2.2 Angles Formed by Parallel Lines

GOAL

Prove properties of angles formed by parallel lines and a transversal, and use these properties to solve problems.

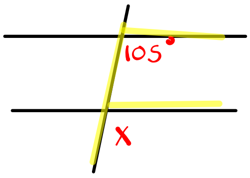
EXPLORE...

- Parallel bars are used in therapy to help people recover from injuries to their legs or spine. How could the manufacturer ensure that the bars are actually parallel?

SAMPLE ANSWER

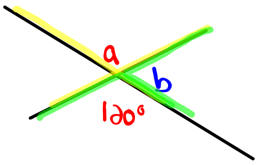
Here are three possible solutions:

- Solution 1: Install one of the parallel bars. Place the second bar in the approximate position, and then place another straight object, such as a metre stick, across both bars. Measure the corresponding angles formed by this transversal, and adjust the second bar until the corresponding angles are equal. Check for accuracy by moving the metre stick to a different position, intersecting at a different angle. Measure the corresponding angles again, ensuring that they are equal.
- Solution 2: The manufacturer could provide a paper template with the parallel bars. The template would indicate the placement of the bars and the locations for the supporting feet of the bars. The manufacturer could ensure that the template is correct by drawing a set of bars on the template master, drawing a transversal, and then measuring corresponding angles to check that they are equal. The installer could lay the template on the floor and install each bar in the correct position as indicated by the template.
- Solution 3: Install the first bar, making sure it is the same distance from the floor in several locations using a tape measure. Install the second bar ensuring the distance from the floor to the bar is the same as the first bar in several locations. Measure the distance between the two bars in several locations and adjust so that this distance is the same in each location.



"F pattern"

$x = 105$ → corresponding angles are equal



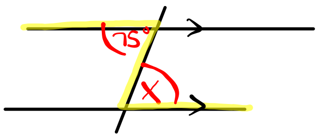
$a = 120°$ → vertically opposite angles are equal.

$b + 120° = 180°$ → supplementary angles add to 180°

$b = 180° - 120°$

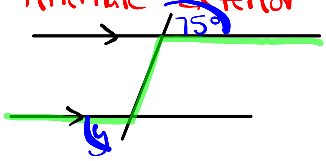
$b = 60°$

Alternate Interior



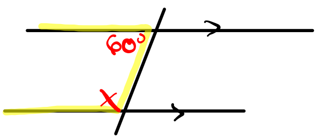
$x = 75°$ → Alternate Interior angles are equal

Alternate Exterior



$y = 75°$ → Alternate Exterior angles are equal

Co-Interior



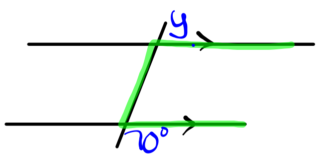
$x + 60° = 180°$

$x = 180° - 60°$

$x = 120°$

Co-Interior angles are supplementary

Co-Exterior



$y + 70° = 180°$

$y = 180 - 70°$

$y = 110°$

Co-Exterior angles are supplementary

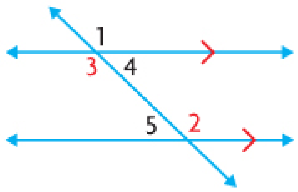
APPLY the Math

EXAMPLE 1 Reasoning about conjectures involving angles formed by transversals

Make a conjecture that involves the interior angles formed by parallel lines and a transversal. Prove your conjecture.

Tuyet's Solution

My conjecture: When a transversal intersects a pair of parallel lines, the **alternate interior angles** are equal.



I drew two parallel lines and a transversal as shown, and I numbered the angles. I need to show that $\angle 3 = \angle 2$.

Statement	Justification
$\angle 1 = \angle 2$	Corresponding angles
$\angle 1 = \angle 3$	Vertically opposite angles
$\angle 3 = \angle 2$	Transitive property

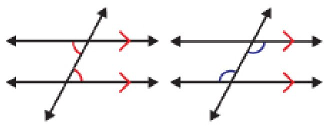
Since I know that the lines are parallel, the corresponding angles are equal.

When two lines intersect, the opposite angles are equal.

$\angle 2$ and $\angle 3$ are both equal to $\angle 1$, so $\angle 2$ and $\angle 3$ are equal to each other.

My conjecture is proved.

alternate interior angles
Two non-adjacent interior angles on opposite sides of a transversal.

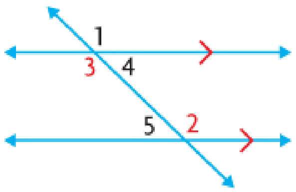


EXAMPLE 1 Reasoning about conjectures involving angles formed by transversals

Make a conjecture that involves the interior angles formed by parallel lines and a transversal. Prove your conjecture.

Ali's Solution

My conjecture: When a transversal intersects a pair of parallel lines, the interior angles on the same side of the transversal are supplementary.



I need to show that $\angle 3$ and $\angle 5$ are supplementary.

$\angle 1 = \angle 2$

Since the lines are parallel, the corresponding angles are equal.

$\angle 2 + \angle 5 = 180°$

These angles form a straight line, so they are supplementary.

$\angle 1 + \angle 5 = 180°$

Since $\angle 2 = \angle 1$, I could substitute $\angle 1$ for $\angle 2$ in the equation.

$\angle 1 = \angle 3$

Vertically opposite angles are equal. Since $\angle 1 = \angle 3$, I could substitute $\angle 3$ for $\angle 1$ in the equation.

$\angle 3 + \angle 5 = 180°$

My conjecture is proved.

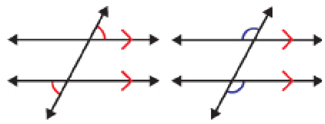
EXAMPLE 1 Reasoning about conjectures involving angles formed by transversals

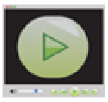
Make a conjecture that involves the interior angles formed by parallel lines and a transversal. Prove your conjecture.

Your Turn

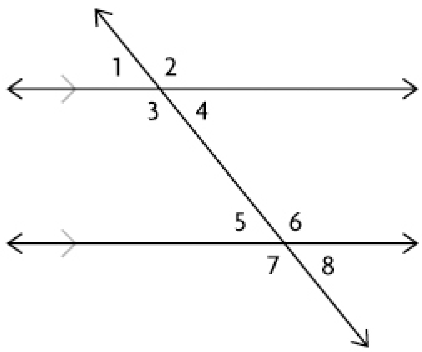
Naveen made the following conjecture: "**Alternate exterior angles** are equal." Prove Naveen's conjecture.

alternate exterior angles
Two exterior angles formed between two lines and a transversal, on opposite sides of the transversal.



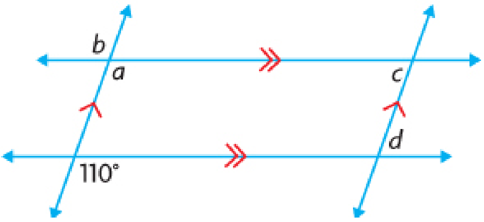


Answer

	I drew two parallel lines and a transversal, and I numbered the angles as shown. I need to show that $\angle 1$ and $\angle 8$ are equal.
$\angle 1 = \angle 5$	Since the lines are parallel, the corresponding angles are equal.
$\angle 5 = \angle 8$	Vertically opposite angles are equal.
$\angle 1 = \angle 8$	Transitive property
Naveen's conjecture is proved.	

EXAMPLE 2 Using reasoning to determine unknown angles

Determine the measures of a, b, c, and d.



Kebeh's Solution

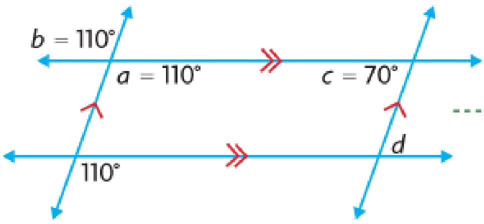
$\angle a = 110°$

The 110° angle and $\angle a$ are corresponding. Since the lines are parallel, the 110° angle and $\angle a$ are equal.

$\angle a = \angle b$
$\angle b = 110°$

Vertically opposite angles are equal.

$\angle c + \angle a = 180°$
$\angle c + 110° = 180°$
$\angle c = 70°$



$\angle c$ and $\angle a$ are interior angles on the same side of a transversal. Since the lines are parallel, $\angle c$ and $\angle a$ are supplementary.

I updated the diagram.

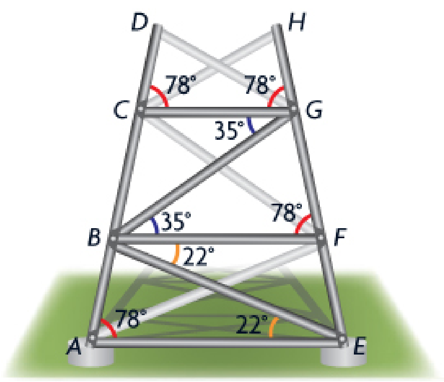
$\angle c = \angle d$
$\angle d = 70°$

$\angle c$ and $\angle d$ are alternate interior angles. Since the lines are parallel, $\angle c$ and $\angle d$ are equal.

The measures of the angles are:
$\angle a = 110°$; $\angle b = 110°$;
$\angle c = 70°$; $\angle d = 70°$.

EXAMPLE 3 Using angle properties to prove that lines are parallel

One side of a cellphone tower will be built as shown. Use the angle measures to prove that braces CG, BF, and AE are parallel.



Morteza's Solution: Using corresponding angles

$\angle BAE = 78°$ and $\angle DCG = 78°$ — Given

$AE \parallel CG$ — When corresponding angles are equal, the lines are parallel.

$\angle CGH = 78°$ and $\angle BFG = 78°$ — Given

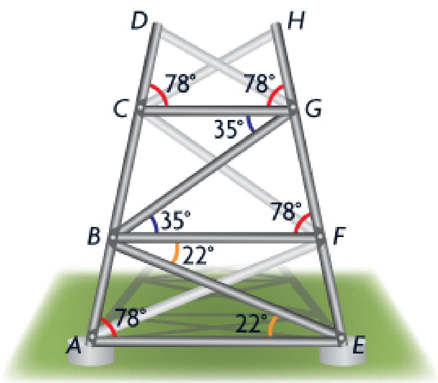
$CG \parallel BF$ — When corresponding angles are equal, the lines are parallel.

$AE \parallel CG$ and $CG \parallel BF$ — Since AE and BF are both parallel to CG, all three lines are parallel to each other.

The three braces are parallel.

EXAMPLE 3 Using angle properties to prove that lines are parallel

One side of a cellphone tower will be built as shown. Use the angle measures to prove that braces *CG*, *BF*, and *AE* are parallel.



Jennifer's Solution: Using alternate interior angles

Statement	Justification
$\angle CGB = 35°$ and $\angle GBF = 35°$	Given
$CG \parallel BF$	Alternate interior angles
$\angle FBE = 22°$ and $\angle BEA = 22°$	Given
$BF \parallel AE$	Alternate interior angles
$CG \parallel BF$ and $BF \parallel AE$	Transitive property

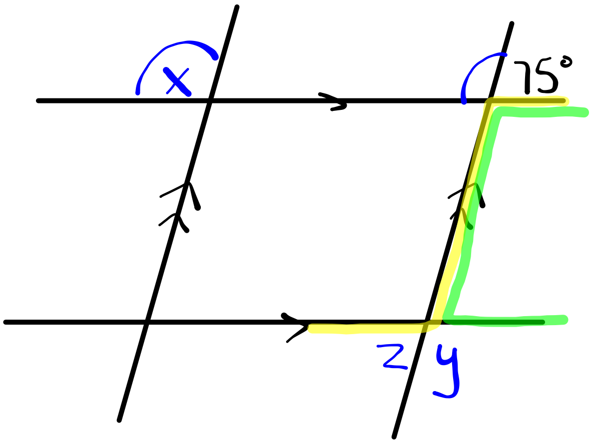
The three braces are parallel.

When alternate interior angles are equal, the lines are parallel.

When alternate interior angles are equal, the lines are parallel.

Since *CG* and *AE* are both parallel to *BF*, they must also be parallel to each other.

Example:



$x = 105°$

$y = 105°$

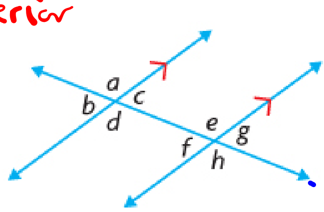
$z = 75°$

In Summary

Key Idea

- When a transversal intersects two parallel lines,
  - i) the corresponding angles are equal. F
  - ii) the alternate interior angles are equal. Z
  - iii) the alternate exterior angles are equal. Z
  - iv) the interior angles on the same side of the transversal are supplementary. C

  or exterior



i) $a = e, b = f$
$c = g, d = h$

ii) $c = f, d = e$

iii) $a = h, b = g$

iv) $c + e = 180°$
$d + f = 180°$

Need to Know

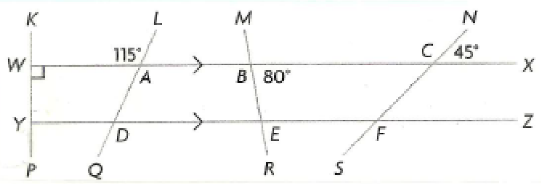
- If a transversal intersects two lines such that
  - i) the corresponding angles are equal, or
  - ii) the alternate interior angles are equal, or
  - iii) the alternate exterior angles are equal, or
  - iv) the interior angles on the same side of the transversal are supplementary,

  then the lines are parallel.

<u>Assignment:</u> **pgs. 78 - 82**
1, 2, 3, 4, 10, 12, 15, 20

SOLUTIONS => 2.2 Angles Formed by Parallel Lines

1. Determine the measures of <WYD, < YDA, < DEB, and < EFS. Give your reasoning for each measure.



$\angle WYD = 180° - 90°$
$= 90°$

* Interior angles on the same side of the transversal are supplementary.

$\angle YDA = 115°$

* Corresponding angles are equal.

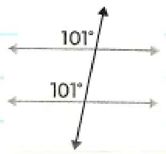
$\angle DEB = 80°$

* Alternate interior angles are equal.

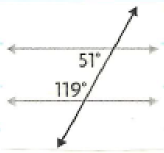
$\angle EFS = 45°$

* Alternate exterior angles are equal.

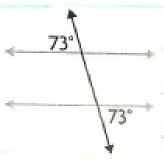
2. For each diagram, decide if the given angle measures prove that the blue lines are parallel. Justify your decisions.

a) 

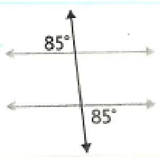
Yes, these lines are parallel as the corresponding angles are equal.

b) 

No, these lines are not parallel as the interior angles on the same side of the transversal are not supplementary.
$(119° + 51° = 170°)$

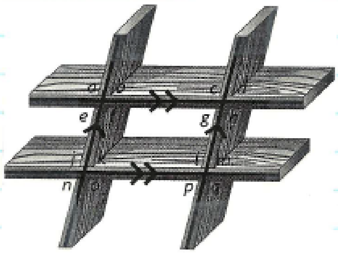
c) 

Yes, these lines are parallel since the alternate exterior angles are equal.

d) 

Yes, these lines are parallel since the alternate exterior angles are equal.

3. A shelving unit is built with two pairs of parallel planks. Explain why each of the following statements is true.



a) $\angle k = \angle p$
Alternate interior angles are equal.

b) $\angle a = \angle j$
Corresponding angles are equal.

c) $\angle j = \angle q$
Alternate exterior angles are equal

d) $\angle g = \angle d$
Vertically opposite angles are equal.

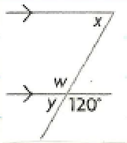
e) $\angle b = \angle m$
$\{\angle b = \angle k = \angle m\}$
Corresponding angles are equal.

f) $\angle e = \angle p$
$\{\angle e = \angle n = \angle p\}$
Corresponding angles are equal.

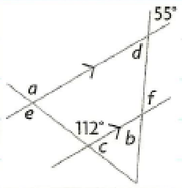
g) $\angle n = \angle d$
↳ $\angle n = \angle p$
Corresponding Angles
↳ $\angle p = \angle d$
Alternate Exterior Angles are equal.

h) $\angle f + \angle k = 180°$
Interior Angles on the same side of the transversal are supplementary.

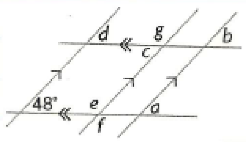
4. Determine the measures of the indicated angles.

a) 

<w = 120° (vertically opposite angles)
<y = 180° − 120°
= 60° (supplementary angles)
<x = 60° (corresponding angles)

b) 

<d = 55° (vertically opposite angles)
<f = 55° (corresponding angles)
<b = 55° (Corresponding /opposite)
<c = 180° − 112°
= 68° (supplementary angles)
<a = 112° (corresponding angles)
<e = 112° (vertically opposite angles)

c) 

<d = 48° (corresponding angles)
<e = 180° − 48°
= 132° (interior angles)
<g = 132° (corresponding angles)
<f = 132° (vertically opposite angles)
<a = 48° (corresponding angles)
<b = 48° (corresponding angles)
<c =48°

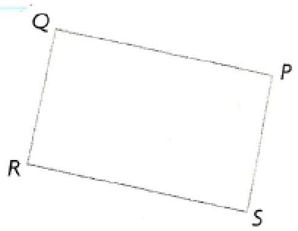
10. Jason wrote the following proof.
Identify his errors, and correct his proof.

Given: $QP \perp QR$

$QR \perp RS$

$QR \parallel PS$

Prove: $QPSR$ is a parallelogram.



Jason's Proof

Statement	Justification
$\angle PQR = 90°$ and $\angle QRS = 90°$	Lines that are perpendicular meet at right angles.
$QP \parallel RS$	Since the interior angles on the same side of a transversal are equal, QP and RS are parallel. $\Rightarrow$ ERROR!
$QR \parallel PS$	Given
$QPSR$ is a parallelogram	$QPSR$ has two pairs of parallel sides.

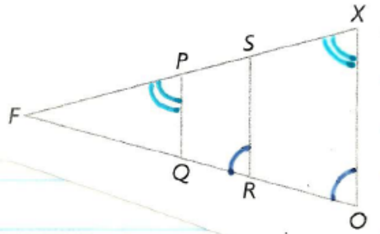
CORRECT PROOF

Statement	Justification
$\angle PQR = 90°$ and $\angle QRS = 90°$	Lines that are perpendicular meet at right angles.
$QP \parallel RS$	Since interior angles on the same side of a transversal are supplementary, QP and RS are parallel.
$QR \parallel PS$	Given
QPSR is a parallelogram	QPSR has two pairs of parallel sides.

12. Given: $\triangle FOX$ is isosceles.

$\angle FOX = \angle FRS$
$\angle FXO = \angle FPQ$

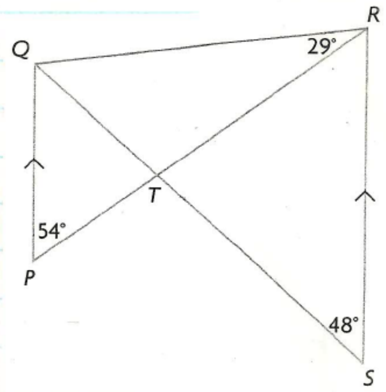
Prove: $PQ \parallel SR$ and $SR \parallel XO$



PROOF

Statement	Justification
SR ∥ XO	< FOX and < FRS are equal Corresponding angles.
PQ ∥ XO	< FPQ and < FXO are equal corresponding angles.
SR ∥ PQ	Transitive Property

15. Determine the measures of all the unknown angles in this diagram, given $PQ \parallel RS$.



$\angle TRS = 54^\circ$ (alternate interior)

$\angle RTS = 180^\circ - 54^\circ - 48^\circ$
$= 78^\circ$ (angle sum of Δ)

$\angle QTP = 78^\circ$ (vertically opposite)

$\angle PQT = 180^\circ - 54^\circ - 78^\circ$
$= 48^\circ$ (angle sum of Δ)

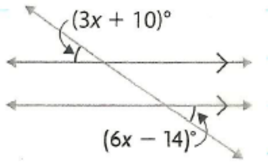
$\angle QTR = 180^\circ - 78^\circ$
$= 102^\circ$ (supplementary)

$\angle TQR = 180^\circ - 102^\circ - 29^\circ$
$= 49^\circ$

$\angle PTS = 102^\circ$ (vertically opposite)

20. Solve for x.

a)



$3x+10=6x-14$

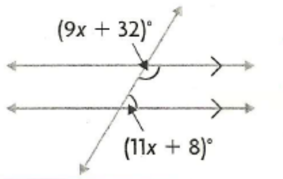
$3x-6x=-14-10$

$\dfrac{-3x}{-3}=\dfrac{-24}{-3}$

$x=8$

* Alternate exterior angles are equal.

b)



$(9x+32)^\circ+(11x+8)^\circ=180^\circ$

$9x+11x+32+8=180^\circ$

$20x+40=180^\circ$

$20x=180^\circ-40^\circ$

$\dfrac{\cancel{20}x}{\cancel{20}}=\dfrac{140^\circ}{20}$

$x=7$

* Interior angles on the same side of the transversal are supplementary.

Attachments

PM11-2s2.gsp

2s2e3 finalt.mp4

2s2e1 final.mp4

2s2e2 final.mp4

PM11-2s2-review.gsp