

Exercise 7.6

Differentiating Inverse Trig Functions:

$$u = \frac{\cos x}{1 + \sin x} \quad \frac{du}{dx} = \frac{-\sin x (1 + \sin x) - \cos x (\cos x)}{(1 + \sin x)^2}$$

$$\frac{du}{dx} = \frac{-\sin x - \sin^2 x - \cos^2 x}{(1 + \sin x)^2}$$

$$\textcircled{1} \text{ i) } y = \sin^{-1} \left(\frac{\cos x}{1 + \sin x} \right)$$

$$y' = \frac{1}{\sqrt{1 - \left(\frac{\cos x}{1 + \sin x} \right)^2}} \cdot \frac{-\sin x - \sin^2 x - \cos^2 x}{(1 + \sin x)^2}$$

$$y' = \frac{-\sin x - \sin^2 x - \cos^2 x}{(1 + \sin x)^2 \sqrt{1 - \frac{\cos^2 x}{(1 + \sin x)^2}}}$$

$$y' = \frac{-(\sin x + \sin^2 x + \cos^2 x)}{(1 + \sin x)^2 \sqrt{1 - \frac{\cos^2 x}{(1 + \sin x)^2}}}$$

Pythagorean identity
 $\sin^2 x + \cos^2 x = 1$

$$y' = \frac{-(\cancel{\sin x + 1})}{(1 + \sin x)^2 \sqrt{1 - \frac{\cos^2 x}{(1 + \sin x)^2}}}$$

$$y' = \frac{-1}{(1 + \sin x)^2 \left(\frac{1 - \cos^2 x}{(1 + \sin x)^2} \right)}$$

$$y' = \frac{-1}{\sqrt{(1 + \sin x)^2 - \cos^2 x}}$$

$(1 + \sin x)(1 + \sin x)$
 $1 + \sin x + \sin x + \sin^2 x$

$$y' = \frac{-1}{\sqrt{1 + 2\sin x + \sin^2 x - \cos^2 x}}$$

$$y' = \frac{-1}{\sqrt{\sin^2 x + \cos^2 x + 2\sin x + \sin^2 x - \cos^2 x}}$$

$$y' = \frac{-1}{\sqrt{2\sin^2 x + 2\sin x}}$$

$$y' = \frac{-1}{\sqrt{2\sin x(\sin x + 1)}}$$

7.6

$$\textcircled{1} \Rightarrow y = \frac{\sqrt{1-x^2}}{x} + \sin^{-1} \sqrt{x}$$

$$y = \frac{(1-x^2)^{1/2}}{x} + \sin^{-1} \sqrt{x}$$

$$\frac{dy}{dx} = \frac{\frac{1}{2}(1-x^2)^{-1/2}(-2x) - 1(1-x^2)^{1/2}}{x^2} + \frac{1}{\sqrt{1-x^2}} \cdot 1$$

$$\frac{dy}{dx} = \frac{-x^2(1-x^2)^{-1/2} - 1(1-x^2)^{1/2}}{x^2} + \frac{1}{\sqrt{1-x^2}}$$

$$\frac{dy}{dx} = \frac{(1-x^2)^{-1/2} \left[-x^2 - 1 + x^2 \right]}{x^2} + \frac{1}{\sqrt{1-x^2}}$$

$$\frac{dy}{dx} = \frac{-1}{x^2 \sqrt{1-x^2}} + \frac{1}{\sqrt{1-x^2}}$$

$$\frac{dy}{dx} = \frac{-1}{x^2 \sqrt{1-x^2}} + \frac{x^2}{x^2 \sqrt{1-x^2}}$$

$$\frac{dy}{dx} = \frac{x^2 - 1}{x^2 \sqrt{1-x^2}}$$

7.6

$$\textcircled{1} \text{ p) } y = \sin^{-1} x + \cos^{-1} \sqrt{1-x^2}$$

$$u = (1-x^2)^{\frac{1}{2}}$$

$$\frac{du}{dx} = \frac{1}{2}(1-x^2)^{-\frac{1}{2}} (-2x)$$

$$\frac{du}{dx} = \frac{-x}{\sqrt{1-x^2}}$$

$$y' = \frac{1}{\sqrt{1-x^2}} \cdot 1 + \frac{-1}{\sqrt{1-(\sqrt{1-x^2})^2}} \cdot \frac{-x}{\sqrt{1-x^2}}$$

$$y' = \frac{1}{\sqrt{1-x^2}} + \frac{x}{\sqrt{1-x^2} \sqrt{1-(1-x^2)}}$$

$$y' = \frac{1}{\sqrt{1-x^2}} + \frac{x}{\sqrt{1-x^2} \cdot \sqrt{x^2}}$$

$$y' = \frac{1}{\sqrt{1-x^2}} + \frac{\cancel{x}}{\sqrt{1-x^2} \cdot \cancel{x}}$$

$$y' = \frac{1}{\sqrt{1-x^2}} + \frac{1}{\sqrt{1-x^2}}$$

$$y' = \frac{2}{\sqrt{1-x^2}}$$

Ch. 1 Practice Test

⑩ 5, 36, 67, 98, 129, 160 (Arithmetic)

Given:

$$\begin{array}{l}
 \underline{t_1 = 5} \\
 t_n = a + (n-1)d \\
 t_1 = a + (1-1)d \\
 t_1 = a + 0d \\
 \underline{t_1 = a} \\
 \boxed{5 = a}
 \end{array}
 \left|
 \begin{array}{l}
 \underline{t_6 = 160} \\
 t_n = a + (n-1)d \\
 t_6 = a + (6-1)d \\
 \underline{t_6 = a + 5d} \\
 \boxed{160 = a + 5d}
 \end{array}
 \right.
 \begin{array}{l}
 160 = a + 5d \\
 160 = 5 + 5d \\
 \underline{155 = 5d} \\
 \underline{5} \quad \underline{5} \\
 \boxed{31 = d}
 \end{array}$$

b) general term

$$t_n = a + (n-1)d$$

$$t_n = 5 + (n-1)(31)$$

$$t_n = 5 + 31n - 31$$

$$\boxed{t_n = 31n - 26}$$

Series + Sequence

$$\textcircled{1} \quad \underline{80000}, \quad \underline{\quad}, \quad \underline{\quad}, \quad \underline{\quad}, \quad \underline{117128}$$

$$a = 80000$$

$$t_n = ar^{n-1}$$

$$t_5 = 117128$$

$$\frac{117128}{80000} = \frac{80000r^{5-1}}{80000}$$

$$n = 5$$

$$r = ?$$

$$\text{AROI} = ?$$

$$(1.4641)^{\frac{1}{4}} = (r^4)^{\frac{1}{4}} \quad \left| \quad \sqrt[4]{1.4641} = \sqrt[4]{r^4} \right.$$

$$\boxed{1.1 = r} \quad \left| \quad \pm 1.1 = r \right.$$

$$1.1 = r$$

$$\text{AROI} = 100(1.1 - 1) = 10\%$$

Series + Sequence

$$\textcircled{5} \quad a) \sum_{n=1}^5 n^2 + 1$$

$$= [(1)^2 + 1] + [(2)^2 + 1] + [(3)^2 + 1] + [(4)^2 + 1] + [(5)^2 + 1]$$

$$= 2 + 5 + 10 + 17 + 26$$

$$= 60$$

$$b) \sum_{n=1}^{\infty} 3 \left(\frac{1}{2} \right)^{n-1}$$

$$S_n = \frac{a}{1-r}$$

$$S_n = \frac{3}{1 - \frac{1}{2}}$$

$$a = 3$$

$$r = \frac{1}{2}$$

$$S_n = \frac{3}{\frac{2}{2} - \frac{1}{2}}$$

$$S_n = \frac{3}{\frac{1}{2}} = 3 \times 2 = \textcircled{6}$$

Series + Sequence

$$\textcircled{5} \text{ c) } \underline{1} + 5 + 9 + \dots + \underline{77}$$

$\begin{array}{c} \vee \quad \vee \\ 4 \quad 4 \end{array}$

$$a = 1$$

$$t_n = 77$$

$$d = 4$$

① Find n :

$$t_n = a + (n-1)d$$

$$77 = 1 + (n-1)4$$

$$76 = 4(n-1)$$

$$19 = n-1$$

$$\boxed{20 = n}$$

② Find S_{20}

$$S_{20} = \frac{20}{2} [1 + 77]$$

$$= 10(78)$$

$$\boxed{= 780}$$

$$\textcircled{5} \text{ b) } \sum_{n=1}^{\infty} \underline{3} \left(\frac{1}{2} \right)^{n-1} = 3 + \frac{3}{2} + \frac{3}{4} + \dots$$

$\begin{array}{c} \vee \quad \vee \\ \frac{1}{2} \quad \frac{1}{2} \end{array}$

$$a = 3$$

$$r = \frac{1}{2}$$

$$S_n = \frac{3}{1 - \frac{1}{2}} = \frac{3}{\frac{1}{2}} = 3 \cdot 2 = \textcircled{6}$$

$$\textcircled{4} \text{ c) } \lim_{n \rightarrow \infty} (-1)^{n+1} n^2$$

$$= 1, -4, 9, -16, 25, -36, 49, -64, 81, -100$$

= Diverging (has no limit)

Series & Sequence

⑥ Find "a", "r", and S_5 $t_n = ar^{n-1}$
 $\frac{1}{9} \div 9 = \frac{1}{9} \times \frac{1}{9} = \frac{1}{81}$

$$\begin{array}{l|l} \underline{t_3} = 9 & \underline{t_7} = \frac{1}{9} \\ t_3 = ar^{3-1} & t_7 = ar^{7-1} \\ \underline{t_3} = ar^2 & \underline{t_7} = ar^6 \\ 9 = ar^2 & \frac{1}{9} = ar^6 \\ ar^2 = 9 & ar^6 = \frac{1}{9} \end{array}$$

Elimination

$$\begin{array}{l} ar^6 = \frac{1}{9} \\ ar^2 = 9 \\ r^4 = \frac{1}{81} \end{array} \left. \begin{array}{l} \\ \\ \end{array} \right\} \begin{array}{l} ar^2 = 9 \\ a\left(\frac{1}{3}\right)^2 = 9 \\ a\left(\frac{1}{9}\right) = 9 \end{array}$$

~~$\frac{a}{9} = 9 \cdot 9$~~

$a = 81$

when $r = \frac{1}{3}$

$$S_n = \frac{a(r^n - 1)}{r - 1}$$

$$S_5 = \frac{81\left(\left(\frac{1}{3}\right)^5 - 1\right)}{\frac{1}{3} - 1}$$

$$S_5 = 81 \left(\frac{\frac{1}{243} - \frac{243}{243}}{\frac{1}{3} - \frac{3}{3}} \right)$$

$$S_5 = 81 \left(\frac{-\frac{242}{243}}{-\frac{2}{3}} \right)$$

$$S_5 = 81 \left(\frac{121}{243} \right) \left(\frac{-3}{-2} \right)$$

$$S_5 = 121$$

when $r = -\frac{1}{3}$

$$S_5 = \frac{81\left(\left(-\frac{1}{3}\right)^5 - 1\right)}{-\frac{1}{3} - 1}$$

$$S_5 = 81 \left(\frac{-\frac{1}{243} - \frac{243}{243}}{-\frac{1}{3} - \frac{3}{3}} \right)$$

$$S_5 = 81 \left(\frac{\frac{61}{243}}{-\frac{4}{3}} \right)$$

$$S_5 = 61$$

Series & Sequence

$$t_n = a + (n-1)d$$

$$S_n = \frac{n}{2}[2a + (n-1)d]$$

$$\textcircled{1} \quad t_{12} = 15$$

$$t_{12} = a + (12-1)d$$

$$t_{12} = a + 11d$$

$$15 = a + 11d$$

$$a + 11d = 15$$

$$S_{15} = 105$$

$$S_{15} = \frac{15}{2}[2a + (15-1)d]$$

$$S_{15} = 7.5[2a + 14d]$$

$$S_{15} = 15a + 105d$$

$$105 = 15a + 105d$$

$$15a + 105d = 105$$

$$a + 7d = 7$$

Elimination

$$a + 11d = 15$$

$$\Leftrightarrow a + 7d = 7$$

$$\frac{4d = 8}{4 \quad 4}$$

$$d = 2$$

↳

$$a + 7d = 7$$

$$a + 7(2) = 7$$

$$a + 14 = 7$$

$$a = -7$$

$$t_1 = -7$$

$$t_2 = -5$$

$$t_3 = -3$$

Series & Sequence

—, $\frac{1}{5}$, —, —, 25 , —, —

$$t_2 = \frac{1}{5}, \quad t_5 = 25$$

$$t_2 = ar^{2-1}, \quad t_5 = ar^{5-1}$$

$$t_2 = ar^1, \quad t_5 = ar^4$$

$$\frac{1}{5} = ar, \quad 25 = ar^4$$

$$ar = \frac{1}{5}, \quad ar^4 = 25$$

$$\frac{ar^4 = 25}{ar = \frac{1}{5}}$$

$$r^3 = 125$$

$$r = 5$$

$$ar = \frac{1}{5}$$

$$a(5) = \frac{1}{5}$$

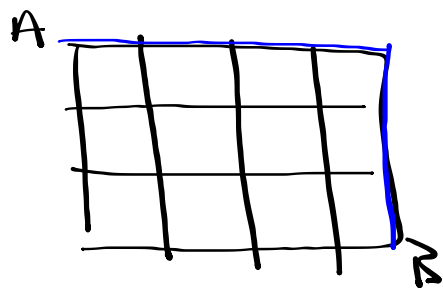
$$5a = \frac{1}{5}$$

$$a = \frac{1}{25}$$

Permutations + Combinations

MISSISSIPPI, $n=11$

$$\frac{11!}{4!4!2!} =$$



RRRR DDD $n=7$

$$\frac{7!}{4!3!} =$$

Permutations + Combinations

binomial theorem

- used to expand $(x + y)^n$, $n \in \mathbb{N}$
- each term has the form ${}_n C_k (x)^{n-k} (y)^k$, where $k + 1$ is the term number

You can use the **binomial theorem** to expand any power of a binomial expression.

$$(x + y)^n = {}_n C_0 (x)^n (y)^0 + {}_n C_1 (x)^{n-1} (y)^1 + {}_n C_2 (x)^{n-2} (y)^2 + \dots \\ + {}_n C_{n-1} (x)^1 (y)^{n-1} + {}_n C_n (x)^0 (y)^n$$

In this chapter, all binomial expansions will be written in descending order of the exponent of the first term in the binomial.

The following are some important observations about the expansion of $(x + y)^n$, where x and y represent the terms of the binomial and $n \in \mathbb{N}$:

- the expansion contains $n + 1$ terms
- the number of objects, k , selected in the combination ${}_n C_k$ can be taken to match the number of factors of the second variable selected; that is, it is the same as the exponent on the second variable
- the general term, t_{k+1} , has the form

$${}_n C_k (x)^{n-k} (y)^k$$

↑
the same

- the sum of the exponents in any term of the expansion is n

Permutations + Combinations

- ① 3 digit # from 0, 1, 2, 8, and 9
with no repetitions

$$\underline{4} \times \underline{4} \times \underline{3} = 48 \quad C$$

↑
can't
use 0
as digit
one

$$\textcircled{15} \left(y - \frac{2}{y^2}\right)^5 \quad \begin{array}{l} n=5 \\ x=y \\ y = \frac{-2}{y^2} \end{array}$$

$${}^5C_0(y)^5\left(\frac{-2}{y^2}\right)^0 + {}^5C_1(y)^4\left(\frac{-2}{y^2}\right)^1 + {}^5C_2(y)^3\left(\frac{-2}{y^2}\right)^2 + {}^5C_3(y)^2\left(\frac{-2}{y^2}\right)^3 + {}^5C_4(y)^1\left(\frac{-2}{y^2}\right)^4 + {}^5C_5(y)^0\left(\frac{-2}{y^2}\right)^5$$

$$1(y^5)(1) + 5(y^4)\left(\frac{-2}{y^2}\right) + 10(y^3)\left(\frac{4}{y^4}\right) + 10(y^2)\left(\frac{-8}{y^6}\right) + 5(y)\left(\frac{16}{y^8}\right) + 1(1)\left(\frac{-32}{y^{10}}\right)$$

$$y^5 - \frac{10y^4}{y^2} + \frac{40y^3}{y^4} - \frac{80y^2}{y^6} + \frac{80y}{y^8} - \frac{32}{y^{10}}$$

$$\boxed{y^5 - 10y^2 + \frac{40}{y} - \frac{80}{y^4} + \frac{80}{y^7} - \frac{32}{y^{10}}}$$

Permutations + Combinations

$$\textcircled{5} \quad (\underline{2x^2} + \underline{3y})^7 \rightarrow 3^{\text{rd}} \text{ term}$$

$$\underline{7}C_2 (2x^2)^5 (3y)^2$$

$$21 (32x^{10}) (9y^2)$$

$$\boxed{6048x^{10}y^2}$$

Permutations + Combinations

$$\textcircled{a} \left(x^2 - \frac{x}{2}\right)^5$$

$${}_5C_0(x^2)^5\left(-\frac{x}{2}\right)^0 + {}_5C_1(x^2)^4\left(-\frac{x}{2}\right)^1 + {}_5C_2(x^2)^3\left(-\frac{x}{2}\right)^2 + {}_5C_3(x^2)^2\left(-\frac{x}{2}\right)^3 + {}_5C_4(x^2)^1\left(-\frac{x}{2}\right)^4 + {}_5C_5(x^2)^0\left(-\frac{x}{2}\right)^5$$

$$1(x^{10})(1) + 5(x^8)\left(-\frac{x}{2}\right) + 10(x^6)\left(\frac{x^2}{4}\right) + 10(x^4)\left(-\frac{x^3}{8}\right) + 5(x^2)\left(\frac{x^4}{16}\right) + 1(1)\left(-\frac{x^5}{32}\right)$$

$$x^{10} - \frac{5x^9}{2} + \frac{5x^8}{4} - \frac{5x^7}{8} + \frac{5x^6}{16} - \frac{x^5}{32}$$

Perm/Comb

$$\textcircled{2} \quad \left(\cancel{x^2} + \frac{x}{3} \right)^3 \quad a = x^2 \quad b = -\frac{x}{3} \quad n = 3$$

$${}^3C_0 (\cancel{x^2})^3 \left(\frac{-x}{3} \right)^0 + {}^3C_1 (\cancel{x^2})^2 \left(\frac{-x}{3} \right)^1 + {}^3C_2 (\cancel{x^2})^1 \left(\frac{-x}{3} \right)^2 + {}^3C_3 (\cancel{x^2})^0 \left(\frac{-x}{3} \right)^3$$

$$1(x^6)(1) + 3(x^4) \left(\frac{-x}{3} \right) + 3(x^2) \left(\frac{x^2}{9} \right) + 1(1) \left(\frac{-x^3}{27} \right)$$

$$x^6 - \frac{3x^5}{3} + \frac{3x^4}{9} - \frac{1x^3}{27}$$

$$\boxed{x^6 - x^5 + \frac{1}{3}x^4 - \frac{1}{27}x^3}$$

Permutations + Combinations

Expand: $(x - \frac{1}{2})^4$

$${}^4C_0(x)^4(\frac{1}{2})^0 + {}^4C_1(x)^3(\frac{1}{2})^1 + {}^4C_2(x)^2(\frac{1}{2})^2 + {}^4C_3(x)^1(\frac{1}{2})^3 + {}^4C_4(x)^0(\frac{1}{2})^4$$

$$(1)(x^4)(1) + (4)(x^3)(\frac{1}{2}) + (6)(x^2)(\frac{1}{4}) + (4)(x)(\frac{1}{8}) + (1)(1)(\frac{1}{16})$$

$$x^4 - \frac{4x^3}{2} + \frac{6x^2}{4} - \frac{4x}{8} + \frac{1}{16}$$

$$x^4 - 2x^3 + \frac{3x^2}{2} - \frac{1}{2}x + \frac{1}{16}$$

Permutations + Combinations

④ c) Case 1 (All Black)

$${}_{26}C_5 = \boxed{65\,780}$$

Case 2 (4 Black + 1 Red)

$${}_{26}C_4 \times {}_{26}C_1 = 14\,950 \times 26 = \boxed{388\,700}$$

Case 3 (3 black + 2 red)

$${}_{26}C_3 \times {}_{26}C_2 = 2600 \times 325 = \boxed{845\,000}$$

$$\text{Total} = 845\,000 + 388\,700 + 65\,780 = \boxed{1\,299\,480}$$

Permutations + Combinations

⑤ 14 Letters

- 5 are burnt
- 9 are good

3 good + 2 bad
 $9C_3 \times 5C_2$

$$84 \times 10$$

$$840$$

Limit Review:

$$x^2 - 25$$

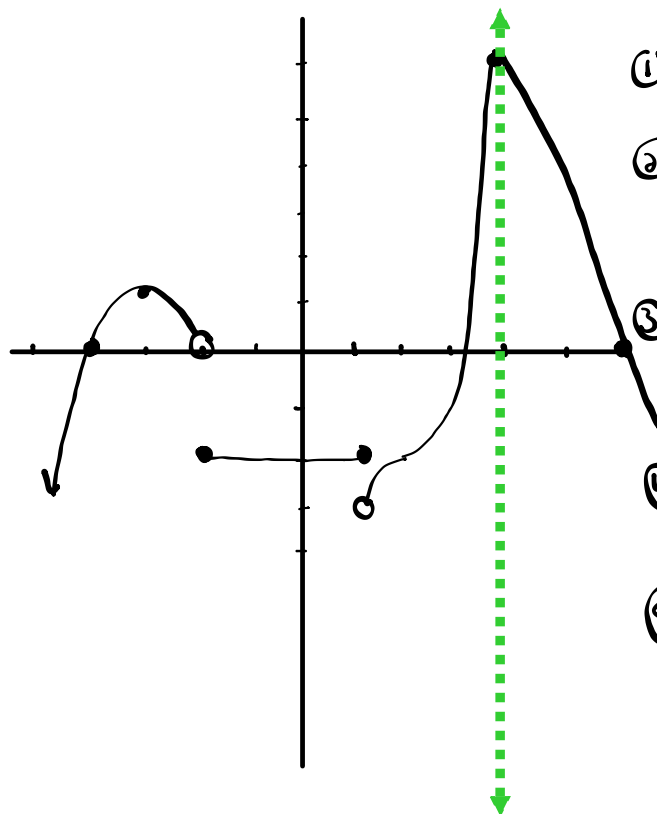
$$(x+5)(x-5)$$

$$\textcircled{1} \text{ g) } \lim_{x \rightarrow b} \frac{x^2 - b^2}{x^4 - b^4}$$

$$\lim_{x \rightarrow b} \frac{(x^2 - b^2)}{(x^4 + b^4)(x^2 - b^2)}$$

$$\lim_{x \rightarrow b} \frac{\cancel{(x^2 - b^2)}}{(x^4 + b^4)\cancel{(x^2 - b^2)}} = \frac{1}{(2b^4)(2b^2)} = \frac{1}{4b^6}$$

$$\textcircled{2} \lim_{x \rightarrow 0} \frac{x^3 + 1}{x^2 + 3x + 2} = \frac{1}{2}$$



$$\textcircled{1} f(-2) = -2$$

$$\textcircled{2} \lim_{x \rightarrow -2^-} f(x) = 0$$

$$\textcircled{3} \lim_{x \rightarrow 1^+} f(x) = -3$$

$$\textcircled{4} f(4) = 6$$

$$\textcircled{5} f(1) = -2$$

Limits

$$a^3 + b^3 = (a+b)(a^2 - ab + b^2)$$

$$a^3 - b^3 = (a-b)(a^2 + ab + b^2)$$

Limits

$$\textcircled{1} \text{ c) } \lim_{a \rightarrow b} \frac{\underline{(a+2b)^2} - \underline{9b^2}}{a-b}$$

$$\lim_{a \rightarrow b} \frac{\underline{(a+2b+3b)} \underline{(a+2b-3b)}}{(a-b)}$$

$$\lim_{a \rightarrow \underline{b}} \frac{\underline{(a+5b)} \cancel{(a-b)}}{\cancel{(a-b)}} = 6b$$

$$\textcircled{1} \text{ f) } \lim_{x \rightarrow 0} \frac{\overset{5(x+5)}{\cancel{5(x+5)}} \frac{1}{x+5} - \frac{1}{5} \overset{5(x+5)}{\cancel{5(x+5)}}}{(x^2+5x)(5(x+5))} \quad \text{CD: } 5(x+5)$$

$$\lim_{x \rightarrow 0} \frac{\overset{\curvearrowright}{5} - \overset{\curvearrowright}{1(x+5)}}{x(x+5)(5(x+5))}$$

$$\lim_{x \rightarrow 0} \frac{5-x-5}{5x(x+5)^2}$$

$$\lim_{x \rightarrow \underline{0}} \frac{\cancel{-x}}{5x \cancel{(x+5)}^2} = \frac{-1}{5(25)} = \frac{-1}{125}$$

Limits

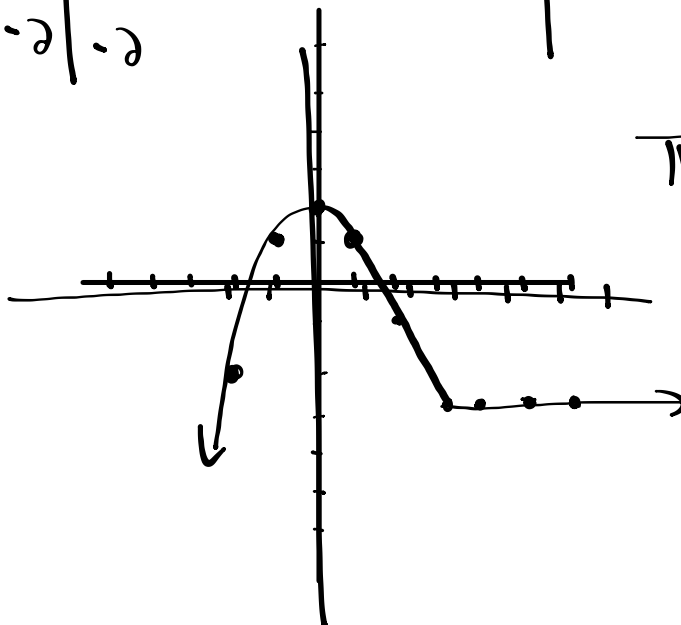
$$\textcircled{2} \text{ Let } f(x) = \begin{cases} -x^2 + 2, & \text{if } x < 1 \\ 1, & \text{if } x = 1 \\ -2x + 3, & \text{if } 1 < x \leq 3 \\ -3, & \text{if } x > 3 \end{cases}$$

$-x^2 + 2$	
x	y
1	1
0	2
-1	1
-2	-2

1	
x	y
1	1

$-2x + 3$	
x	y
1	1
2	-1
3	-3

-3	
x	y
3	-3
4	-3
5	-3
6	-3



The function is continuous

Limits

$$\lim_{x \rightarrow 0} \frac{\cancel{3(x+3)} \frac{1}{\cancel{x+3}} - \frac{1}{\cancel{3}} \cancel{3(x+3)}}{x \cdot 3(x+3)}$$

$$\text{CD: } 3(x+3)$$

$$\lim_{x \rightarrow 0} \frac{3 - (x+3)}{3x(x+3)}$$

$$\lim_{x \rightarrow 0} \frac{3 - x - 3}{3x(x+3)}$$

$$\lim_{x \rightarrow 0} \frac{\cancel{-x}}{\underline{3x}(\underline{x+3})} = \frac{-1}{3(3)} = \frac{-1}{9}$$

Differentiation

$$\textcircled{5} \quad y = (x^2 - 3)^8 \quad @ \quad x = 2$$

(i) Find y :

$$y = (2^2 - 3)^8$$

$$y = (4 - 3)^8$$

$$y = \underline{1}$$

(ii) Find y' :

$$y = (x^2 - 3)^8$$

$$y' = 8(x^2 - 3)^7 (2x)$$

$$y' = 16x(x^2 - 3)^7$$

(iii) Find slope (m):

$$y' = 16(2)[(2)^2 - 3]^7$$

$$y' = 32[4 - 3]^7$$

$$y' = 32 \quad \leftarrow m = 32$$

$$\textcircled{6} \quad y - y_1 = m(x - x_1)$$

$$y - 1 = 32(x - 2)$$

$$y - 1 = 32x - 64$$

$$\boxed{y = 32x - 63}$$

$$\text{or } \boxed{32x - y - 63 = 0}$$

$$\textcircled{7} \quad f(x) = \left(\frac{2x+1}{x-1}\right)^5$$

$$f'(x) = 5 \left(\frac{2x+1}{x-1}\right)^4 \left[\frac{2(x-1) - 1(2x+1)}{(x-1)^2} \right]$$

$$f'(x) = 5 \frac{(2x+1)^4}{(x-1)^4} \cdot \frac{-3}{(x-1)^2}$$

$$f'(x) = \frac{-15(2x+1)^4}{(x-1)^6}$$

Differentiation

Product: $(f \cdot g)'(x) = f'(x)g(x) + f(x)g'(x)$

$$y = \underbrace{(3x^2-5)}_{f(x)} \underbrace{(4x^3+3x)}_{g(x)}$$

$$y' = 6x(4x^3+3x) + (3x^2-5)(12x^2+3)$$

$$y' = 24x^4 + 18x^2 + 36x^4 + 9x^2 - 60x^2 - 15$$

$$y' = 60x^4 - 33x^2 - 15$$

Quotient: $\left(\frac{f}{g}\right)'(x) = \frac{f'(x)g(x) - f(x)g'(x)}{[g(x)]^2}$

$$y = \frac{x+2}{3x+5}$$

$$y' = \frac{1(3x+5) - 3(x+2)}{(3x+5)^2}$$

$$y' = \frac{6x^2 + 10x - 3x^2 - 6}{(3x+5)^2} = \frac{3x^2 + 10x - 6}{(3x+5)^2}$$

Chain: $(f \circ g)'(x) = f'(g(x)) \cdot g'(x)$

$$y = \sqrt{4x^2-6x} = (4x^2-6x)^{\frac{1}{2}}$$

$$y' = \frac{1}{2}(4x^2-6x)^{-\frac{1}{2}}(8x-6)$$

$$y' = \frac{12x-6}{2(4x^2-6x)^{\frac{1}{2}}} = \frac{3(2x-1)}{\sqrt{4x^2-6x}}$$

$$\frac{6x-3}{\sqrt{4x^2-6x}}$$

Combo:

$$y = (3x^2+5)^3 \sqrt{4x-2} = \underbrace{(3x^2+5)^3}_{f(x)} \underbrace{(4x-2)^{\frac{1}{2}}}_{g(x)}$$

$$y' = 3(3x^2+5)^2(6x) \underbrace{(4x-2)^{-\frac{1}{2}}}_{\frac{1}{2}(4x-2)^{-\frac{1}{2}}(4)}$$

$$y' = 18x(3x^2+5)^2(4x-2)^{-\frac{1}{2}} + 2(3x^2+5)^3(4x-2)^{-\frac{1}{2}}$$

$$y' = 2(3x^2+5)^2(4x-2)^{-\frac{1}{2}} [9x(4x-2) + (3x^2+5)]$$

$$y' = 2(3x^2+5)^2(4x-2)^{-\frac{1}{2}}(39x^2-18x+5)$$

$$y' = \frac{2(3x^2+5)^2(39x^2-18x+5)}{\sqrt{4x-2}}$$

$$\frac{(3x^2+5)^2}{(3x^2+5)^2} = (3x^2+5)^0 = 1 \quad \left| \quad \frac{(4x-2)^{\frac{1}{2}}}{(4x-2)^{\frac{1}{2}}} = (4x-2)^{\frac{1}{2}-\frac{1}{2}} = (4x-2)^0 = 1 \right.$$

Differentiation

$$\frac{-1}{2}x^3 = \frac{-3}{2}$$

$$\textcircled{2} \text{ b) } f(x) = \frac{3}{\sqrt{x}} = \frac{3}{x^{1/2}} = 3x^{-1/2}$$

$$\frac{-1}{2} - 1$$

$$\frac{-1}{2} \cdot \frac{-2}{2} = \frac{-3}{2}$$

$$f'(x) = \frac{-3}{2}x^{-3/2} = \frac{-3}{2x^{3/2}}$$

$$\textcircled{4} \text{ b) } y = \frac{\sqrt{x}}{3+x^2} = \frac{x^{1/2}}{3+x^2} \quad \begin{matrix} f \\ g \end{matrix} \quad \frac{f'g - fg'}{(g)^2}$$

$$y' = \frac{\frac{1}{2}x^{-1/2}(3+x^2) - x^{1/2}(2x)}{(3+x^2)^2}$$

$$x^{1/2} \cdot x^1$$

$$x^{1/2+1}$$

$$x^{3/2+2}$$

$$x^{7/2}$$

$$y' = \frac{\frac{1}{2}x^{-1/2}(3+x^2) - 2x^{3/2}}{(3+x^2)^2}$$

$$y' = \frac{\frac{3+x^2}{2x^{1/2}} - 2x^{3/2} \cdot 2x^{1/2}}{2x^{1/2}(3+x^2)^2}$$

Complex Fraction:
CD: $2x^{1/2}$

$$y' = \frac{3+x^2 - 4x^2}{2\sqrt{x}(3+x^2)^2}$$

$$2x^{3/2} \cdot 2x^{1/2}$$

$$4x^{3/2+1/2} = 4x^{4/2}$$

$$y' = \frac{3-3x^2}{2\sqrt{x}(3+x^2)^2}$$

$$\textcircled{6} \text{ b) } y = \frac{16}{\sqrt{x-1}} = \frac{16}{(x-1)^{1/2}} = 16(x-1)^{-1/2}$$

$$y' = -8(x-1)^{-3/2} (1) = -8(x-1)^{-3/2} = \frac{-8}{(x-1)^{3/2}}$$

$$= \frac{-8}{\sqrt{(x-1)^3}}$$

Differentiation

$$f(x) = 3x^2 + 2x - 7$$

$$f(x+h) = 3(x+h)^2 + 2(x+h) - 7$$

$$= 3(x^2 + 2xh + h^2) + 2x + 2h - 7$$

$$= 3x^2 + 6xh + 3h^2 + 2x + 2h - 7$$

$$f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$$

$$f'(x) = \lim_{h \rightarrow 0} \frac{3x^2 + 6xh + 3h^2 + 2x + 2h - 7 - (3x^2 + 2x - 7)}{h}$$

$$f'(x) = \lim_{h \rightarrow 0} \frac{6xh + 3h^2 + 2h}{h}$$

$$f'(x) = \lim_{h \rightarrow 0} \frac{h(6x + 3h + 2)}{h} = 6x + 2$$

Differentiation

$$f(x) = \frac{(3x^2+5)^3}{\sqrt{2x-7}}$$

$$\frac{f'g - fg'}{g^2}$$

$$f'(x) = \frac{\overbrace{3(3x^2+5)^2}^{f'}}{\overbrace{(6x)(2x-7)^{1/2}}^g} - \frac{\overbrace{(3x^2+5)^3}^f \overbrace{(\frac{1}{2})(2x-7)^{-1/2}}^{g'}}{(2x-7)^2}$$

$$f'(x) = \frac{18x(3x^2+5)^2(2x-7)^{1/2} - (3x^2+5)^3(2x-7)^{-1/2}}{(2x-7)^2} \quad \leftarrow \text{Factor}$$

$$f'(x) = \frac{(3x^2+5)^2(2x-7)^{-1/2} \left[18x(2x-7) - (3x^2+5) \right]}{(2x-7)^2}$$

$$f'(x) = \frac{(3x^2+5)^2(33x^2-126x-5)}{(2x-7)^{3/2}}$$

Differentiation

$$\textcircled{3} \quad y = \sqrt[7]{2x^2 + \sqrt{x^2 - 8x} \sqrt{3-x}} = \left[2x^2 + (x^2 - 8x)^{\frac{1}{2}} (3-x)^{\frac{1}{2}} \right]^{\frac{1}{7}}$$

$$y' = \frac{1}{7} \left[2x^2 + (x^2 - 8x)^{\frac{1}{2}} (3-x)^{\frac{1}{2}} \right]^{-\frac{6}{7}} \left[4x + \frac{1}{2} (x^2 - 8x)^{-\frac{1}{2}} (7x^6 - (8(3-x)^{\frac{1}{2}} + 8x \left(\frac{1}{2}\right) (3-x)^{-\frac{1}{2}}) (1)) \right]$$

Find the point (x, y) on the curve $y = x^2 + 5x$
 where the slope of the tangent line equals 9

(i) Find derivative

$$y = x^2 + 5x$$

$$y' = 2x + 5$$

(ii) Solve for x:

$$y' = 2x + 5$$

$$9 = 2x + 5$$

$$4 = 2x$$

$$\underline{2 = x}$$

(iii) Solve for y:

$$y = x^2 + 5x$$

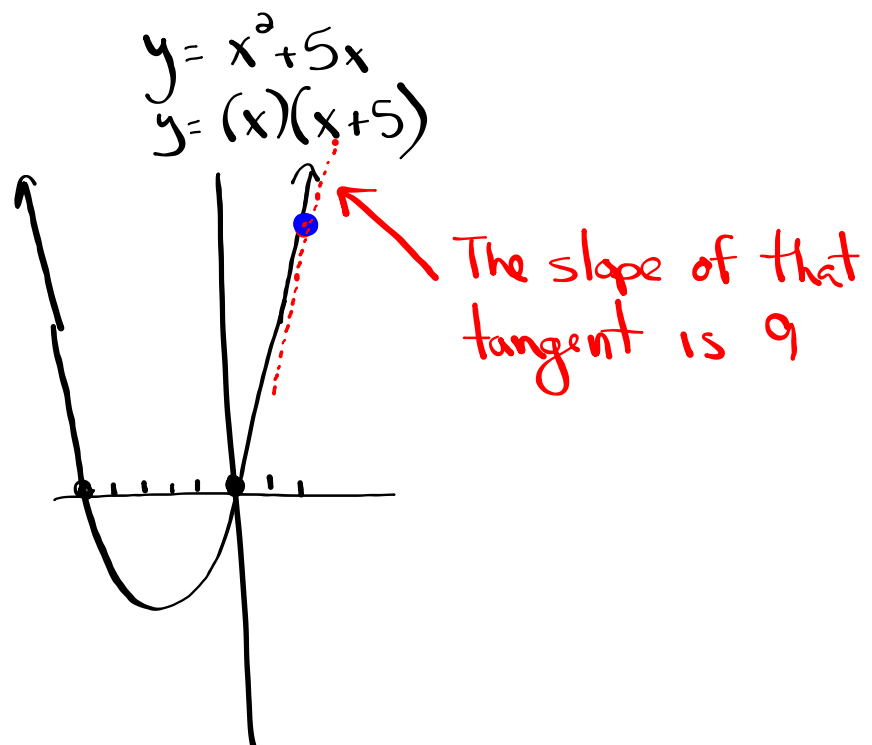
$$y = (2)^2 + 5(2)$$

$$y = 4 + 10$$

$$\underline{y = 14}$$

(iv) Point:

$$(2, 14)$$



Derivatives Exam Review:

$$\textcircled{4} \text{ b) } y = \frac{\sqrt{x}}{3+x^2} = \frac{x^{1/2}}{(3+x^2)}$$

$$y' = \frac{(3+x^2)\left(\frac{1}{2}x^{-1/2}\right) - x^{1/2}(2x)}{(3+x^2)^2}$$

$$y' = \frac{\frac{3}{2}x^{-1/2} + \frac{1}{2}x^{3/2} - 2x^{3/2}}{2(3+x^2)^2}$$

$$y' = \frac{3x^{-1/2} + x^{3/2} - 4x^{3/2}}{2(3+x^2)^2}$$

$$y' = \frac{x^{-1/2}(3+x^2-4x^2)}{2(3+x^2)^2}$$

$$y' = \frac{3-3x^2}{2x^{1/2}(3+x^2)^2} = \frac{3(1-x^2)}{2\sqrt{x}(3+x^2)^2}$$

Curve Sketching:

- ① Plot all points: x -int, y -int, max, min, I.P.,
- ② Plot all asymptotes (Check behaviour near VA.)
- ③ use intervals of inc/dec and concavity to connect everything

Sketch $f(x) = x^3 - 12x^2 + 36x$
 $f'(x) = 3x^2 - 24x + 36$
 $f''(x) = 6x - 24$

① x-int $0 = x^3 - 12x^2 + 36x$
 $0 = x(x^2 - 12x + 36)$
 $0 = x(x-6)(x-6)$
 $x=0 \mid x-6=0$
 $x=6$
 $(0,0) \quad (6,0)$

② y-int ($x=0$)
 $y = (0)^3 - 12(0)^2 + 36(0)$
 $y = 0$
 $(0,0)$

③ Intervals of Inc/Dec

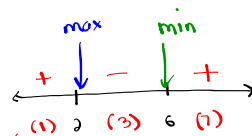
$$f'(x) = 3x^2 - 24x + 36$$

$$f'(x) = 3(x^2 - 8x + 12)$$

$$f'(x) = 3(x-6)(x-2)$$

$$0 = 3(x-6)(x-2)$$

$$CV: x=6, x=2$$



Increasing on $(-\infty, 2) \cup (6, \infty)$
 Decreasing on $(2, 6)$

④ max/min:

$$\text{max @ } x=2$$

$$f(x) = x^3 - 12x^2 + 36x$$

$$f(2) = (2)^3 - 12(2)^2 + 36(2)$$

$$f(2) = 8 - 48 + 72$$

$$f(2) = 32$$

$$(2, 32)$$

$$\text{min @ } x=6$$

$$f(x) = x^3 - 12x^2 + 36x$$

$$f(6) = (6)^3 - 12(6)^2 + 36(6)$$

$$f(6) = 216 - 432 + 216$$

$$f(6) = 0$$

$$(6, 0)$$

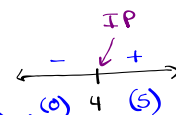
⑤ Intervals of Concavity:

$$f''(x) = 6x - 24$$

$$f''(x) = 6(x-4)$$

$$0 = 6(x-4)$$

$$CV: x=4$$



CD on $(-\infty, 4)$
 CU on $(4, \infty)$

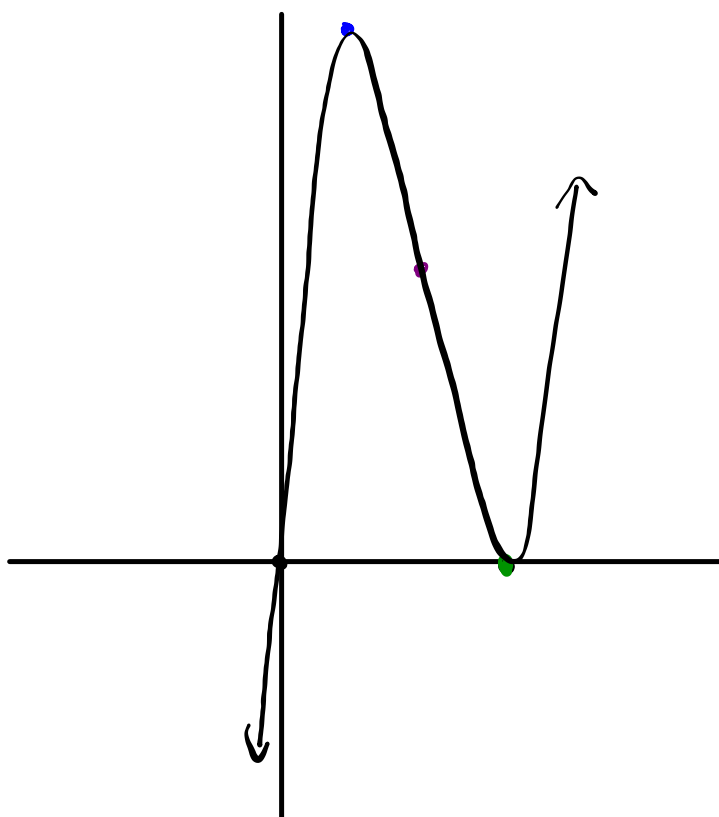
⑥ Inflection Point @ $x=4$

$$f(x) = x^3 - 12x^2 + 36x$$

$$f(4) = (4)^3 - 12(4)^2 + 36(4)$$

$$f(4) = 64 - 192 + 144$$

$$f(4) = 16$$



Curve Sketching

Example:

Examine the function $f(x) = 3x^5 - 5x^3$ with respect to...

- Intercepts $f(x)$
- ~~Symmetry~~
- Asymptotes (No asymptotes for polynomial functions)
- Intervals of Increase or Decrease $f'(x)$
- Local Maximum and Minimum values $f(x)$
- ~~$f''(x)$~~ Concavity and Points of Inflection $f''(x)$
- Sketch the Curve

$$f(x) = 3x^5 - 5x^3 \quad f'(x) = 15x^4 - 15x^2 \quad f''(x) = 60x^3 - 30x$$

$$f(x) = x^3(3x^2 - 5) \quad f'(x) = 15x^2(x^2 - 1) \quad f''(x) = 30x(x^2 - 1)$$

$$f'(x) = 15x^2(x-1)(x+1)$$

① x-int ($y=0$)

$$f(x) = x^3(3x^2 - 5)$$

$$0 = x^3(3x^2 - 5)$$

$$x^3 = 0 \quad 3x^2 - 5 = 0$$

$$x = 0 \quad \frac{3x^2}{3} = \frac{5}{3}$$

$$(0, 0) \quad x^2 = \frac{5}{3}$$

$$x = \pm\sqrt{\frac{5}{3}}$$

$$(1.29, 0)$$

$$+ (-1.29, 0)$$

② y-int ($x=0$)

$$f(x) = 3x^5 - 5x^3$$

$$f(0) = 3(0)^5 - 5(0)^3$$

$$f(0) = 0$$

$$(0, 0)$$

③ Intervals of Inc/Dec.

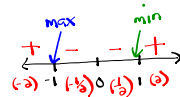
$$f'(x) = 15x^2(x-1)(x+1)$$

$$0 = 15x^2(x-1)(x+1)$$

$$15x^2 = 0 \quad x-1=0 \quad x+1=0$$

$$x^2 = 0 \quad x = 1 \quad x = -1$$

$$x = 0$$

Increasing on $(-\infty, -1) \cup (1, \infty)$ Decreasing on $(-1, 0) \cup (0, 1)$
or $(-1, 1)$ cv: $x = -1, 0, 1$ ④ max @ $x = -1$

$$f(x) = 3x^5 - 5x^3$$

$$f(-1) = 3(-1)^5 - 5(-1)^3$$

$$f(-1) = -3 + 5$$

$$f(-1) = 2$$

$$(-1, 2)$$

⑤ min @ $x = 1$

$$f(x) = 3x^5 - 5x^3$$

$$f(1) = 3(1)^5 - 5(1)^3$$

$$f(1) = 3 - 5$$

$$f(1) = -2$$

$$(1, -2)$$

⑥ Intervals of Concavity:

$$f''(x) = 30x(x^2 - 1)$$

$$0 = 30x(x^2 - 1)$$

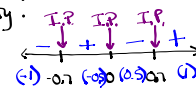
$$30x = 0 \quad x^2 - 1 = 0$$

$$x = 0 \quad x^2 = 1$$

$$x^2 = \frac{1}{3}$$

$$x = \pm\sqrt{\frac{1}{3}}$$

$$x = \pm 0.577$$

cv: $x = -0.577, 0, 0.577$ CD on $(-\infty, -0.577) \cup (0, 0.577)$ CU on $(-0.577, 0) \cup (0.577, \infty)$

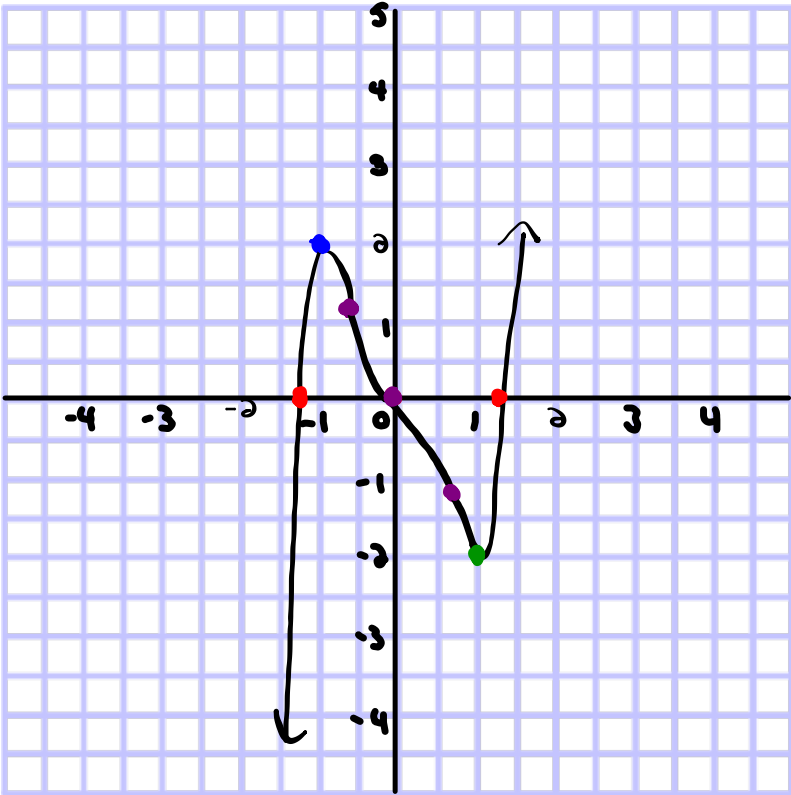
⑦ Inflection Points

$$f(x) = 3x^5 - 5x^3$$

$$f(-0.577) = 3(-0.577)^5 - 5(-0.577)^3 = -0.504 + 1.715 = 1.2 \quad (-0.577, 1.2)$$

$$f(0) = 3(0)^5 - 5(0)^3 = 0 - 0 = 0 \quad (0, 0)$$

$$f(0.577) = 3(0.577)^5 - 5(0.577)^3 = 0.504 - 1.715 = -1.2 \quad (0.577, -1.2)$$



Curve Sketching

Examine the function $f(x) = \frac{x^2}{1-x^2}$ with respect to... $f'(x) = \frac{2x}{(1-x^2)^2}$

- Intercepts $f(x)$
- ~~Symmetry~~
- Asymptotes
- Intervals of Increase or Decrease
- Local Maximum and Minimum values
- Concavity and Points of Inflection
- Sketch the Curve

$$f''(x) = \frac{2(1+3x^2)}{(1-x^2)^3}$$

① x-int ($y=0$)

$$f(x) = \frac{x^2}{1-x^2}$$

$$0 = \frac{x^2}{1-x^2}$$

$$0 = x^2$$

$$0 = x$$

$$(0,0)$$

② y-int ($x=0$)

$$f(x) = \frac{x^2}{1-x^2}$$

$$f(0) = \frac{0^2}{1-0^2}$$

$$f(0) = \frac{0}{1} = 0$$

$$(0,0)$$

③ Vertical Asymptote:
(zeros of the denominator)

$$f(x) = \frac{x^2}{1-x^2}$$

$$\text{VA: } 1-x^2=0$$

$$(1-x)(1+x)=0$$

$$1-x=0 \quad | \quad 1+x=0$$

$$\boxed{1-x} \quad | \quad \boxed{x=-1}$$

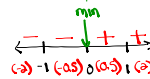
④ Horizontal Asymptote:

$$\lim_{x \rightarrow \infty} \frac{x^2}{1-x^2} = \frac{1}{-1} = -1$$

$$\boxed{y=-1}$$

⑤ Intervals of Inc/Dec.

$$f'(x) = \frac{2x}{(1-x^2)^2}$$



$$\begin{array}{l} 2x=0 \\ x=0 \end{array} \left| \begin{array}{l} (1-x^2)^2=0 \\ 1-x^2=0 \\ 1-x^2 \\ \pm 1=x \end{array} \right.$$

Increasing on $(0, \infty)$
Decreasing on $(-\infty, 0)$

$$\text{CR: } x=-1, 0, 1$$

⑥ min @ $x=0$

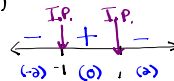
$$f(x) = \frac{x^2}{1-x^2}$$

$$f(0) = \frac{0^2}{1-0^2} = 0$$

$$(0,0)$$

⑦ Intervals of concavity

$$f''(x) = \frac{2(1+3x^2)}{(1-x^2)^3}$$



$$\begin{array}{l} 2(1+3x^2)=0 \\ 1+3x^2=0 \\ 3x^2=-1 \\ x^2=-\frac{1}{3} \\ \text{Not Possible} \end{array} \left| \begin{array}{l} (1-x^2)^3=0 \\ 1-x^2=0 \\ 1-x^2 \\ \pm 1=x \end{array} \right.$$

CO on $(-\infty, -1) \cup (1, \infty)$
CU on $(-1, 1)$

(Numerator is always positive)

⑧ Inflection Points:

when $x=-1$

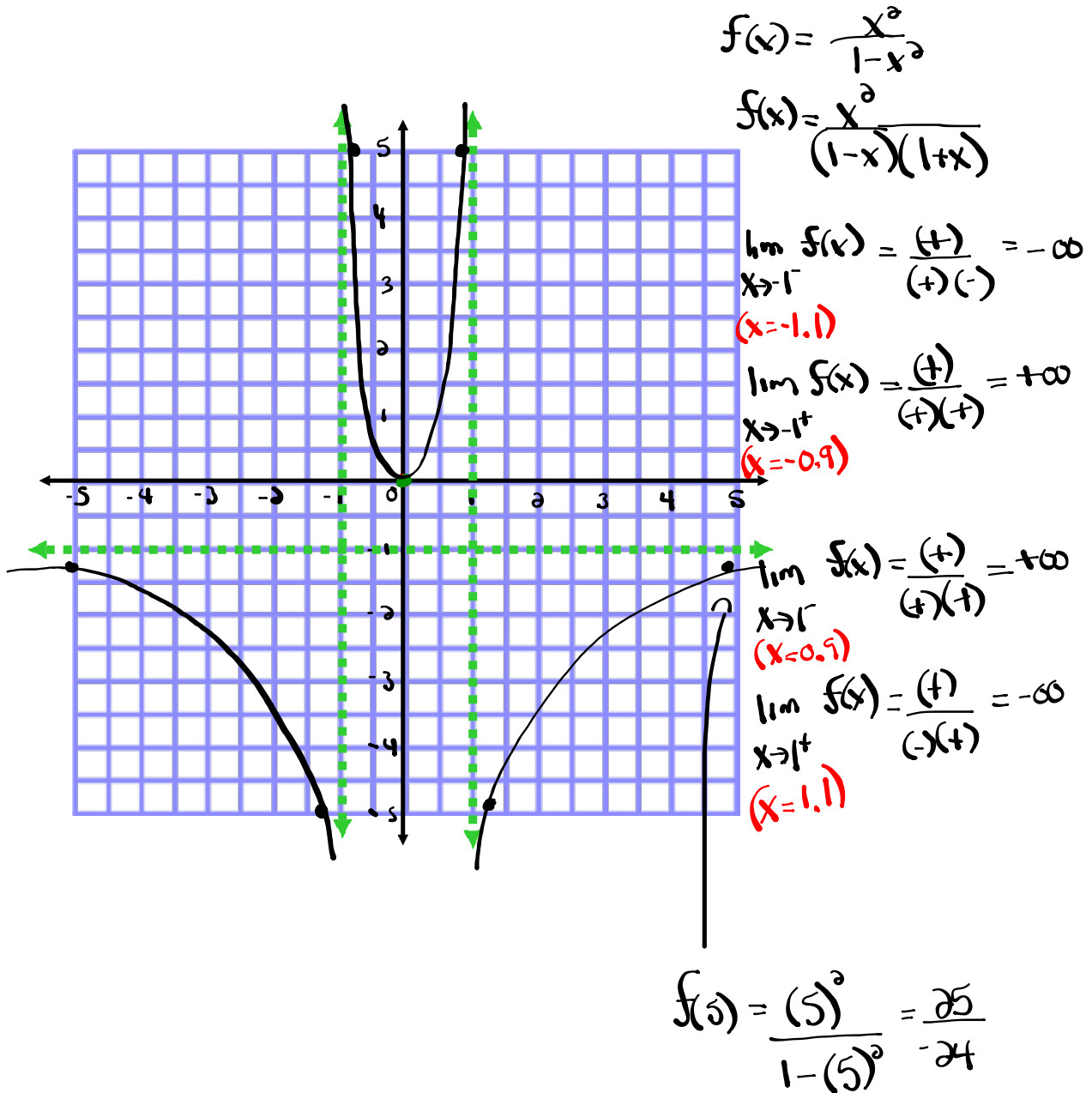
$$f(x) = \frac{x^2}{1-x^2}$$

$$f(-1) = \frac{(-1)^2}{1-(-1)^2} = \frac{1}{0} = \text{und.}$$

when $x=1$

$$f(x) = \frac{x^2}{1-x^2}$$

$$f(1) = \frac{(1)^2}{1-(1)^2} = \frac{1}{0} = \text{und.}$$



Synthetic Sub:

$$-27 + 45 + 6 - 24$$

$$x^3 + 5x^2 - 2x - 24$$

$$(2)^3 + 5(2)^2 - 2(2) - 24$$

$$8 + 20 - 4 - 24$$

$$0$$

2	1	5	-2	-24
↓		2	14	24
x=2	1	7	12	0

Find an x value
that makes the
polynomial equal 0

$$(x-2)(x^2 + 7x + 12) \leftarrow \begin{array}{l} \text{simple} \\ \text{trinomial} \end{array} \quad \begin{array}{l} \underline{3} + \underline{4} = 7 \\ \underline{3} \times \underline{4} = 12 \end{array}$$

$$(x-2)(x+3)(x+4)$$

Curve Sketching:

$$f(x) = \frac{x^2 + 5x + 6}{x^2 - 9} = \frac{(x+2)\cancel{(x+3)}}{(x-3)\cancel{(x+3)}} = \frac{x+2}{x-3}$$

x int:

$$x+2=0$$

$$x=-2$$

$(-2, 0)$

y int:

$$f(0) = \frac{0+2}{0-3}$$

$$f(0) = -\frac{2}{3}$$

$(0, -\frac{2}{3})$

VA:

$$x-3=0$$

$$x=3$$

HA: $\lim_{x \rightarrow \infty} \frac{x+2}{x-3} = 1$

$$y=1$$

Hole:

$$x+3=0$$

$$x=-3$$

$$f(-3) = \frac{-3+2}{-3-3}$$

$$f(-3) = \frac{1}{6}$$