the dimensions of the two tabletops. How can you determine if your conjecture is valid?



Conjecture

The dimensions of the tabletop are the same

* You can determine the validity of this conjecture by using a ruler to check the measurements.

2. Examine the number pattern. Make a conjecture about this pattern. What steps can you take to determine if your conjecture is valid?

Pattern:

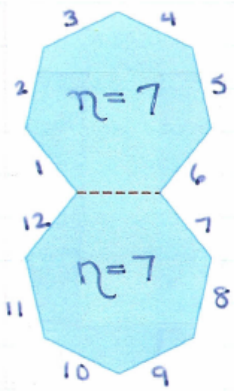
$1^2 = 1$
$11^2 = 121$
$111^2 = 12321$
$1111^2 = 1234321$

Conjecture:

This pattern will continue until 123456789876543 21 and after that it will change.

* To determine if this conjecture is valid, you would have to use a spreadsheet. (Calculator does not have enough space).

3. If two congruent regular heptagons are positioned so that they share a side, a dodecagon (12-sided polygon) is formed If two congruent regular hexagons are positioned so that they share a side, a decagon is formed. If two congruent regular pentagons are positioned so that they share a side, an octagon is formed. Make a conjecture about positioning two congruent regular quadrilaterals so that they share a side. Determine whether your conjecture is valid. Record your evidence.

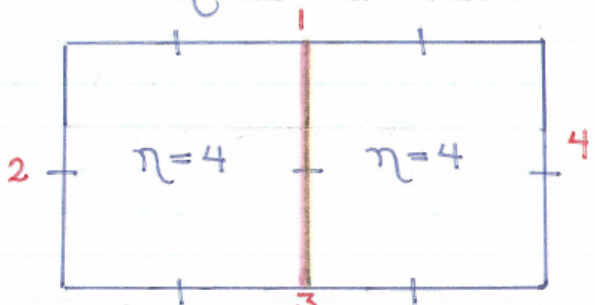


$\Rightarrow 2n-2$
$= 2(7)-2$
$= 14-2$
$= 12$ sides

Conjecture (According to the above pattern)

When two regular congruent polygons are positioned so that they share a common side, the resulting polygon will have $2n-2$ sides, where n = # of sides in one polygon.

This is invalid if you join two quadrilateral's:



$\hookrightarrow 2n-2$
$= 2(4)-2$
$= 8-2$
$= 6$

Since $6 \neq 4$, this conjecture does not hold true.