

Questions from Homework

Find the derivative of each function.

Remember!

$$f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$$

1. $f(x) = 8x^2 - 10$

$$f'(x) = 16x$$

2. $f(x) = 2x^2 + 14x - 7$

$$f'(x) = 4x + 14$$

3. $f(x) = x^3$

$$f'(x) = 3x^2$$

4. $f(x) = \frac{x+4}{2x+3}$

$$f'(x) = \frac{-5}{(2x+3)^2}$$

Remember!

$$f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$$

- ① Find $f(x+h)$
- ② Fill in formula
- ③ Solve

$$\textcircled{1} f(x) = 8x^2 - 10$$

$$\textcircled{1} f(x+h) = 8(x+h)^2 - 10$$

$$f(x+h) = 8(x+h)(x+h) - 10$$

$$f(x+h) = 8(x^2 + 2xh + h^2) - 10$$

$$f(x+h) = 8x^2 + 16xh + 8h^2 - 10$$

$$\textcircled{2} f'(x) = \lim_{h \rightarrow 0} \frac{8x^2 + 16xh + 8h^2 - 10 - (8x^2 - 10)}{h}$$

$$f'(x) = \lim_{h \rightarrow 0} \frac{\cancel{8x^2} + 16xh + \cancel{8h^2} - \cancel{10} - \cancel{8x^2} + \cancel{10}}{h}$$

$$f'(x) = \lim_{h \rightarrow 0} \frac{16xh + 8h^2}{h}$$

$$f'(x) = \lim_{h \rightarrow 0} \frac{\cancel{h}(16x + 8h)}{\cancel{h}}$$

$$f'(x) = 16x + 8(0)$$

$$f'(x) = 16x$$

Remember!

$$f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$$

① Find $f(x+h)$

② Fill in formula

③ Solve.

2. $f(x) = 2x^2 + 14x - 7$

$$\begin{aligned} \textcircled{1} f(x+h) &= 2(x+h)^2 + 14(x+h) - 7 \\ &= 2(x^2 + 2xh + h^2) + 14x + 14h - 7 \\ &= 2x^2 + 4xh + 2h^2 + 14x + 14h - 7 \end{aligned}$$

$$f'(x) = \lim_{h \rightarrow 0} \frac{2x^2 + 4xh + 2h^2 + 14x + 14h - 7 - (2x^2 + 14x - 7)}{h}$$

$$= \lim_{h \rightarrow 0} \frac{\cancel{2x^2} + 4xh + \cancel{2h^2} + \cancel{14x} + 14h - \cancel{7} - \cancel{2x^2} - \cancel{14x} + \cancel{7}}{h}$$

$$= \lim_{h \rightarrow 0} \frac{4xh + 2h^2 + 14h}{h}$$

← Common factor

$$= \lim_{h \rightarrow 0} \frac{\cancel{h}(4x + 2h + 14)}{\cancel{h}}$$

$$= 4x + 14$$

$$f'(x) = 4x + 14$$

Remember!

$$f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$$

① Find $f(x+h)$

② Fill in formula

③ Solve.

③ $f(x) = x^3$

$$\begin{aligned} f(x+h) &= (x+h)^3 \\ &= (x+h)(x+h)(x+h) \\ &= (x+h)(x^2 + 2xh + h^2) \\ &= x^3 + 2x^2h + xh^2 + x^2h + 2xh^2 + h^3 \\ &= x^3 + 3x^2h + 3xh^2 + h^3 \end{aligned}$$

$$f'(x) = \lim_{h \rightarrow 0} \frac{x^3 + 3x^2h + 3xh^2 + h^3 - x^3}{h}$$

$$f'(x) = \lim_{h \rightarrow 0} \frac{3x^2h + 3xh^2 + h^3}{h}$$

$$f'(x) = \lim_{h \rightarrow 0} \frac{h(3x^2 + 3xh + h^2)}{h} = 3x^2$$

$$f'(x) = 3x^2$$

$f(x) = x^3$

$f(x+h) = (x+h)^3$

$$f'(x) = \lim_{h \rightarrow 0} \frac{(x+h)^3 - x^3}{h}$$

$$= \lim_{h \rightarrow 0} \frac{[(x+h) - x][(x+h)^2 + x(x+h) + x^2]}{h}$$

$$= \lim_{h \rightarrow 0} \frac{h[(x+h)^2 + x(x+h) + x^2]}{h} = x^2 + x^2 + x^2$$

$$f'(x) = 3x^2$$

Remember!

$$f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$$

- ① Find $f(x+h)$
- ② Fill in formula
- ③ Solve.

④ $f(x) = \frac{x+4}{2x+3}$

① $f(x+h) = \frac{(x+h)+4}{2(x+h)+3}$

$f(x+h) = \frac{x+h+4}{2x+2h+3}$

② $f'(x) = \lim_{h \rightarrow 0} \frac{\frac{x+h+4}{2x+2h+3} - \frac{x+4}{2x+3}}{h}$

$f'(x) = \lim_{h \rightarrow 0} \frac{(2x+3)(x+h+4) - (x+4)(2x+2h+3)}{h(2x+3)(2x+2h+3)}$

$f'(x) = \lim_{h \rightarrow 0} \frac{\cancel{2x^2} + \cancel{2xh} + \cancel{8x} + \cancel{3x} + \cancel{3h} + \cancel{12} - (\cancel{2x^2} + \cancel{2xh} + \cancel{3x} + \cancel{8x} + \cancel{8h} + \cancel{12})}{h(2x+3)(2x+2h+3)}$

$f'(x) = \lim_{h \rightarrow 0} \frac{-5h}{h(2x+3)(2x+2h+3)}$

$f'(x) = \frac{-5}{(2x+3)(2x+2(0)+3)}$

$f'(x) = \frac{-5}{(2x+3)^2}$

Try this one!

Remember!

$$f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$$

Differentiate the following function using the *Limit Definition of the Derivative*

$$f(x) = \sqrt{x+3} \quad f(x+h) = \sqrt{(x+h)+3} = \sqrt{x+h+3}$$

$$f'(x) = \lim_{h \rightarrow 0} \frac{(\sqrt{x+h+3} - \sqrt{x+3})}{h} \cdot \frac{(\sqrt{x+h+3} + \sqrt{x+3})}{(\sqrt{x+h+3} + \sqrt{x+3})}$$

$$= \lim_{h \rightarrow 0} \frac{x+h+3 - (x+3)}{h(\sqrt{x+h+3} + \sqrt{x+3})}$$

Homework