Warm-Up

Solving Polynomial Inequalities

Express answers using interval notation.

$$x^3 - 3x^2 - 4x + 12 \le 0$$

Where does the function have y-value that are less than or equal to zero.

$y = (x^3 - 3x^2)(-4x + 12)$ factor by grouping

$y = x^2(x-3) - 4(x-3)$

$y = (x-3)(\underline{x^2 - 4})$ diff of squares

$y = (x-3)(x+2)(x-2)$

$0 = (x-3)(x+2)(x-2)$

$x - 3 = 0$ | $x + 2 = 0$ | $x - 2 = 0$

$x = 3$ | $x = -2$ | $x = 2$

Roots: $x = -2, 2, 3$

$-$ $+$ $-$ $+$

(-3) -2 (0) 2 (2.5) 3 (4)

$$\boxed{(-\infty, -2] + [2, 3]}$$

Absolute Maxima/Minima

A function f has an absolute (or global) maximum at c if $f(c) \geq f(x)$ for all x in the domain D of f.

- The number $f(c)$ is called the maximum value of f on D.

A function f has an absolute (or global) minimum at c if $f(c) \leq f(x)$ for all x in the domain D of f.

- The number $f(c)$ is called the minimum value of f on D.

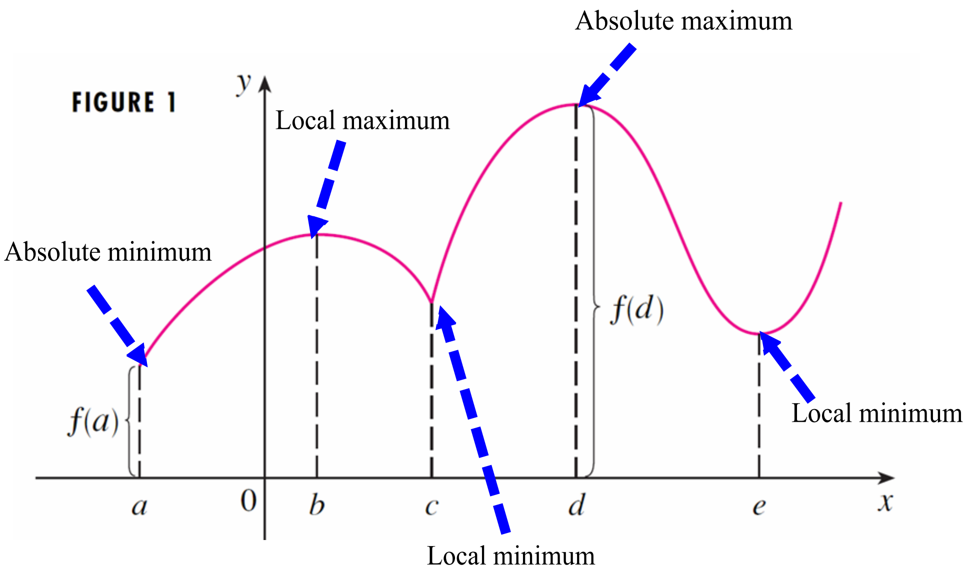
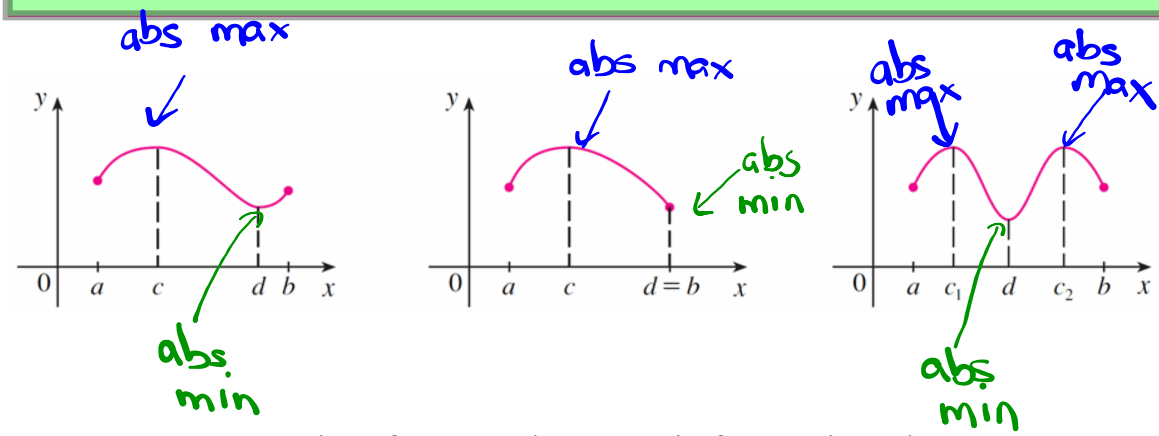


FIGURE 1

3 The Extreme Value Theorem If f is continuous on a closed interval $[a, b]$, then f attains an absolute maximum value $f(c)$ and an absolute minimum value $f(d)$ at some numbers c and d in $[a, b]$.



Here are a couple of examples to reinforce that the function must be continuous over a closed interval.

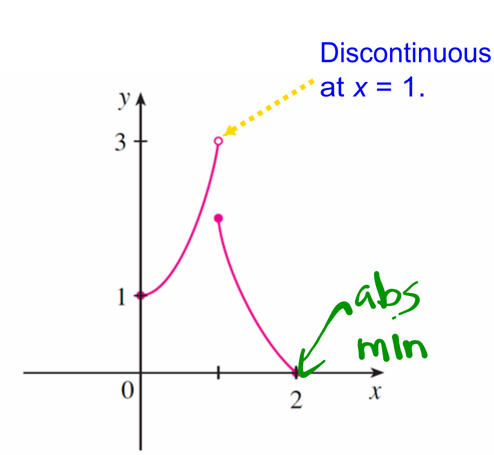


FIGURE 6
This function has minimum value $f(2) = 0$, but no maximum value.

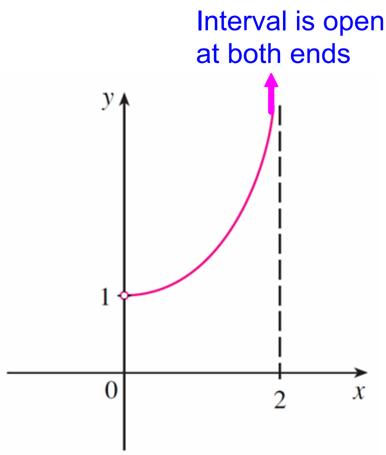


FIGURE 7
This continuous function g has no maximum or minimum.

How do we find extreme values?

4 Fermat's Theorem If f has a local maximum or minimum at c, and if $f'(c)$ exists, then $f'(c) = 0$.

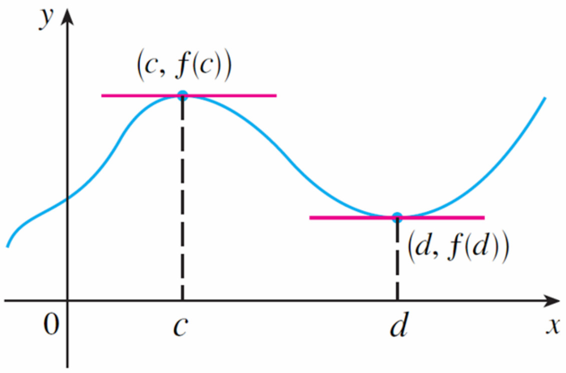


FIGURE 8

There can also be an extreme value where $f'(c)$ does not exist.

In general, functions whose graphs have "corners" or "kinks" are not differentiable there.

Look at the function $f(x) = |x|$

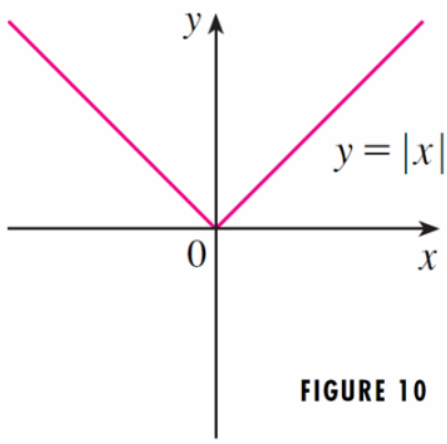


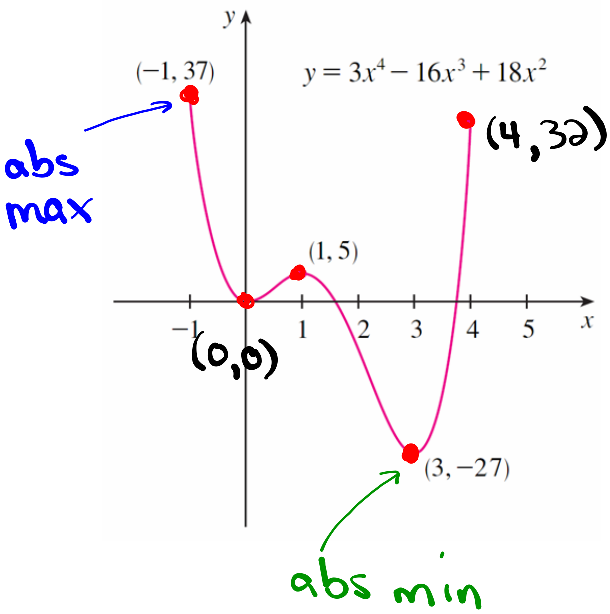
FIGURE 10

Example:

Given the function...

$$f(x) = 3x^4 - 16x^3 + 18x^2 \;,\; -1 \le x \le 4$$ closed interval

Determine the absolute maximum and minimum values of the function.



$3(256) - 16(64) + 18(16)$

$768 - 1024 + 288$

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Finding Absolute Maxima/Minima

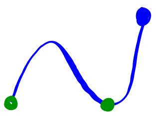
The Closed Interval Method To find the *absolute* maximum and minimum values of a continuous function f on a closed interval $[a, b]$:

1. Find the values of f at the critical numbers of f in (a, b). $f(c)$, $f(d)$, where c, d are critical values
2. Find the values of f at the endpoints of the interval. $f(a)$ and $f(b)$
3. The largest of the values from Steps 1 and 2 is the absolute maximum value; the smallest of these values is the absolute minimum value.

Determine the absolute max and absolute min for the following function.

$a=-4 \quad b=1$

$f(x) = x^3 + 6x^2 + 9x + 2 \qquad -4 \leq x \leq 1$



$f'(x) = 3x^2 + 12x + 9$

$f'(x) = 3(x^2 + 4x + 3)$

$f'(x) = 3(x+1)(x+3)$

$0 = 3(x+1)(x+3)$

cv: $x+1=0$, $x=-1$ | $x+3=0$, $x=-3$

$f(x) = x^3 + 6x^2 + 9x + 2$

$f(-1) = (-1)^3 + 6(-1)^2 + 9(-1) + 2$

$f(-1) = -1 + 6 - 9 + 2$

$f(-1) = -2$

$(-1, -2)$ abs min

$f(x) = x^3 + 6x^2 + 9x + 2$

$f(-3) = (-3)^3 + 6(-3)^2 + 9(-3) + 2$

$f(-3) = -27 + 54 - 27 + 2$

$f(-3) = 2$

$(-3, 2)$

$f(-4) = (-4)^3 + 6(-4)^2 + 9(-4) + 2$

$f(-4) = -64 + 96 - 36 + 2$

$f(-4) = -2$

$(-4, -2)$ abs min

$f(1) = (1)^3 + 6(1)^2 + 9(1) + 2$

$f(1) = 1 + 6 + 9 + 2$

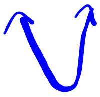
$f(1) = 18$

$(1, 18)$ abs max

Quadratics

Determine the absolute max or absolute min for the following function.

Recall Parabolas have either a max or a min at the vertex!



$$y = x^2 + 4x + 10$$

$y' = 2x + 4$

$0 = 2x + 4$

$0 = 2(x+2)$

cv: $x + 2 = 0$

$x = -2$

$y = x^2 + 4x + 10$

$y = (-2)^2 + 4(-2) + 10$

$y = 6$

$y - 10 = x^2 + 4x$

$y - 10 + 4 = x^2 + 4x + 4$

$y - 6 = (x+2)^2$

$y = (x+2)^2 + 6$

vertex $(-2, 6)$ min

Homework

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