

Chapter Review for Final Exam:

Ch.1 → (Inverse Functions)

Ch.2 → (Radical Functions)

Ch.7 → (Exponential Functions) $y = 2^x$ Ch.8 → (Logarithmic Functions) $y = \log_2 x$

Ch.4 → (Trig + Unit Circle)

Ch.5 → (Trig Functions)

Ch.6 → (Trig Identities)

Ch.2	Ch.7	Ch.8	Ch.5																																														
$y = \sqrt{x}$	$y = 3^x$	$y = \log_3 x$	$y = \sin x$																																														
<table border="1"> <thead> <tr><th>x</th><th>y</th></tr> </thead> <tbody> <tr><td>0</td><td>0</td></tr> <tr><td>1</td><td>1</td></tr> <tr><td>4</td><td>2</td></tr> <tr><td>9</td><td>3</td></tr> </tbody> </table>	x	y	0	0	1	1	4	2	9	3	<table border="1"> <thead> <tr><th>x</th><th>y</th></tr> </thead> <tbody> <tr><td>-2</td><td>1/9</td></tr> <tr><td>-1</td><td>1/3</td></tr> <tr><td>0</td><td>1</td></tr> <tr><td>1</td><td>3</td></tr> <tr><td>2</td><td>9</td></tr> </tbody> </table>	x	y	-2	1/9	-1	1/3	0	1	1	3	2	9	<table border="1"> <thead> <tr><th>x</th><th>y</th></tr> </thead> <tbody> <tr><td>1/9</td><td>-2</td></tr> <tr><td>1/3</td><td>-1</td></tr> <tr><td>1</td><td>0</td></tr> <tr><td>3</td><td>1</td></tr> <tr><td>9</td><td>2</td></tr> </tbody> </table>	x	y	1/9	-2	1/3	-1	1	0	3	1	9	2	<table border="1"> <thead> <tr><th>x</th><th>y</th></tr> </thead> <tbody> <tr><td>0°</td><td>0</td></tr> <tr><td>90°</td><td>1</td></tr> <tr><td>180°</td><td>0</td></tr> <tr><td>270°</td><td>-1</td></tr> <tr><td>360°</td><td>0</td></tr> </tbody> </table>	x	y	0°	0	90°	1	180°	0	270°	-1	360°	0
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$$y = a f[b(x-h)] + k \quad \text{Ch. 1}$$

$$y = a \sqrt{b(x-h)} + k \quad \text{Ch. 2}$$

$$y = a e^{b(x-h)} + k \quad \text{Ch. 7}$$

$$y = a \log_c [b(x-h)] + k \quad \text{Ch. 8}$$

$$y = a \sin[b(x-h)] + k \quad \text{Ch. 5}$$

Ch. 2 (Radical Functions)

$$D: \{x \mid x \geq h, x \in \mathbb{R}\} \quad \text{if } b > 0 \quad R: \{y \mid y \geq k, y \in \mathbb{R}\} \quad \text{if } a > 0$$

$$\{x \mid x \leq h, x \in \mathbb{R}\} \quad \text{if } b < 0 \quad \{y \mid y \leq k, y \in \mathbb{R}\} \quad \text{if } a < 0$$

Ch. 7 (Exponential Functions)

$$D: \{x \mid x \in \mathbb{R}\} \quad R: \{y \mid y > k, y \in \mathbb{R}\} \quad \text{if } a > 0$$

$$\{y \mid y < k, y \in \mathbb{R}\} \quad \text{if } a < 0$$

$$HA: y = k$$

Ch. 8 (Logarithmic Functions)

$$D: \{x \mid x > h, x \in \mathbb{R}\} \quad \text{if } b > 0 \quad R: \{y \mid y \in \mathbb{R}\}$$

$$\{x \mid x < h, x \in \mathbb{R}\} \quad \text{if } b < 0$$

$$VA: x = h$$

Ch. 5 (Sinusoidal Functions)

$$D: \{x \mid x \in \mathbb{R}\}$$

$$\text{or } \{0 \mid 0 \in \mathbb{R}\}$$

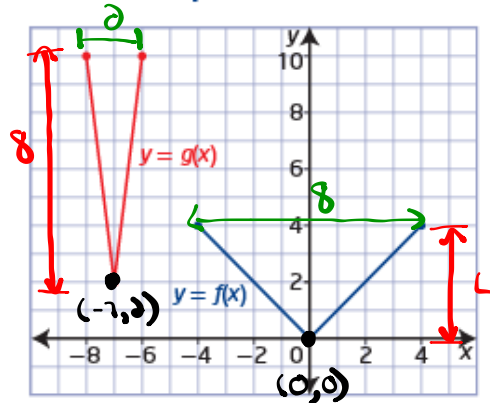
$$R: \{y \mid \min \leq y \leq \max, y \in \mathbb{R}\}$$

Chapter 1:

Example 3

Write the Equation of a Transformed Function Graph

The graph of the function $y = g(x)$ represents a transformation of the graph of $y = f(x)$. Determine the equation of $g(x)$ in the form $y = af(b(x - h)) + k$. Explain your answer.



Solution

① Reflection: None

② VSF = $\frac{8}{4} = 2$ ($a=2$)

③ HSF = $\frac{2}{8} = \frac{1}{4}$ ($b=4$)

④ HT: $(0,0) \rightarrow (-7,2)$ 7 units left ($h=-7$)

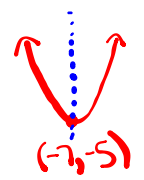
⑤ VT: $(0,0) \rightarrow (-7,2)$ 2 units up ($k=2$)

⑥ Equation: $g(x) = 2f[4(x+7)] + 2$

Ch. 1

Find the inverse:

- ① Replace $f(x)$ with y
- ② Switch x 's and y 's
- ③ Solve for y
- ④ Replace y with $f^{-1}(x)$
(if it is a function)

Ex: $f(x) = (x+7)^2 - 5 \rightarrow$ Parabola  Fails HLT
Inverse is not a function
axis of sym: $x = -7$

$$\text{Ex: } f(x) = (x+7)^2 - 5$$

$$y = (x+7)^2 - 5$$

$$x = (y+7)^2 - 5$$

$$x+5 = (y+7)^2$$

$$\pm \sqrt{x+5} = y+7$$

$$-7 \pm \sqrt{x+5} = y$$

$$y = -7 \pm \sqrt{x+5} \rightarrow \curvearrowright$$

$$f^{-1}(x) = -7 + \sqrt{x+5}, \text{ if } x \geq -5$$

$$f^{-1}(x) = -7 - \sqrt{x+5}, \text{ if } x \leq -5$$

Chapter 2 Radical Functions

Solve for x: $(\sqrt{x+8})^2 = (x+6)^2$

$$x+8 = (x+6)(x+6)$$

$$x+8 = x^2 + 12x + 36$$

$$0 = x^2 + 11x + 28 \quad \begin{array}{l} 4 + 7 = 11 \\ 4 \times 7 = 28 \end{array}$$

$$0 = (x+4)(x+7)$$

$$\begin{array}{l|l} x+4=0 & x+7=0 \\ x=-4 & x=-7 \end{array}$$

is a solution
↓

Test $x=-4$

$$\sqrt{x+8} = x+6$$

$$\sqrt{-4+8} \quad | \quad -4+6$$

$$\sqrt{4} \quad | \quad 2 \checkmark$$

$$2 \checkmark$$

is not a solution
↓
(extraneous)

Test $x=-7$

$$\sqrt{x+8} = x+6$$

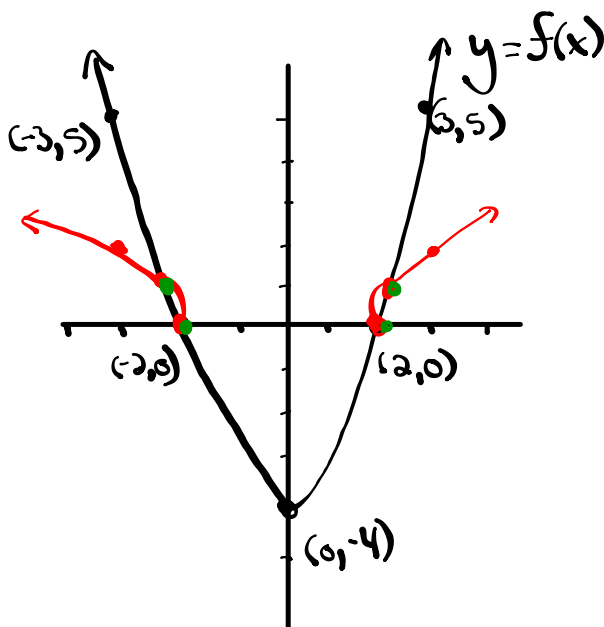
$$\sqrt{-7+8} \quad | \quad -7+6$$

$$\sqrt{1} \quad | \quad -1 \quad \times$$

$$1$$

Ch. 2

③ Using the graph of $y = f(x)$, sketch the graph of $y = \sqrt{f(x)}$. state the domain and range of each.



$$y = f(x)$$

$$D: \{x \mid x \in \mathbb{R}\} \text{ or } (-\infty, \infty)$$

$$R: \{y \mid y \geq -4, y \in \mathbb{R}\} \text{ or } [-4, \infty)$$

$$y = \sqrt{f(x)}$$

$$D: \{x \mid x \leq -2, x \geq 2, x \in \mathbb{R}\} \\ \text{or } (-\infty, -2] \cup [2, \infty)$$

$$R: \{y \mid y \geq 0, y \in \mathbb{R}\} \text{ or } [0, \infty)$$

Ch. 7 → Exponential Functions

6. Solve the following equations (be sure to test your answers).

(a) $2^{2x+2} + 7 = 71$

(b) $9^{2x+1} = 81(27^x)$

a) $2^{2x+2} + 7 = 71$

$2^{2x+2} = 64$

~~$2^{2x+2} = (2)^6$~~

$2x+2 = 6$

$\frac{2x}{2} = \frac{4}{2}$

$x = 2$

b) $9^{2x+1} = 81(27^x)$

$(3^2)^{2x+1} = (3^4)(3^3)^x$

$3^{4x+2} = 3^4 \cdot 3^{3x}$

~~$3^{4x+2} = 3^{3x+4}$~~

$4x+2 = 3x+4$

$x = 2$

* Be sure to test your answers!

Ch. 8 → Logarithmic Functions

4. Rewrite each expression as a single logarithm.

$$3 \log_5 x + \frac{1}{2} \log_5 (x-1) - \log_5 (x^2 + 1)$$

$$\log_5 x^3 + \log_5 (x-1)^{\frac{1}{2}} - \log_5 (x^2 + 1)$$

$$\log_5 x^3 (x-1)^{\frac{1}{2}} - \log_5 (x^2 + 1)$$

$$\log_5 \left[\frac{x^3 (x-1)^{\frac{1}{2}}}{x^2 + 1} \right] \quad \text{or} \quad \log_5 \left[\frac{x^3 \sqrt{x-1}}{x^2 + 1} \right]$$

Ch. 8

7. Solve the following equation (be sure to test your answers).

$$\log_{10}(x+2) + \log_{10}(x-1) = 1$$

$$\log_{10}((x+2)(x-1)) = 1$$

$$\log_{10}(x^2 + x - 2) = 1 \quad (\text{log. form})$$

\uparrow \uparrow \uparrow
 Base Ans Exp.

$$10^1 = x^2 + x - 2 \quad (\text{exp. form})$$

$$10 = x^2 + x - 2$$

$$0 = x^2 + x - 12 \quad \begin{array}{l} -3 + 4 = 1 \\ -3 \times 4 = -12 \end{array}$$

$$0 = (x-3)(x+4)$$

$$x-3=0 \quad | \quad x+4=0$$

$$\boxed{x=3} \quad | \quad x=-4$$

is a solution | extraneous

* Be sure to test your answers!

Ch. 7 or Ch. 8 Base = $\frac{1}{2}$ or 0.5 $A_0 = 60 \text{ mg}$

2. Cobalt-60, which has a half-life of 5.3 years, is used in medical radiology. A sample of 60 mg of the material is present today.

$$\uparrow \text{exp} = \frac{t}{5.3}$$

a) Write an equation to express the mass of cobalt-60 (in mg), as a function of time, t in years. [2]

$$y = (\text{Initial Amount})(\text{Base})^{\text{exp.}}$$

$$y = (60)(0.5)^{t/5.3}$$

b) What amount will be present in 10.6 years? $t = 10.6$ [2]

$$y = (60)(0.5)^{\frac{10.6}{5.3}}$$

$$y = (60)(0.5)^2$$

$$y = (60)(0.25) = \boxed{15 \text{ mg}}$$

c) How long will it take for the amount of cobalt-60 to decay to 12.5% of its initial amount? [3]

(i) 12.5% of initial Amount:

$$= 0.125 \times 60$$

$$= 7.5 \text{ mg} \quad (y = 7.5 \text{ mg})$$

(ii) Solve for t :

$$y = (60)(0.5)^{t/5.3}$$

$$\frac{7.5}{60} = \frac{60(0.5)^{t/5.3}}{60}$$

$$0.125 = (0.5)^{t/5.3}$$

$$\cancel{(0.5)}^3 = \cancel{(0.5)}^{t/5.3}$$

$$* \frac{\log(0.125)}{\log(0.5)} = \underline{\underline{3}}$$

$$(5.3) 3 = \frac{t}{5.3} \quad (\cancel{5.3})$$

$$\boxed{15.9 \text{ years} = t}$$

Ch. 4 → Special Angles

2. Solve for all values of θ in the specified domain.

$$\tan^2 \theta + \tan \theta = 0, 0 \leq \theta \leq 2\pi \quad (\text{Radians})$$

$$\tan \theta (\tan \theta + 1) = 0$$

$$\tan \theta = 0 \quad (\text{Unit Circle})$$

$$\theta = 0, \pi, 2\pi$$

$$\tan \theta + 1 = 0$$

$$\tan \theta = -1 \quad (\text{Special Triangle})$$

(i) Find $\bar{\theta}$:

$$\bar{\theta} = \tan^{-1}(-1)$$

$$\bar{\theta} = \frac{3\pi}{4}$$

(ii) where is $\tan \theta < 0$

S	A
T	C

(iii) Find θ :

Q2	Q4
$\theta = \pi - \bar{\theta}$	$\theta = 2\pi - \bar{\theta}$
$\theta = \pi - \frac{3\pi}{4}$	$\theta = 2\pi - \frac{3\pi}{4}$
$\theta = \frac{4\pi}{4} - \frac{\pi}{4} = \frac{3\pi}{4}$	$\theta = \frac{8\pi}{4} - \frac{\pi}{4} = \frac{7\pi}{4}$

$$e. \cos^2 \theta + \frac{1}{2} \cos \theta = 0, 0^\circ \leq \theta < 360^\circ \quad (\text{Degrees})$$

$$\cos \theta (\cos \theta + \frac{1}{2}) = 0$$

$$\cos \theta = 0 \quad (\text{Unit Circle})$$

$$\theta = 90^\circ, 270^\circ$$

$$\cos \theta + \frac{1}{2} = 0$$

$$\cos \theta = -\frac{1}{2} \quad (\text{Special Triangles})$$

(i) Find $\bar{\theta}$:

$$\bar{\theta} = \cos^{-1}(-\frac{1}{2})$$

$$\bar{\theta} = 120^\circ$$

(ii) where is $\cos \theta < 0$

S	A
T	C

(iii) Find θ :

Q2	Q3
$\theta = 180^\circ - \bar{\theta}$	$\theta = 180^\circ + \bar{\theta}$
$\theta = 180^\circ - 120^\circ = 60^\circ$	$\theta = 180^\circ + 120^\circ = 300^\circ$

Ch. 4 → Special Angles:

$$\frac{5 \tan^2\left(\frac{5\pi}{4}\right)}{6 \sin\left(\frac{5\pi}{6}\right) + 4 \sin\left(\frac{4\pi}{3}\right)}$$

$$\frac{5(1)^2}{6\left(\frac{1}{2}\right) + 4\left(-\frac{\sqrt{3}}{2}\right)}$$

$$\frac{5}{3 - 4\sqrt{3}}$$

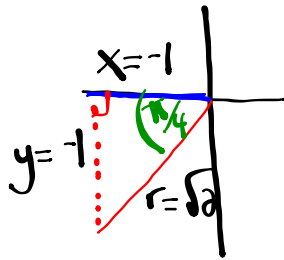
$$\frac{5}{(3 - 2\sqrt{3})(3 + 2\sqrt{3})}$$

$$\frac{15 + 10\sqrt{3}}{9 + 6\sqrt{3} - 6\sqrt{3} - 4(3)}$$

$$\frac{15 + 10\sqrt{3}}{-3}$$

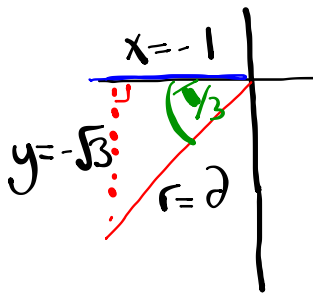
$$\boxed{\frac{-15 - 10\sqrt{3}}{3}}$$

(i) $\frac{4\pi}{4}, \frac{5\pi}{4}, \frac{6\pi}{4}$
 π



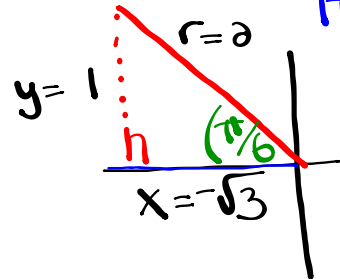
$$\tan\left(\frac{5\pi}{4}\right) = \frac{-1}{-1} = 1$$

(ii) $\frac{3\pi}{3}, \frac{4\pi}{3}, \frac{5\pi}{3}$
 π



$$\sin\left(\frac{4\pi}{3}\right) = -\frac{\sqrt{3}}{2}$$

(iii) $\frac{4\pi}{6}, \frac{5\pi}{6}, \frac{6\pi}{6}$
 π



$$\sin\left(\frac{5\pi}{6}\right) = \frac{1}{2}$$

Ch. 4 → Special Angles

$$\sec(15\pi) + \sqrt{2} \sin\left(\frac{39\pi}{4}\right) \sin\left(\frac{21\pi}{2}\right) - \csc^2\left(\frac{100\pi}{3}\right)$$

$$(-1) + \sqrt{2} \left(\frac{-1}{\sqrt{2}}\right)(1) - \left(\frac{2}{-\sqrt{3}}\right)^2$$

$$-1 - \frac{\sqrt{2}}{\sqrt{2}} - \frac{4}{3}$$

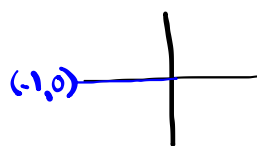
$$-1 - 1 - \frac{4}{3}$$

$$-\frac{2}{1} - \frac{4}{3}$$

$$-\frac{6}{3} - \frac{4}{3}$$

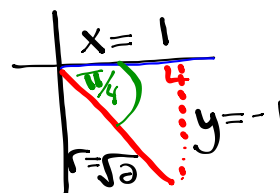
$$\left(-\frac{10}{3}\right)$$

(i) 15π



$$\sec(15\pi) = \frac{1}{-1} = -1$$

(ii) $\frac{38\pi}{4}, \frac{39\pi}{4}, \frac{40\pi}{4}$
 10π



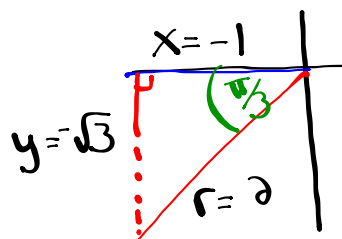
$$\sin\left(\frac{39\pi}{4}\right) = -\frac{1}{\sqrt{2}}$$

(iii) $\frac{20\pi}{2}, \frac{21\pi}{2}, \frac{22\pi}{2}$
 10π 11π



$$\sin\left(\frac{21\pi}{2}\right) = 1$$

(iv) $\frac{99\pi}{3}, \frac{100\pi}{3}, \frac{101\pi}{3}$
 33π



$$\csc\left(\frac{100\pi}{3}\right) = \frac{2}{-\sqrt{3}}$$

Ch. 5 → Trig Functions

2. A weight attached to the end of a spring is bouncing up and down. As it bounces, its distance from the floor varies sinusoidally with time. You start a stopwatch, when the watch reads 0.4 sec, the weight first reaches a high point 50 cm above the floor. The next low point, 30 cm above the floor, occurs at 1.8 sec.

$$y = \cos x \quad x = 17.2$$

(a) Predict the distance the weight will be from the floor when the stopwatch reads 17.2 sec.

$$\max = 50$$

$$\text{Amp} = 10$$

$$P = 2(1.8 - 0.4) = 2.8$$

$$\min = 30$$

$$a = \pm 10$$

$$b = \frac{360}{P} = \frac{360}{2.8} = 128.57$$

$$\sin \text{ axis} = k = \frac{30+50}{2} = 40$$

$$h = 0.4$$

$$y = 10 \cos[128.57(x - 0.4)] + 40$$

$$y = 10 \cos[128.57(17.2 - 0.4)] + 40$$

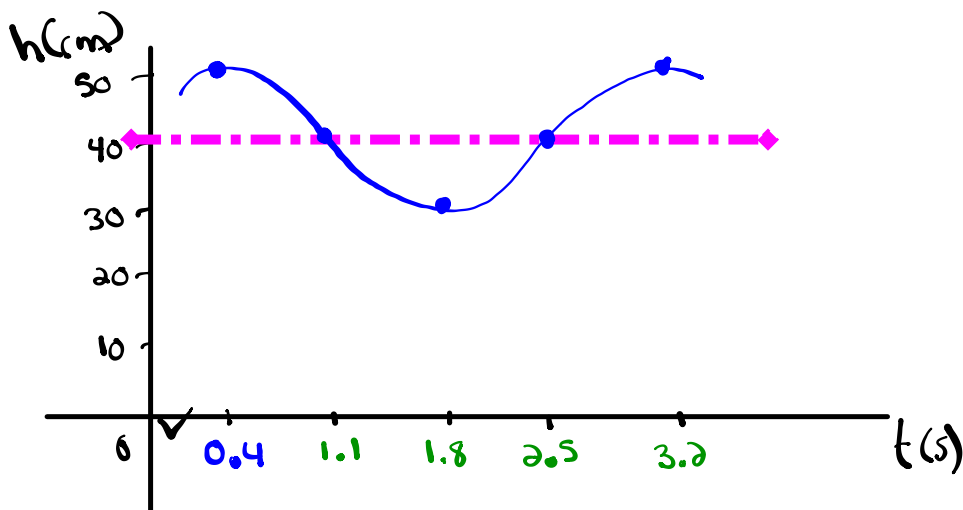
$$y = 49.99 \text{ cm}$$

(b) How high was the weight above the floor when the stopwatch was initially started?

$$(x = 0)$$

$$y = 10 \cos[128.57(0 - 0.4)] + 40$$

$$y = 46.23 \text{ cm}$$



$$\frac{P}{4} = \frac{2.8}{4} = 0.7$$

Ch. 6 → Trig Identities

$$\frac{1}{\sec^2 \theta \cot \theta} = \frac{\sin \theta - \sin^3 \theta}{\cos \theta} \quad \leftarrow \text{Factor}$$

$$\boxed{\frac{1}{\sec^2 \theta}} \cdot \boxed{\frac{1}{\cot \theta}}$$

$$\cos^2 \theta \cdot \tan \theta$$

$$\cos^2 \theta \cdot \frac{\sin \theta}{\cancel{\cos \theta}}$$

$$\boxed{\sin \theta \cos \theta}$$

$$\frac{\sin \theta \boxed{(1 - \sin^2 \theta)}}{\cos \theta}$$

$$\frac{\sin \theta \cancel{\cos^2 \theta}}{\cancel{\cos \theta}}$$

$$\boxed{\sin \theta \cos \theta}$$

Pre-Calculus 12A

Chapter 1 Exam Review

1. Given the function $y = f(x)$ write the equation of the form $y = af(b(x-h)) + k$ that would result from the following transformations:

A horizontal stretch about the y-axis by a factor of $\frac{1}{4}$ and a horizontal reflection in the y-axis. A vertical stretch about the x-axis by a factor of 3, and a translation of 5 units to the right and 2 units up.

$$a = 3$$

$$b = -4$$

$$h = 5$$

$$k = 2$$

$$y = 3f[-4(x-5)] + 2$$

2. Determine the inverse of the function $f(x) = (x-3)^2 - 2$.

$$f(x) = (x-3)^2 - 2$$

$$y = (x-3)^2 - 2$$

$$x = (y-3)^2 - 2$$

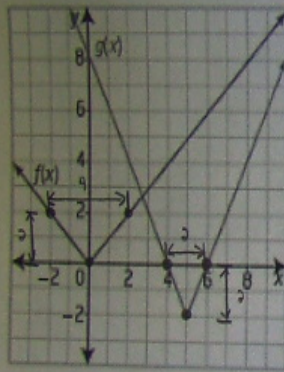
$$x+2 = (y-3)^2$$

$$\pm\sqrt{x+2} = y-3$$

$$3 \pm \sqrt{x+2} = y$$

$$y = 3 \pm \sqrt{x+2}$$

3. Write the equation for the graph of $g(x)$ as a transformation of the equation for the graph of $f(x)$.



① Reflections: none

② V.S.F. = $\frac{2}{2} = 1 \rightarrow a = 1$

③ H.S.F. = $\frac{2}{4} = \frac{1}{2} \rightarrow b = 2$

④ HT: $(0,0) \rightarrow (5, -2) \quad h = 5$

⑤ VT: $(0,0) \rightarrow (5, -2) \quad k = -2$

⑥ Equation: $g(x) = 1f[2(x-5)] - 2$

4. The key point $(12, -18)$ is on the graph of $y = f(x)$. Calculate its image point under the following transformation:

4. The key point $(12, -18)$ is on the graph of $y = f(x)$. Calculate its image point under the following transformation:

$$a) y+3 = -\frac{1}{3}f(2x+12)$$

$$y = -\frac{1}{3}f[2(x+6)] - 3$$

$$a = -\frac{1}{3} \quad b = 2 \quad h = -6 \quad k = -3$$

$$(x, y) \rightarrow \left[\frac{1}{2}x - 6, -\frac{1}{3}y - 3 \right]$$

$$(12, -18) \rightarrow \boxed{(0, 3)}$$

$$b) 2y - 4 = 6f(6x - 12) + 4$$

$$2y = 6f[6(x-2)] + 8$$

$$y = 3f[6(x-2)] + 4$$

$$a = 3 \quad b = 6 \quad h = 2 \quad k = 4$$

$$(x, y) \rightarrow \left[\frac{1}{6}x + 2, 3y + 4 \right]$$

$$(12, -18) \rightarrow \boxed{(4, -50)}$$

Radical Functions Exam Review

1. Given that $2y + 8 = -4\sqrt{-x + 3}$, complete the chart shown below. When identifying translations be sure that you indicate both the number of units and direction of the shift.

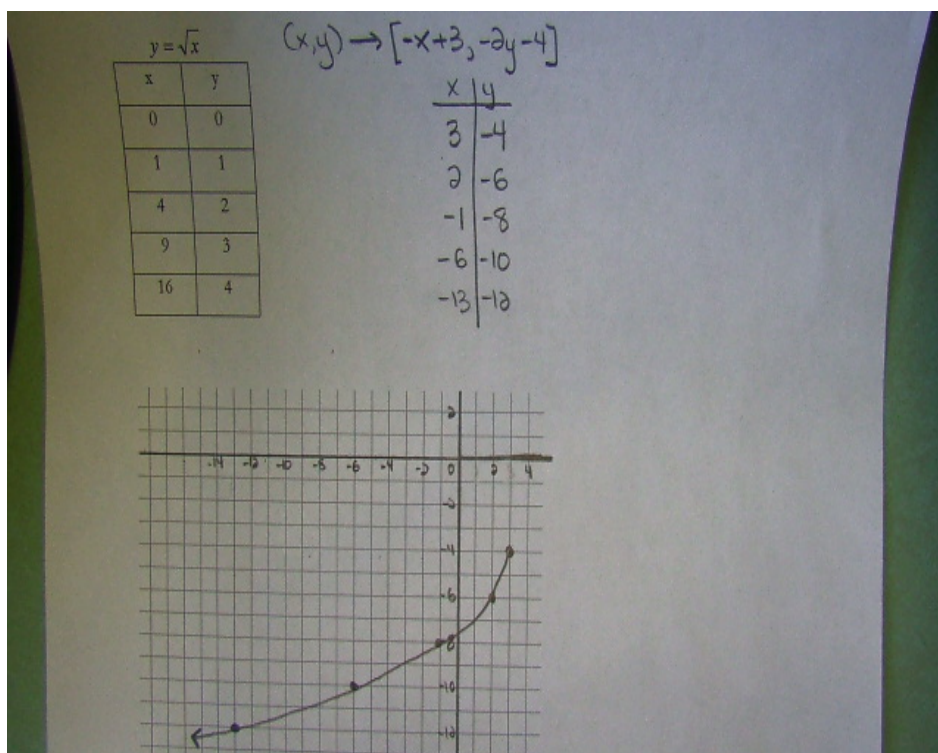
$$y = -2\sqrt{-(x-3)} - 4$$

$$\begin{aligned} a &= -2 \\ b &= -1 \\ h &= 3 \\ k &= -4 \end{aligned}$$

Reflected in x -axis	<input checked="" type="radio"/> YES or NO (circle correct solution)
Reflected in y -axis	<input checked="" type="radio"/> YES or NO (circle correct solution)
Horizontal translation of...	3 units right
Vertical translation of...	4 units down
Horizontally stretched by a factor of...	1 or (no stretch)
Vertically stretched by a factor of...	2
Domain	$\{x \mid x \leq 3, x \in \mathbb{R}\}$
Range	$\{y \mid y \leq -4, y \in \mathbb{R}\}$

Write a mapping rule and sketch the curve in the space below.

$$y = \sqrt{x} \quad (x, y) \rightarrow [-x + 3, -2y - 4]$$



2. Solve the following radical equation. $\sqrt{2x-6}+3=x$

$$\sqrt{2x-6} = x-3$$

$$2x-6 = x^2 - 6x + 9$$

$$0 = x^2 - 8x + 15$$

$$0 = (x-3)(x-5)$$

$$x-3=0 \quad | \quad x-5=0$$

$$\boxed{x=3} \quad | \quad \boxed{x=5}$$

Test $x=3$

$$\sqrt{2(3)-6} + 3 \quad | \quad 3$$

$$\sqrt{6-6} + 3$$

$$\sqrt{0} + 3$$

$$0 + 3$$

3

is a solution

Test $x=5$

$$\sqrt{2(5)-6} + 3 \quad | \quad 5$$

$$\sqrt{10-6} + 3$$

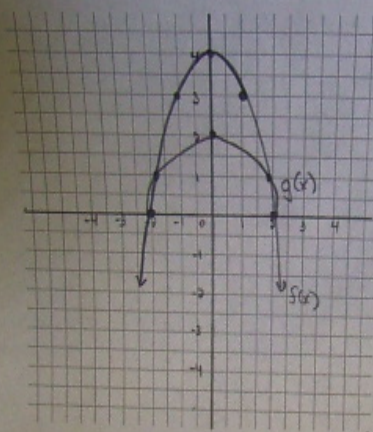
$$\sqrt{4} + 3$$

$$2 + 3$$

5

is a solution

3. Sketch the graph of $f(x) = -x^2 + 4$ on the grid provided. Then sketch the graph of $g(x) = \sqrt{f(x)}$ on the same grid. State the domain and range of each function.



$$f(x) = -x^2 + 4$$

x	y
-2	0
-1	3
0	4
1	3
2	0

$$D: \{x \mid x \in \mathbb{R}\}$$

$$R: \{y \mid y \leq 4, y \in \mathbb{R}\}$$

$$g(x) = \sqrt{-x^2 + 4}$$

x	y
-2	$\sqrt{0} = 0$
-1	$\sqrt{3} = 1.71$
0	$\sqrt{4} = 2$
1	$\sqrt{3} = 1.71$
2	$\sqrt{0} = 0$

$$D: \{x \mid -2 \leq x \leq 2, x \in \mathbb{R}\}$$

$$R: \{y \mid 0 \leq y \leq 2, y \in \mathbb{R}\}$$

Exponential Functions Exam Review

1. Given the exponential function: $\frac{3}{4}(y-1) = 6(3)^{4(x+2)} + 9$

(a) Express this function in standard form.

$$y-1 = 8(3)^{4(x+2)} + 12$$

$$y = 8(3)^{4(x+2)} + 13$$

$$a=8 \quad b=4 \quad h=-2 \quad k=13$$

(b) Complete the chart shown below.

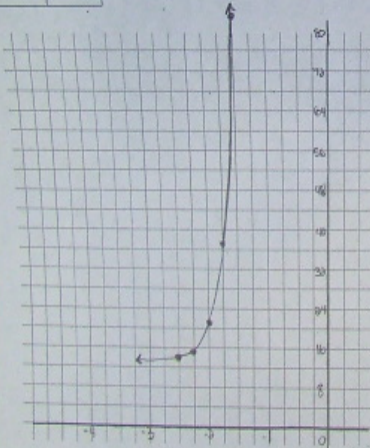
Reflected in x -axis	YES or <u>NO</u> (circle correct solution)
Reflected in y -axis	YES or <u>NO</u> (circle correct solution)
Horizontal translation of...	2 units left
Vertical translation of...	13 units up
Horizontally stretched by a factor of...	$\frac{1}{4}$
Vertically stretched by a factor of...	8
x -intercept (show work)	No x -intercept
y -intercept (show work)	$y = 52501$ or $(0, 52501)$
Horizontal Asymptote	$y = 13$
Domain	$\{x x \in \mathbb{R}\}$
Range	$\{y y > 13, y \in \mathbb{R}\}$

Write a mapping rule and sketch the curve in the space below.

$y = 3^x$ $(x, y) \rightarrow [\frac{1}{4}x - 2, 8y + 13]$

x	y
-2	$\frac{1}{9}$
-1	$\frac{1}{3}$
0	1
1	3
2	9

x	y
$(-2, \frac{1}{9}) \rightarrow (-\frac{5}{2}, \frac{105}{9})$ (13.8)	
$(-1, \frac{1}{3}) \rightarrow (-\frac{7}{4}, \frac{47}{3})$ (15.6)	
$(0, 1) \rightarrow (-2, 21)$	
$(1, 3) \rightarrow (-\frac{7}{4}, 37)$	
$(2, 9) \rightarrow (-\frac{3}{2}, 85)$	



x-intercept (y=0)

$$0 = 8(3)^{\frac{1}{4}(x-2)} + 13$$

$$\frac{-13}{8} = \frac{8(3)^{\frac{1}{4}(x-2)}}{8}$$

$$-1.625 = (3)^{\frac{1}{4}(x-2)}$$

Not Possible

y-intercept (x=0)

$$y = 8(3)^{\frac{1}{4}(0-2)} + 13$$

$$y = 8(3)^{-\frac{1}{2}} + 13$$

$$y = 8(6\sqrt{6}) + 13$$

$$y = 50488 + 13$$

$$y = 50501$$

2. Radioactive carbon-14 has a half-life of 5750 years. When an organism dies, the amount of C-14 present decays exponentially. By measuring the radioactivity of the remains of a fossilized organism and comparing it with the radioactivity of a living organism, archaeologists can approximate the age of the artifact. An antique dealer was selling a piece of wood purported to come from a chariot used by Caesar in ancient Rome. Archaeologists found the wood to contain 0.4 mg of C-14, compared with the 0.68 mg found in a new piece of wood. Caesar was assassinated 2048 years ago, could this dealer's claim possibly be true? Provide mathematical proof to back up your claim!

Given:

Base = 0.5

Initial Amount (A_0) = 0.68

Final Amount (A_f) = 0.4

Exponent = $\frac{x}{5750}$

$x = 2048$

$$\text{Equation: } y = 0.68(0.5)^{\frac{x}{5750}}$$

$$y = 0.68(0.5)^{\frac{2048}{5750}}$$

$$y = 0.68(0.78)$$

$$\boxed{y = 0.53 \text{ mg}}$$

$$\text{or } 0.4 = 0.68(0.5)^{\frac{x}{5750}}$$

$$0.5882 = (0.5)^{\frac{x}{5750}}$$

$$(0.5)^{0.7665} = (0.5)^{\frac{x}{5750}}$$

$$0.7665 = \frac{x}{5750}$$

$$\boxed{x = 4409 \text{ years}}$$

3. Solve each of the following:

3. Solve each of the following:

$$a) 64^{x-3} = (16)^{x-1} \left(\frac{1}{4}\right)^{2x}$$

$$(4^3)^{x-3} = (4^2)^{x-1} (4^{-1})^{2x}$$

$$4^{3x-9} = 4^{2x-2} \cdot 4^{-2x}$$

$$4^{3x-9} = 4^{-2}$$

$$3x-9 = -2$$

$$3x = 7$$

$$\boxed{x = \frac{7}{3}}$$

5750

$$\boxed{x = 4402 \text{ years}}$$

$$b) \left(\frac{1}{27}\right)^{x+2} = (3)^{2x-1} (81)^x$$

$$(3^{-3})^{x+2} = (3)^{2x-1} (3^4)^x$$

$$3^{-3x-6} = 3^{2x-1} \cdot 3^{4x}$$

$$3^{-3x-6} = 3^{6x-1}$$

$$-3x-6 = 6x-1$$

$$-5 = 9x$$

$$\boxed{-\frac{5}{9} = x}$$

1. Express the following as a single logarithm in simplest form:

$$8 \log_3 \sqrt{x} - \frac{2}{3} \left[9 \log_3 x^{-2} + 6 \left(\log_3 x^4 - \frac{3}{4} \log_3 \sqrt{x} \right) \right]$$

$$8 \log_3 x^{\frac{1}{2}} - \frac{2}{3} \left[9 \log_3 x^{-2} + 6 \log_3 x^4 - \frac{18}{4} \log_3 x^{\frac{1}{2}} \right]$$

$$8 \log_3 x^{\frac{1}{2}} - 6 \log_3 x^{-2} - 4 \log_3 x^4 + 3 \log_3 x^{\frac{1}{2}}$$

$$\log_3 x^4 - \log_3 x^{-12} - \log_3 x^6 + \log_3 x^{\frac{3}{2}}$$

$$\log_3 \left(\frac{x^4 \cdot x^{\frac{3}{2}}}{x^{-12} \cdot x^6} \right)$$

$$\log_3 \left(\frac{x^{\frac{11}{2}} \cdot x^{\frac{3}{2}}}{x^{-6}} \right)$$

$$\log_3 x^{\frac{3}{5}} \rightarrow \boxed{\frac{3}{5} \log_3 x}$$

2. Given that $\log_r x = -6$, $\log_r y = -3$, and $\log_r z = 8$, evaluate the expression $\log_r \left(\frac{x^2 z}{r^3 y} \right)$.

$$\log_r x^2 + \log_r z^1 - \log_r r^3 - \log_r y^1$$

$$5 \log_r x + 3 \log_r z + 3 \log_r r - 5 \log_r y$$

$$5(-6) + 3(8) + 3(1) - 5(-3)$$

$$-30 + 24 + 3 + 15$$

12

3. Solve for x in the following equations...

$$\log_3(2x^2 - x) - \log_3(x+2) = 1$$

$$\log_3\left(\frac{2x^2 - x}{x+2}\right) = 1$$

$$3^1 = \frac{2x^2 - x}{x+2}$$

$$3(x+2) = 2x^2 - x$$

$$0 = 2x^2 - 4x - 6$$

$$0 = 2(x^2 - 2x - 3)$$

$$0 = 2(x-3)(x+1)$$

$$x-3=0 \quad | \quad x+1=0$$

$$\boxed{x=3} \quad | \quad \boxed{x=-1}$$

Test $x=3$

$$\log_3(15) - \log_3(5) = 1$$

$$\log_3\left(\frac{15}{5}\right)$$

$$\log_3 3$$

$$1$$

$x=3$ is a solution

Test $x=-1$

$$\log_3(3) - \log_3(1) = 1$$

$$1 - 0$$

$$1$$

$x=-1$ is a solution

4. Given the logarithmic function: $y - 6 = 2 \log_3(3x + 15)$

(a) Express this function in standard form.

$$y = 2 \log_3 [3(x+5)] + 6$$

$$a=2 \quad b=3 \quad h=-5 \quad k=6$$

(b) Complete the chart shown below.

Reflected in x -axis	YES or <u>NO</u> (circle correct solution)
Reflected in y -axis	YES or <u>NO</u> (circle correct solution)
Horizontal translation of...	5 units left
Vertical translation of...	6 units up
Horizontally stretched by a factor of...	$\frac{1}{3}$
Vertically stretched by a factor of ...	2
x -intercept (show work)	$x = \frac{-15.74}{3.28} = -4.997$ or $(-4.997, 0)$
y -intercept (show work)	$y = 9.37$ or $(0, 9.37)$
Horizontal Asymptote	$x = -5$
Domain	$\{x x > -5, x \in \mathbb{R}\}$
Range	$\{y y \in \mathbb{R}\}$

Write a mapping rule and sketch the curve in the space below.

$y = \log_5 x$

x	y
$\frac{1}{25}$	-2
$\frac{1}{5}$	-1
1	0
5	1
25	2

$(x, y) \rightarrow \left[\frac{1}{3}x - 5, 2y + 6\right]$

x	y
$(-4, 2) = -\frac{374}{5}$	2
$(-4, 1) = -\frac{74}{5}$	4
$(-4, 0) = -14/3$	6
$(-3, 2) = -\frac{19}{3}$	8
$(-3, 1) = \frac{10}{3}$	10

x-intercept (y=0)

$$0 = 2 \log_5 [3(x+5)] + 6$$

$$-6 = 2 \log_5 [3(x+5)]$$

$$-3 = \log_5 [3(x+5)]$$

$$5^{-3} = 3(x+5)$$

$$\frac{1}{125} = 3 \left(\frac{x+5}{3}\right)$$

$$\frac{1}{375} = x + 5$$

$$\frac{-1874}{375} = x$$

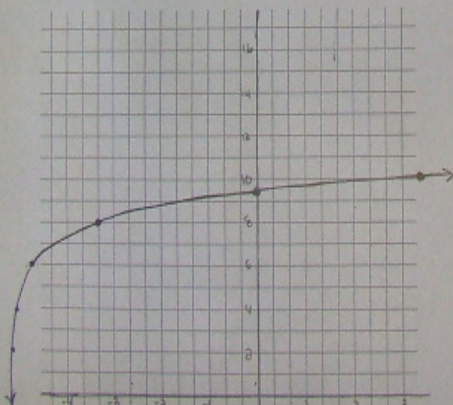
y-intercept (x=0)

$$y = 2 \log_5 [3(0+5)] + 6$$

$$y = 2 \log_5 (15) + 6$$

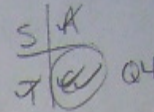
$$y = 2(1.683) + 6$$

$$y = 3.365 + 6$$

$$y = 9.37$$


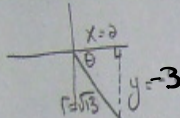
Pre-Calculus 12A
Special Angles Exam Review

$\cos \theta > 0$ and $\sin \theta < 0$



1. Given that $\cos \theta = \frac{2}{\sqrt{13}}$ and $\sin \theta < 0$. Sketch the angle and determine the five remaining trig ratios as radicals in simplest form.

Given:
 $x=2$
 $r=\sqrt{13}$



$$\begin{aligned} x^2 + y^2 &= r^2 \\ 2^2 + y^2 &= (\sqrt{13})^2 \\ 4 + y^2 &= 13 \\ y^2 &= 9 \\ y &= \pm 3 \\ y &= -3 \end{aligned}$$

$$\sin \theta = \frac{-3}{\sqrt{13}} = -\frac{3\sqrt{13}}{13}$$

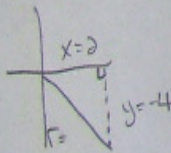
$$\tan \theta = -\frac{3}{2}$$

$$\sec \theta = \frac{\sqrt{13}}{2}$$

$$\csc \theta = -\frac{\sqrt{13}}{3}$$

$$\cot \theta = -\frac{2}{3}$$

2. The point (2, -4) lies on the terminal arm of an angle. Make a sketch of this angle and determine the 6 trigonometric ratios expressed as radicals in simplest form.



$$\begin{aligned} x^2 + y^2 &= r^2 \\ 2^2 + (-4)^2 &= r^2 \\ 4 + 16 &= r^2 \\ 20 &= r^2 \\ \sqrt{20} &= r \\ 2\sqrt{5} &= r \end{aligned}$$

$$\sin \theta = \frac{-4}{2\sqrt{5}} = -\frac{2}{\sqrt{5}} = -\frac{2\sqrt{5}}{5}$$

$$\cos \theta = \frac{2}{2\sqrt{5}} = \frac{1}{\sqrt{5}} = \frac{\sqrt{5}}{5}$$

$$\tan \theta = \frac{-4}{2} = -2$$

$$\csc \theta = -\frac{2\sqrt{5}}{4} = -\frac{\sqrt{5}}{2}$$

$$\sec \theta = \frac{2\sqrt{5}}{2} = \sqrt{5}$$

$$\cot \theta = \frac{2}{-4} = -\frac{1}{2}$$

$\frac{-49\pi}{4} \rightarrow \frac{7\pi}{4}$
 $\frac{-59\pi}{2} \rightarrow \frac{2\pi}{3}$

$5 \csc\left(\frac{43\pi}{6}\right) \sec^2\left(\frac{-49\pi}{4}\right)$
 $3 \tan\left(\frac{-22\pi}{3}\right) - 2 \sin\left(\frac{71\pi}{2}\right)$

$\frac{5(-2) \cdot (\sqrt{3})^2}{3(-\sqrt{3}) - 2(-1)} \rightarrow \frac{-10 \cdot 3}{-3\sqrt{3} + 2} \rightarrow \frac{-30}{2 - 3\sqrt{3}}$
 $\frac{-30}{2 - 3\sqrt{3}} \cdot \frac{2 + 3\sqrt{3}}{2 + 3\sqrt{3}} \rightarrow \frac{-40 - 60\sqrt{3}}{4 - 9(3)}$
 $\rightarrow \frac{-40 - 60\sqrt{3}}{4 - 27} \rightarrow \frac{-40 - 60\sqrt{3}}{-23} \rightarrow \boxed{\frac{40 + 60\sqrt{3}}{23}}$

Solve the following trigonometric equation:
 $5 \sin^2 \theta - 13 \sin \theta + 6 = 0, \quad -360^\circ \leq \theta \leq 720^\circ$
 $5 \sin^2 \theta - 10 \sin \theta + 6 = 0$
 $(\sin \theta - \frac{10}{5})(\sin \theta - \frac{6}{5}) = 0$
 $(\sin \theta - 2)(5 \sin \theta - 6) = 0$
 $\sin \theta - 2 = 0$
 $\sin \theta = 2$
 Not possible

$5 \sin \theta - 6 = 0$
 $\sin \theta = \frac{6}{5}$ (approximate value)
 $\theta = \sin^{-1}(\frac{6}{5})$ (i) where is $\sin \theta > 0$ $\frac{\pi}{2} < \theta < \pi$
 $\theta = 37^\circ$

(ii)

Q1	Q2
$\theta = 0$	$\theta = 180^\circ - \theta$
$\theta = 37^\circ$	$\theta = 143^\circ$
$\theta = 37^\circ + 360^\circ = 397^\circ$	$\theta = 143^\circ + 360^\circ = 503^\circ$
$\theta = 37^\circ - 360^\circ = -323^\circ$	$\theta = 143^\circ - 360^\circ = -217^\circ$

Sinusoidal Functions Exam Review

1 rev. in 24 sec

radius = 15

1. A Ferris wheel completes 2 revolutions in 48 seconds and has a diameter of 30 m. If the bottom of the wheel is 2 m above the ground. When a stopwatch is started you notice that your friend is seated at the middle of the wheel and is going up.

$$y = \sin x$$

(a) Find the following:

a: ± 15

P: 24

b: $\frac{360}{24} = 15$

min height: 2m

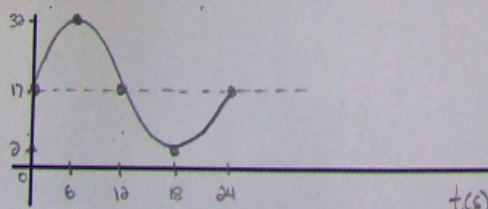
max height: 32m
(min + diameter)

k: 17m
(min + radius)

(b) What is the equation of the graph?

$$y = 15 \sin [15(x)] + 17$$

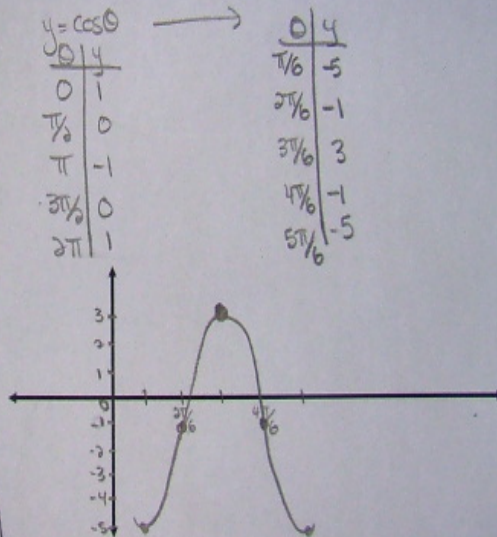
(c) Sketch the graph for one period.



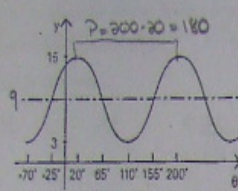
2. Graph the following Sinusoidal Function: (One Period) $\frac{\pi}{3} \div 3 = \frac{\pi}{3} \times \frac{1}{3} = \frac{\pi}{6}$

$-3(y+2) = 12 \cos\left(3\theta - \frac{\pi}{2}\right) - 3$ $y+2 = -4 \cos\left[3\left(\theta - \frac{\pi}{6}\right)\right] + 1$ $a = -4$ $b = 3$ $h = \frac{\pi}{6}$ $k = -1$
 $y = -4 \cos\left[3\left(\theta - \frac{\pi}{6}\right)\right] - 1$

DOMAIN	$\{\theta \mid \theta \in \mathbb{R}\}$
RANGE	$\{y \mid -5 \leq y \leq 3, y \in \mathbb{R}\}$
AMPLITUDE	4
PERIOD	$\frac{2\pi}{3}$
PHASE SHIFT	$\frac{\pi}{6}$ right
VERTICAL TRANSLATION	1 down
EQUATION OF SINUSOIDAL AXIS	$y = -1$
MAPPING NOTATION	$(x, y) \rightarrow \left[\frac{1}{3}\theta + \frac{\pi}{6}, -4y - 1\right]$



3. Find a **positive** sine and a **positive** cosine equation from the graph.



$$a = \pm 6$$

$$P = 180^\circ$$

$$b = \frac{360^\circ}{180^\circ} = 2$$

$$k = 9$$

$$(1) y = \sin \theta (h = 20^\circ)$$

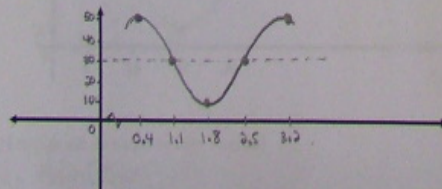
$$y = 6 \sin [2(\theta - 20^\circ)] + 9$$

$$(2) y = \cos \theta (h = 20^\circ)$$

$$y = 6 \cos [2(\theta - 20^\circ)] + 9$$

4. A weight attached to the end of a long spring is bouncing up and down. As it bounces, its distance from the floor varies sinusoidally with time. You start a stopwatch. When the stopwatch reads 0.4 seconds, the weight first reaches a high point 50 cm above the floor. The next low point, 30 cm above the floor, occurs at 1.8 seconds.

(a) Sketch a graph of this sinusoidal function



$$h = 0.4$$

$$\text{max} = 50 \text{ cm}$$

$$\text{min} = 30 \text{ cm}$$

$$K = \frac{50 + 30}{2} = \frac{80}{2} = 40$$

$$a = \pm 10$$

$$P = 2(1.8 - 0.4) = 2.8$$

$$b = \frac{360}{2.8} = 128.57$$

(b) Write an equation to define the graph.

$$y = 10 \cos [128.57(x - 0.4)] + 40$$

y =

(c) What was the distance from the floor when you started the stopwatch? ($x=0$)

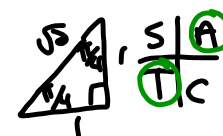
$$y = 10 \cos [128.57(0 - 0.4)] + 40$$

$$y = 46.2 \text{ cm}$$

Extra questions worked out

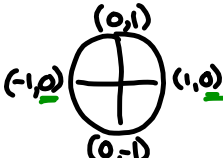
$0 \leq \theta \leq 2\pi$
Common factor

② a) $\sin \theta = \sin \theta \tan \theta$
 $0 = \sin \theta \tan \theta - \sin \theta$
 $0 = (\sin \theta)(\tan \theta - 1)$

$\sin \theta = 0$ | $\tan \theta - 1 = 0$
 $\theta = 0, \pi, 2\pi$ | $\tan \theta = 1$ 
 $\theta_R = \frac{\pi}{4}$

Where is $\tan \theta$ positive

Q1	Q3
$\theta = \theta_R$	$\theta = \pi + \theta_R$
$\theta = \frac{\pi}{4}$	$\theta = \pi + \frac{\pi}{4}$
	$\theta = \frac{5\pi}{4}$



Solutions are: $0, \frac{\pi}{4}, \pi, \frac{5\pi}{4}, 2\pi$

③ b) $3 \sin^2 \theta - 2 \sin \theta - 1 = 0$, $0 \leq \theta \leq 360^\circ$
 $(3 \sin^2 \theta - 3 \sin \theta + \sin \theta - 1) = 0$ $\frac{-3 \times 1}{-3} = -3$
 $3 \sin \theta (\sin \theta - 1) + 1(\sin \theta - 1) = 0$ $\frac{-3}{-3} + \frac{1}{1} = -2$
 $(3 \sin \theta + 1)(\sin \theta - 1) = 0$

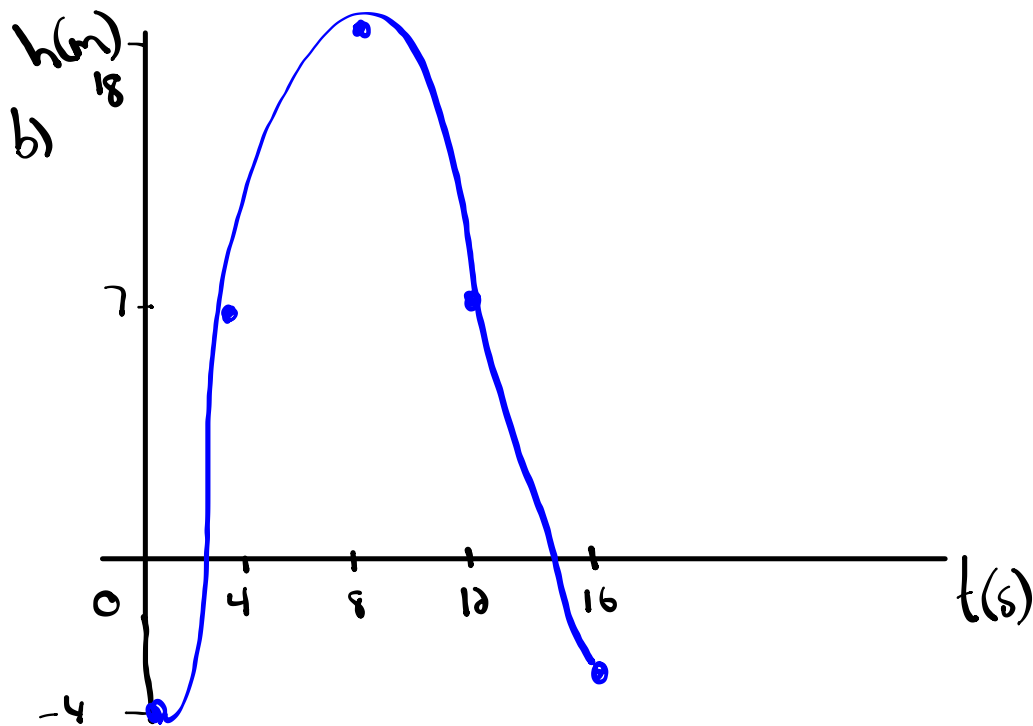
$3 \sin \theta + 1 = 0$ | $\sin \theta - 1 = 0$
 $\sin \theta = -\frac{1}{3}$ | $\sin \theta = 1$ \rightarrow Unit Circle
 $\theta_R = \sin^{-1}(\frac{1}{3})$ | $\theta = 90^\circ$
 $\theta_R = 19$

Where is sine negative: $\frac{S}{T/C}$

Q3	Q4
$\theta = 180^\circ + \theta_R$	$\theta = 360^\circ - \theta_R$
$\theta = 180^\circ + 19$	$\theta = 360^\circ - 19$
$\theta = 199$	$\theta = 340$

$$\begin{aligned} \textcircled{1} \quad \text{Amp} &= 11 & P &= 16 & \text{min} &= -4 \\ a &= \pm 11 & b &= \frac{360}{16} = 22.5 & \text{max} &= -4 + 22 = 18 \\ & & & & K &= -4 + 11 = 7 \\ & & & & h &= 0 \end{aligned}$$

a) equation: $y = -11\cos[22.5(x)] + 7$



④ $\max = 68$

$\min = 24$

$$k = \frac{68 + 24}{2} = 46$$

$\text{Amp} = 68 - 46 = 22$

$a = \pm 22$

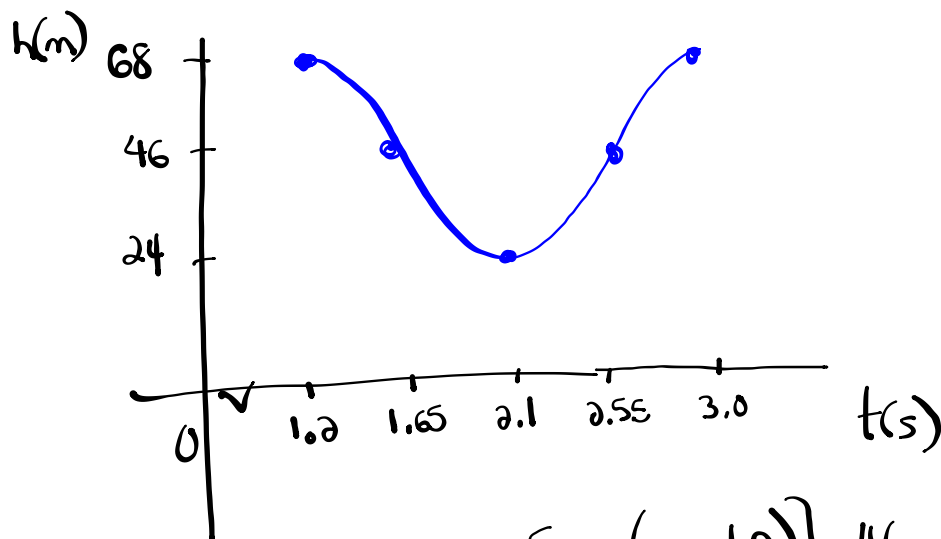
$P = 2(2.1 - 1.2)$

$P = 1.8$

$$b = \frac{360}{1.8} = 200$$

$$h = \underline{1.2}$$

$$\frac{P}{4} = \frac{1.8}{4} = 0.45$$



$$y = 22 \cos[200(x - 1.2)] + 46$$

$$\textcircled{5} \text{ c) } y = \frac{1}{2} \cos(\theta + \underline{\pi}) - \underline{4}$$

$$a = \frac{1}{2}$$

$$(x, y) \rightarrow \left(\frac{1}{2}x - \pi, \frac{1}{2}y - 4 \right)$$

$$b = 1$$

$$P = \frac{2\pi}{b} = \frac{2\pi}{1} = 2\pi$$

$$c = -\pi$$

$$d = -4$$

$$y = \cos \theta$$

x	y
0	1
$\frac{\pi}{2}$	0
π	-1
$\frac{3\pi}{2}$	0
2π	1

$$\rightarrow$$

x	y
$-\pi$	$-\frac{1}{2}$ -3.5
$-\frac{\pi}{2}$	-4 -4
0	$-\frac{1}{2}$ -4.5
$\frac{\pi}{2}$	-4 -4
π	$-\frac{1}{2}$ -3.5

