The Sine Law

Curriculum Outcomes:

GCO: Geometry (G): Develop Spatial Sense

SCO G3: Solve problems that involve the cosine law and the sine law, including the ambiguous case. [CN, PS, R]

I Can Statements:

I can~ draw a diagram to represent a problem that involves cosine law or sine law.

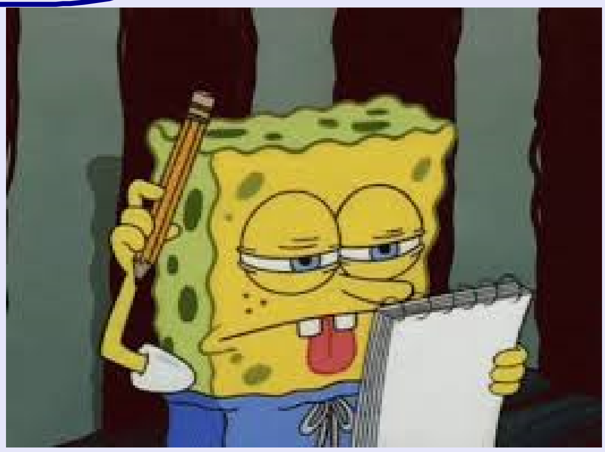
I can~ explain the steps in a given proof of the sine law or cosine law.

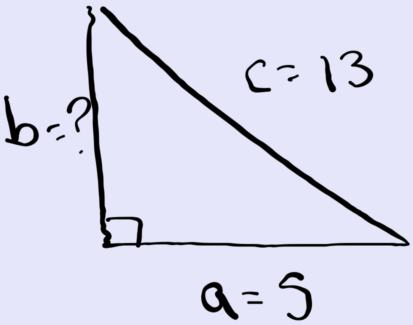
I can~ solve a problem involving the sine law that requires the proper formula.

I can~ solve a contextual problem that involves the cosine law or the sine law.

Since you all know how to find missing angles and side lengths in a right triangle...

what happens if it's not a right triangle!?





hyp

$$a^2 + b^2 = c^2$$

$$(5)^2 + b^2 = (13)^2$$

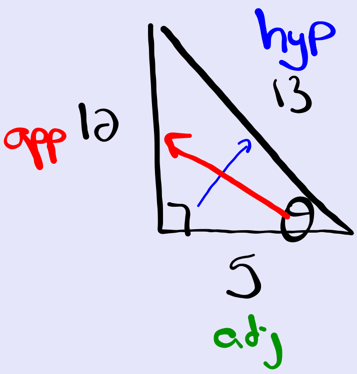
$$25 + b^2 = 169$$

$$b^2 = 169 - 25$$

$$b^2 = 144$$

$$b = \pm 12$$

$$b = 12$$



$$\sin\theta = \frac{o}{h}$$

$$\cos\theta = \frac{a}{h}$$

$$\tan\theta = \frac{o}{a}$$

$$\sin\theta = \frac{12}{13}$$

$$\sin\theta = 0.9231$$

$$\theta = \sin^{-1}(0.9231)$$

$$\theta = 67.38^\circ$$

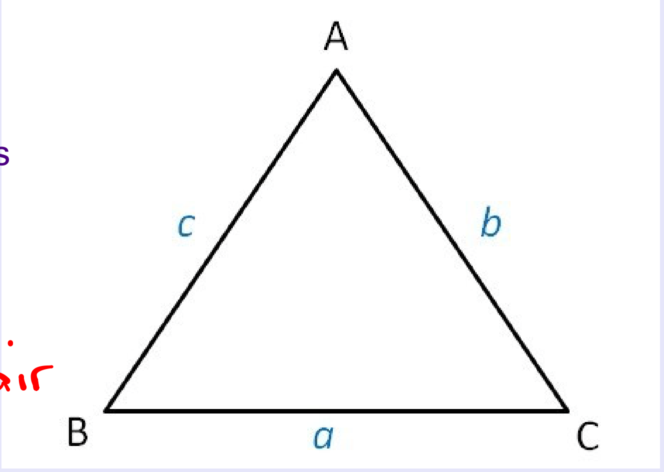
The Sine Law

Option 1: Missing Side

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

Option 2: Missing Angles

$$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$$



**Capital letters represent Angles while the lower case letters represent the Sides.

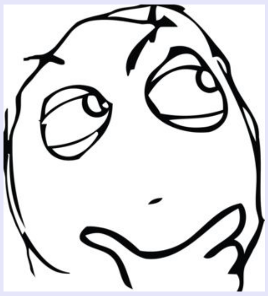
* you need a pair

a and A

b and B

c and C

When will you use the law of Sine?



<u>You will use it when:</u>

1) You are given two angles and a non-included side (AAS)

2) You are given two angles and an included side (ASA)

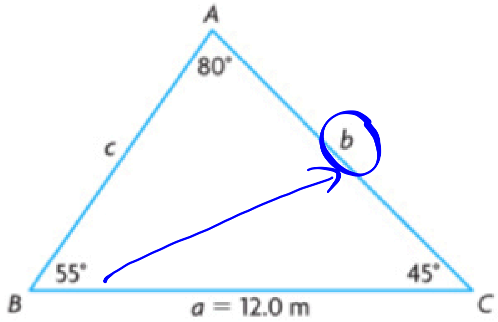
3) You are given two sides and an angle opposite to on the them (SSA)

You need to have 2 "matching pairs"

EXAMPLE 1 | Using reasoning to determine the length of a side

A triangle has angles measuring 80° and 55°. The side opposite the 80° angle is 12.0 m in length. Determine the length of the side opposite the 55° angle to the nearest tenth of a metre.

Elizabeth's Solution



Given:

$a = 12.0\text{m}$

$A = 80°$

$b = ?$

$B = 55°$

$$\frac{a}{\sin A} = \frac{b}{\sin B}$$

$$\frac{12}{\sin 80°} = \frac{b}{\sin 55°}$$

$$\frac{b\cancel{\sin 80°}}{\cancel{\sin 80°}} = \frac{12\sin 55°}{\sin 80°}$$

$$b = \frac{12\sin 55°}{\sin 80°}$$

$$b = \frac{12(0.8192)}{(0.9848)}$$

$b = 9.98\text{ m}$

$b = 10\text{m}$

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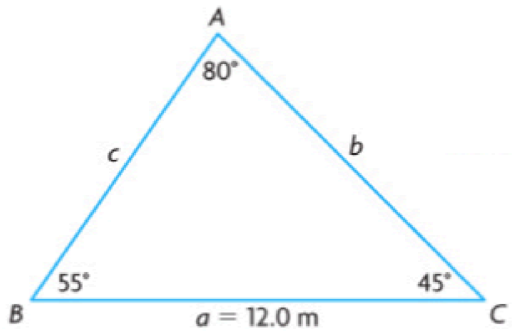
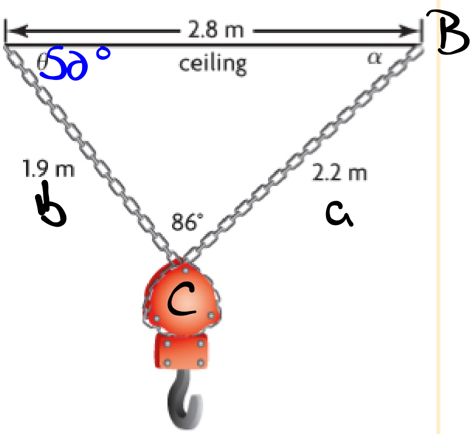


EXAMPLE 2 Solving a problem using the sine law

Toby uses chains attached to hooks on the ceiling and a winch to lift engines at his father's garage. The chains, the winch, and the ceiling are arranged as shown. Toby solved the triangle using the sine law to determine the angle that each chain makes with the ceiling to the nearest degree. He claims that $\theta = 40°$ and $\alpha = 54°$. Is he correct? Explain, and make any necessary corrections.



Given:

$C = 86°$

$c = 2.8$

$A = ?$

$a = 2.2$

$$\frac{\sin A}{a} = \frac{\sin C}{c}$$

$$\frac{\sin A}{2.2} = \frac{\sin 86°}{2.8}$$

$$\frac{\cancel{2.8}\sin A}{\cancel{2.8}} = \frac{2.2\sin 86°}{2.8}$$

$$\sin A = 0.7838$$

$$A = \sin^{-1}(0.7838)$$

$$A = 51.6°$$

$$A = 52°$$

Find $\angle B$

$$A + B + C = 180°$$

$$52° + B + 86° = 180$$

$$B + 138° = 180°$$

$$B = 180° - 138°$$

$$B = 42°$$

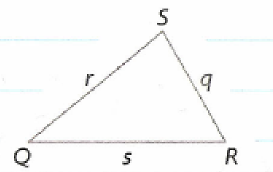
Assignment: pgs. 124 - 127
1, 2, 3, 4, 5, 7, 15

For tomorrow....

we are solving a MURDER........

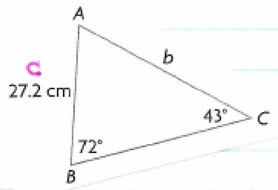
SOLUTIONS ⇒ Proving and Applying the Sine Law

1. Write three equivalent ratios using the sides and angles in the triangle at the right.



$$\frac{q}{\sin Q} = \frac{r}{\sin R} = \frac{s}{\sin S}$$

2a) Determine length b to the nearest tenth of a centimetre.



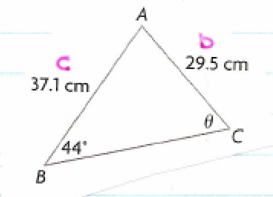
$$\frac{b}{\sin B} = \frac{c}{\sin C}$$

$$\frac{b}{\sin 72^\circ} = \frac{27.2}{\sin 43^\circ}$$

$$\frac{b\cancel{\sin 43^\circ}}{\cancel{\sin 43^\circ}} = \frac{(27.2)(\sin 72^\circ)}{\sin 43^\circ}$$

$$b = 37.9\text{ cm}$$

b) Determine the measure of θ to the nearest degree.



$$\frac{\sin C}{c} = \frac{\sin B}{b}$$

$$\frac{\sin C}{37.1} = \frac{\sin 44^\circ}{29.5}$$

$$\frac{\cancel{29.5}\sin C}{\cancel{29.5}} = \frac{(37.1)(\sin 44^\circ)}{29.5}$$

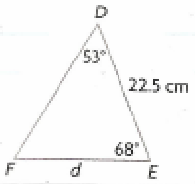
$$\sin C = 0.8736$$

$$C = \sin^{-1}(0.8736)$$

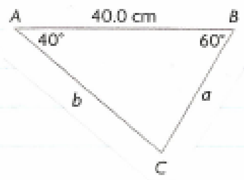
$$C = 61^\circ$$

3. Determine the indicated side lengths to the nearest tenth of a unit and the indicated angle measures to the nearest degree.

a)



b)



Need to find <F first:

$F = 180° - 68° - 53°$

$F = 59°$

$$\frac{d}{\sin D} = \frac{f}{\sin F}$$

$$\frac{d}{\sin 53°} = \frac{22.5}{\sin 59°}$$

$$\frac{d \cancel{\sin 59°}}{\cancel{\sin 59°}} = \frac{22.5 \sin 53°}{\sin 59°}$$

$d = 21.0$ cm

Need to find <C first:

$C = 180° - 40° - 60°$

$C = 80°$

$$\frac{a}{\sin A} = \frac{c}{\sin C}$$

$$\frac{a}{\sin 40°} = \frac{40.0}{\sin 80°}$$

$$\frac{a \cancel{\sin 80°}}{\cancel{\sin 80°}} = \frac{40.0 \sin 40°}{\sin 80°}$$

$a = 26.1$

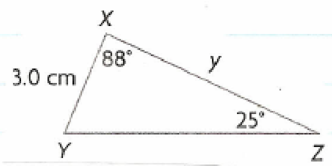
$$\frac{b}{\sin B} = \frac{c}{\sin C}$$

$$\frac{b}{\sin 60°} = \frac{40.0}{\sin 80°}$$

$$\frac{b \cancel{\sin 80°}}{\cancel{\sin 80°}} = \frac{40.0 \sin 60°}{\sin 80°}$$

$b = 35.2$

c)



Need to find <Y first:

$Y = 180° - 88° - 25°$

$Y = 67°$

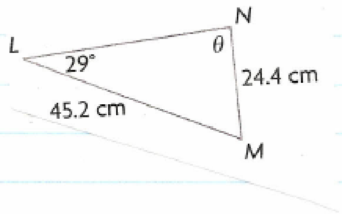
$$\frac{y}{\sin Y} = \frac{z}{\sin Z}$$

$$\frac{y}{\sin 67°} = \frac{3.0}{\sin 25°}$$

$$\frac{y \cancel{\sin 25°}}{\cancel{\sin 25°}} = \frac{3.0 \sin 67°}{\sin 25°}$$

$y = 6.5\text{cm}$

d)



$$\frac{\sin N}{n} = \frac{\sin L}{l}$$

$$\frac{\sin \theta}{45.2} = \frac{\sin 29°}{24.4\text{cm}}$$

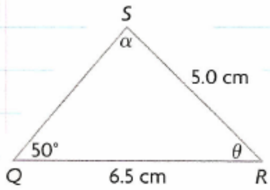
$$\frac{\cancel{24.4} \sin \theta}{\cancel{24.4}} = \frac{45.2 \sin 29°}{24.4}$$

$\sin \theta = 0.8981$

$\theta = \sin^{-1}(0.8981)$

$\theta = 64°$

e)



$$\frac{\sin S}{s} = \frac{\sin Q}{q}$$

$$\frac{\sin S}{6.5} = \frac{\sin 50°}{5.0}$$

$$\frac{\cancel{5.0} \sin S}{\cancel{5.0}} = \frac{6.5 \sin 50°}{5.0}$$

$$\sin S = 0.9959$$

$$S = \sin^{-1}(0.9959)$$

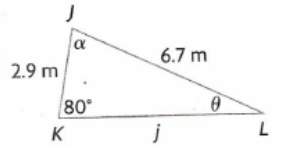
$$S = 85°$$

To find $\angle R$:

$$\theta = 180° - 50° - 85°$$

$$= 45°$$

f)



$$\frac{\sin L}{l} = \frac{\sin K}{k}$$

$$\frac{\sin L}{2.9} = \frac{\sin 80^\circ}{6.7}$$

$$\frac{\cancel{6.7}\sin L}{\cancel{6.7}} = \frac{2.9\sin 80^\circ}{6.7}$$

$$\sin L = 0.4263$$

$$L = \sin^{-1}(0.4263)$$

$$L = 25^\circ$$

To find $\angle J$:

$$J = 180^\circ - 80^\circ - 25^\circ$$

$$J = 75^\circ$$

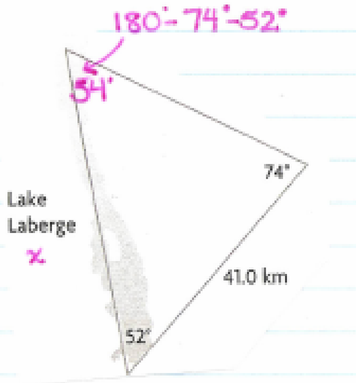
$$\frac{j}{\sin J} = \frac{k}{\sin K}$$

$$\frac{j}{\sin 75^\circ} = \frac{6.7}{\sin 80^\circ}$$

$$\frac{j\,\cancel{\sin 80^\circ}}{\cancel{\sin 80^\circ}} = \frac{6.7\sin 75^\circ}{\sin 80^\circ}$$

$$j = 6.6\text{ m}$$

4. Scott is studying the effects of environmental changes on fish populations in his summer job. As part of his research, he needs to know the distance between two points on Lake Laberge, Yukon. Scott makes the measurements shown and uses the sine law to determine the lake's length as 36.0 km.



a) Agathe, Scott's research partner, says that his answer is incorrect. Explain how she knows.

The length of the lake is opposite the largest angle of the triangle, therefore it must be the longest side. A length of 36 km would not make it the longest side.

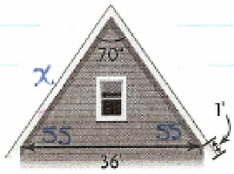
b) Determine the distance between the two points to the nearest tenth of a kilometer.

$$\frac{x}{\sin 74°} = \frac{41.0}{\sin 54°}$$

$$\frac{x \sin 54°}{\cancel{\sin 54°}} = \frac{41.0 \sin 74°}{\sin 54°}$$

$$x = 48.7 \text{ km}$$

5. An architect designed a house and must give more instructions to the builders. The rafters that hold up the roof are equal in length. The rafters extend beyond the supporting wall as shown. How long are the rafters? Express your answer to the nearest inch.



Since the roof is an isosceles triangle, the remaining angles would be:

$$\frac{180° - 70°}{2} = \frac{110°}{2} = 55°$$

$$\frac{x}{\sin 55°} = \frac{36}{\sin 70°}$$

$$\frac{x \cancel{\sin 70°}}{\cancel{\sin 70°}} = \frac{36 \sin 55°}{\sin 70°}$$

$$x = 31.4 \text{ ft}$$

Each rafter would be:

31.4 ft + 1 ft

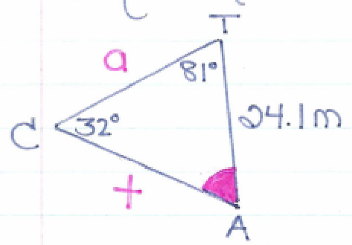
= 32.4 ft

or

32 ft 5 in.

7. In $\triangle CAT$, $\angle C = 32°$, $\angle T = 81°$, and $c = 24.1$ m. Solve the triangle. Round sides to the nearest tenth of a meter.

Sketch:



To find $\angle A$:

$180° - 81° - 32°$
$= 67°$

To find a:

$$\frac{a}{\sin A} = \frac{c}{\sin C}$$

$$\frac{a}{\sin 67°} = \frac{24.1}{\sin 32°}$$

$$\frac{a\cancel{\sin 32°}}{\cancel{\sin 32°}} = \frac{24.1\sin 67°}{\sin 32°}$$

$$a = 41.9 \text{ m}$$

To find t:

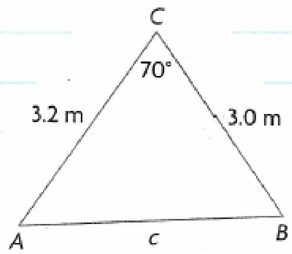
$$\frac{t}{\sin T} = \frac{c}{\sin C}$$

$$\frac{t}{\sin 81°} = \frac{24.1}{\sin 32°}$$

$$\frac{t\cancel{\sin 32°}}{\cancel{\sin 32°}} = \frac{24.1\sin 81°}{\sin 32°}$$

$$t = 44.9 \text{ m}$$

15. Jim says that the sine law cannot be used to determine the length of side c in $\triangle ABC$. Do you agree or disagree? Explain.



I agree with Jim. In order to find "c", you would need to know an angle and its opposite side. In this diagram we do not know "A" or "B".

Assignment: pgs. 124 - 127
1, 2, 3, 4, 5, 7, 15

For tomorrow....

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