2.3 Angle Properties in Triangles

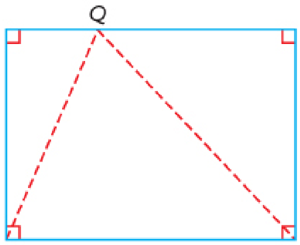
GOAL

Prove properties of angles in triangles, and use these properties to solve problems.

EXPLORE...

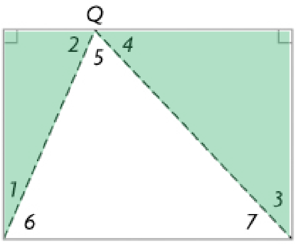
On a rectangular piece of paper, draw lines from two vertices to a point on the opposite side. Cut along the lines to create two right triangles and an acute triangle.

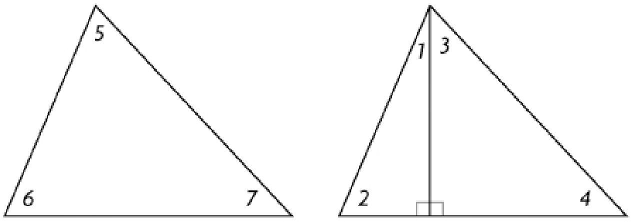
- What do you notice about the three triangles?
- Can you use angle relationships to show that the sum of the measures of the angles in any acute triangle formed this way is 180°?



SAMPLE ANSWER

The two right triangles can be arranged to fit exactly on top of the acute triangle. So, together, the two right triangles form a triangle that is congruent to the acute triangle. This means that the angles in the larger triangle and the composite triangle are equal, so the sums of the measures of the corresponding angles are also equal.





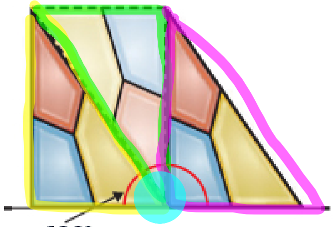
sum of measures of angles in acute triangle = sum of measures of non-right angles in two right triangles

$\angle 5 + \angle 6 + \angle 7 = (\angle 1 + \angle 3) + \angle 2 + \angle 4$	
$\angle 1 + \angle 3 = \angle 5$	Corresponding angles
$\angle 5 + \angle 6 + \angle 7 = \angle 5 + \angle 2 + \angle 4$	Substitution
$\angle 5 + \angle 2 + \angle 4 = 180°$	Angles form a straight line.
$\angle 5 + \angle 6 + \angle 7 = 180°$	

The sum of the measures of the angles in the acute triangle is 180°.

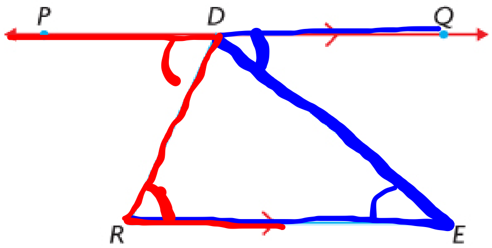
INVESTIGATE the Math

Diko placed three congruent triangular tiles so that a different angle from each triangle met at the same point. She noticed the angles seemed to form a straight line.



? Can you prove that the sum of the measures of the interior angles of any triangle is 180°?

A. Draw an acute triangle, $\triangle RED$. Construct line PQ through vertex D, parallel to RE.



B. Identify pairs of equal angles in your diagram. Explain how you know that the measures of the angles in each pair are equal.

C. What is the sum of the measures of $\angle PDR$, $\angle RDE$, and $\angle QDE$? Explain how you know.

D. Explain why:
$\angle DRE + \angle RDE + \angle RED = 180°$

E. In part A, does it matter which vertex you drew the parallel line through? Explain, using examples.

F. Repeat parts A to E, first for an obtuse triangle and then for a right triangle. Are your results the same as they were for the acute triangle?

Answers

B. $\angle PDR = \angle DRE$; $\angle QDE = \angle RED$ Alternate interior angles

C. 180°, because the three angles form a straight line

D. $\angle PDR + \angle RDE + \angle QDE = 180°$; substitute $\angle DRE$ for $\angle PDR$ and $\angle RED$ for $\angle QDE$.

E. No, it does not matter. I labelled the angles at the parallel line I drew a, b, and c. I labelled the other two angles in the triangle d and e. Two sides of the triangle are transversals for the two parallel lines, so $a = d$ and $c = e$. Since $a + b + c = 180°$, I can substitute to get $d + b + e = 180°$.

F. Yes, my results are the same for these triangles as they were for the acute triangle.

Reflecting

G. Why is Diko's approach not considered to be a proof?

H. Are your results sufficient to prove that the sum of the measures of the angles in any triangle is 180°? Explain.

Answers

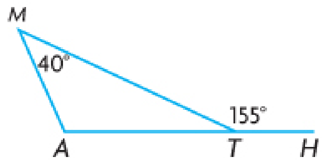
G. Diko only showed that her conjecture appeared to be true for one triangle. She did not prove it for all triangles. It is possible that she did not line up the angles precisely, so the measures of the angles may not have had a sum of exactly 180°.

H. Yes, my results are sufficient, because I proved the conjecture for all possible types of triangles and for any angle measures.

APPLY the Math

EXAMPLE 1 Using angle sums to determine angle measures

In the diagram, $\angle MTH$ is an **exterior angle** of $\triangle MAT$. Determine the measures of the unknown angles in $\triangle MAT$.



Serge's Solution

$$\angle MTA + \angle MTH = 180°$$
$$\angle MTA + (155°) = 180°$$
$$\angle MTA = 25°$$

$\angle MTA$ and $\angle MTH$ are supplementary since they form a straight line.

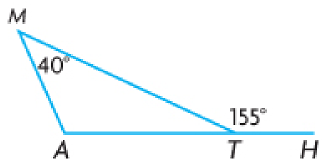
$$\angle MAT + \angle AMT + \angle MTA = 180°$$
$$\angle MAT + (40°) + (25°) = 180°$$
$$\angle MAT = 115°$$

The sum of the measures of the interior angles of any triangle is 180°.

The measures of the unknown angles are:
$\angle MTA = 25°$; $\angle MAT = 115°$.

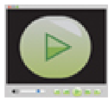
EXAMPLE 1 Using angle sums to determine angle measures

In the diagram, $\angle MTH$ is an **exterior angle** of $\triangle MAT$. Determine the measures of the unknown angles in $\triangle MAT$.



Your Turn

If you are given one interior angle and one exterior angle of a triangle, can you always determine the other interior angles of the triangle? Explain, using diagrams.



Answer

If you are given an exterior angle and one non-adjacent angle, then you can determine the other interior angles.

For example, in $\triangle ABC$,

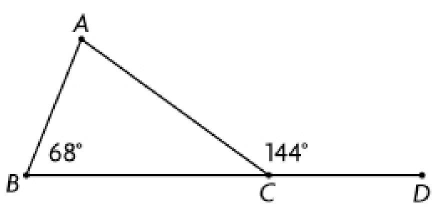
$\angle ACB + 144° = 180°$ The angles form a straight line, so they are supplementary.

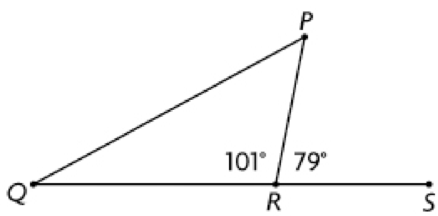
$\angle ACB = 36°$

$\angle CAB + \angle ABC = \angle ACD$ An exterior angle is equal to the sum of the measures of the non-adjacent interior angles.

$\angle CAB + 68° = 144°$

$\angle CAB = 76°$



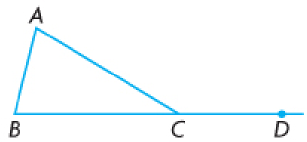


If the interior angle you are given is adjacent to a known exterior angle, then you cannot determine the other angles. For example, in $\triangle PQR$, neither of the non-adjacent interior angles are known, so there is not enough information to determine the unknown interior angles.

EXAMPLE 2 Using reasoning to determine the relationship between the exterior and interior angles of a triangle

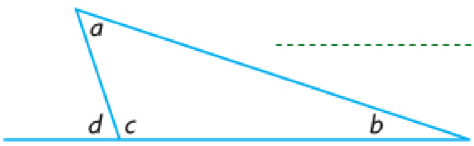
Determine the relationship between an exterior angle of a triangle and its **non-adjacent interior angles**.

non-adjacent interior angles
The two angles of a triangle that do not have the same vertex as an exterior angle.



$\angle A$ and $\angle B$ are non-adjacent interior angles to exterior $\angle ACD$.

Joanna's Solution



I drew a diagram of a triangle with one exterior angle. I labelled the angle measures *a*, *b*, *c*, and *d*.

$\angle d + \angle c = 180°$
$\angle d = 180° - \angle c$

$\angle d$ and $\angle c$ are supplementary. I rearranged these angles to isolate $\angle d$.

$\angle a + \angle b + \angle c = 180°$
$\angle a + \angle b = 180° - \angle c$

The sum of the measures of the angles in any triangle is 180°.

$\angle d = \angle a + \angle b$

Since $\angle d$ and ($\angle a + \angle b$) are both equal to $180° - \angle c$, by the transitive property, they must be equal to each other.

The measure of an exterior angle of a triangle is equal to the sum of the measures of the two non-adjacent interior angles.

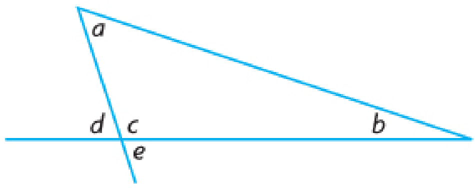
opposite

EXAMPLE 2 Using reasoning to determine the relationship between the exterior and interior angles of a triangle

Determine the relationship between an exterior angle of a triangle and its **non-adjacent interior angles**.

Your Turn

Prove: $\angle e = \angle a + \angle b$

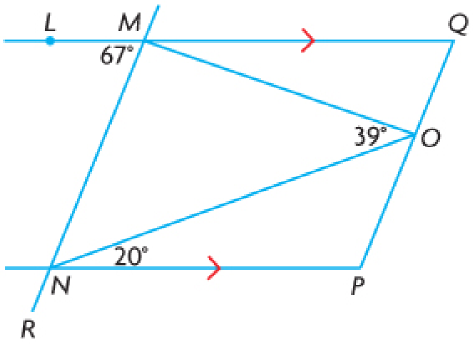


Answer

$\angle d = \angle a + \angle b$ Exterior angle property
$\angle e = \angle d$ Vertically opposite angles
$\angle e = \angle a + \angle b$ Transitive property
I could also prove this by following the same steps as Joanna did in her proof.

EXAMPLE 3 Using reasoning to solve problems

Determine the measures of $\angle NMO$, $\angle MNO$, and $\angle QMO$.



Tyler's Solution

MN is a transversal of parallel lines *LQ* and *NP*. — *MN* intersects parallel lines *LQ* and *NP*.

$\angle MNO + 20° = 67°$
$\angle MNO = 47°$

Since $\angle LMN$ and $\angle MNP$ are alternate interior angles between parallel lines, they are equal.

$\angle NMO + \angle MNO + 39° = 180°$
$\angle NMO + (47°) + 39° = 180°$
$\angle NMO + 86° = 180°$
$\angle NMO = 94°$

The measures of the angles in a triangle add to 180°.

$\angle NMO + \angle QMO + 67° = 180°$
$(94°) + \angle QMO + 67° = 180°$
$161° + \angle QMO = 180°$
$\angle QMO = 19°$

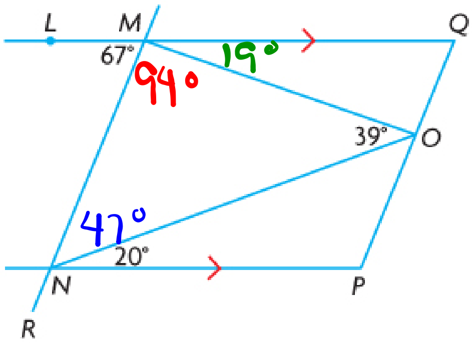
$\angle LMN$, $\angle NMO$, and $\angle QMO$ form a straight line, so their measures must add to 180°.

The measures of the angles are:
$\angle MNO = 47°$; $\angle NMO = 94°$; $\angle QMO = 19°$.

EXAMPLE 3 Using reasoning to solve problems

Determine the measures of $\angle NMO$, $\angle MNO$, and $\angle QMO$.

y x z



① $x + 20° = 67°$

$x = 47°$

$\boxed{\angle MNO = 47°}$

alternate interior angles are equal

② $y + 47° + 39° = 180°$

$y = 180° - 47° - 39°$

$y = 94°$

$\boxed{\angle NMO = 94°}$

the interior angles of a triangle are supplementary

③ $z + 94 + 67 = 180$

$z = 180° - 94° - 67°$

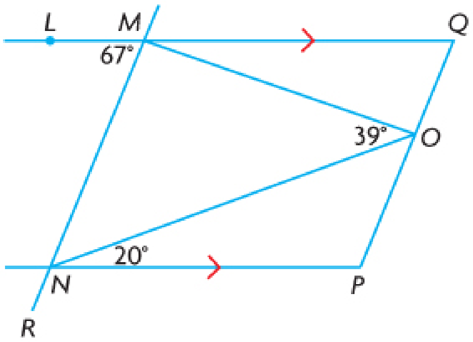
$z = 19°$

$\boxed{\angle QMO = 19°}$

angles that make a straight line are supplementary

EXAMPLE 3 Using reasoning to solve problems

Determine the measures of $\angle NMO$, $\angle MNO$, and $\angle QMO$.



Dominique's Solution

$$\angle NMO + \angle MNO + 39° = 180°$$
$$\angle NMO + \angle MNO = 141°$$

The sum of the measures of the angles in a triangle is 180°.

$$(\angle NMO + \angle QMO) + (\angle MNO + 20°) = 180°$$
$$\angle NMO + \angle MNO + \angle QMO = 160°$$

The angles that are formed by ($\angle NMO + \angle QMO$) and ($\angle MNO + 20°$) are interior angles on the same side of transversal MN. Since $LQ \parallel NP$, these angles are supplementary.

$$(141°) + \angle QMO = 160°$$
$$\angle QMO = 19°$$

I substituted the value of $\angle NMO + \angle MNO$ into the equation.

$$\angle NMO + \angle QMO + 67° = 180°$$
$$\angle NMO + (19°) + 67° = 180°$$
$$\angle NMO = 94°$$

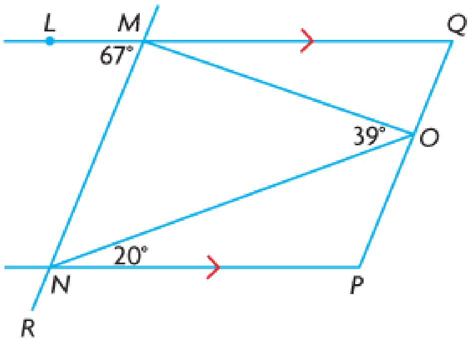
$\angle LMN$, $\angle NMO$, and $\angle QMO$ form a straight line, so the sum of their measures is 180°.

$$\angle NMO + \angle MNO = 141°$$
$$(94°) + \angle MNO = 141°$$
$$\angle MNO = 47°$$

The measures of the angles are:
$\angle QMO = 19°$; $\angle NMO = 94°$; $\angle MNO = 47°$.

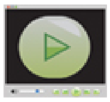
EXAMPLE 3 Using reasoning to solve problems

Determine the measures of $\angle NMO$, $\angle MNO$, and $\angle QMO$.

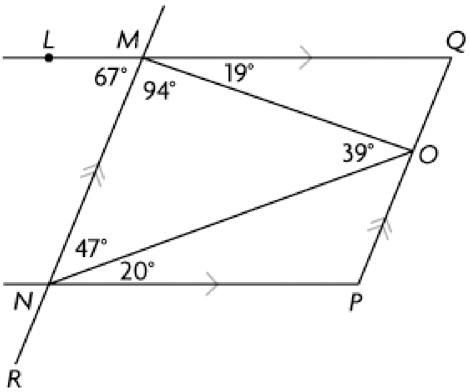


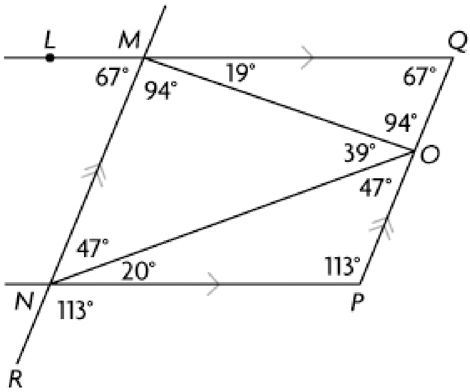
Your Turn

In the diagram for Example 3, $QP \parallel MR$. Determine the measures of $\angle MQO$, $\angle MOQ$, $\angle NOP$, $\angle OPN$, and $\angle RNP$.



Answer





Statement	**Justification**
MQ is a transversal.	*MQ* intersects parallel lines *MR* and *QP*.
$\angle LMN = \angle MQO = 67°$	Corresponding angles
$\angle MOQ + 67° + 19° = 180°$ $\angle MOQ = 94°$	The measures of the angles in a triangle add to 180°.
$\angle MOQ + \angle MON + \angle NOP = 180°$ $94° + 39° + \angle NOP = 180°$ $\angle NOP = 47°$	$\angle MOQ$, $\angle MON$, and $\angle NOP$ form a straight line, so their measures must add to 180°.
$\angle OPN + \angle MNP = 180°$ $\angle OPN = 113°$	Interior angles on the same side of the transversal are supplementary.
$\angle RNP = 113°$	Alternate interior angles

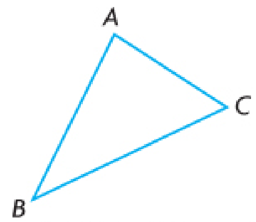
In Summary

Key Idea

- You can prove properties of angles in triangles using other properties that have already been proven.

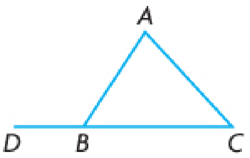
Need to Know

- In any triangle, the sum of the measures of the interior angles is proven to be 180°.



$\angle A + \angle B + \angle C = 180°$

- The measure of any exterior angle of a triangle is proven to be equal to the sum of the measures of the two non-adjacent interior angles.



$\angle DBA = \angle BAC + \angle ACB$

<u>Assignment:</u> **pgs. 90 - 92**
1, 2, 3, 4, 6, 7, 11, 12, 14, 15

SOLUTIONS ⇒ 2.3 Angle Properties in Triangles

1. Harrison drew a triangle and then measured the three interior angles. When he added the measures of these angles, the sum was 180°. Does this prove that the sum of the measures of the angles in any triangle is 180°? Explain.

SOLUTION

No, this only proves that the sum of the angles in that particular triangle is 180°.

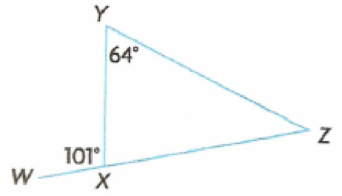
2. Marcel says that it is possible to draw a triangle with two right angles. Do you agree? Explain why or why not.

SOLUTION

I do not agree with Marcel since the sum of the three interior angles in a triangle must be 180°.

3. Determine the following unknown angles.

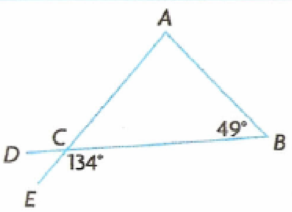
a) $\angle YXZ$, $\angle Z$



$\angle YXZ = 180° - 101°$
$= 79°$
(Supplementary Angles)

$\angle Z = 180° - 64° - 79°$
$= 37°$
(Angle sum of a Δ)

b) $\angle A$, $\angle DCE$

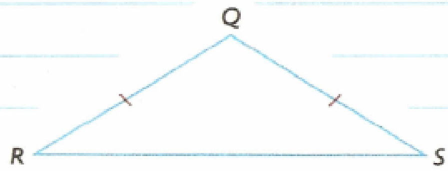


$134° = \angle A + 49°$
$134° - 49° = \angle A$
$85° = \angle A$

{Exterior angle of a triangle}

$\angle DCE = 180° - 134°$
$= 46°$
(Supplementary Angles)

4. If <Q is Known, write an expression for the measure of one of the other two angles.



SOLUTION

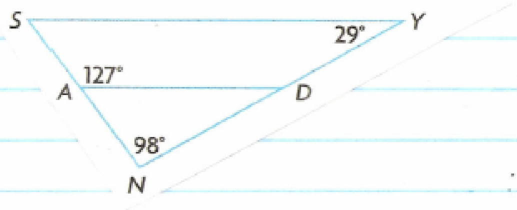
$$< R = \frac{180^\circ - < Q}{2}$$

6. Determine the measures of the exterior angles of an equilateral triangle.

SOLUTION

Since the measure of each interior angle in an equilateral triangle is 60°, each exterior angle would be $180° - 60°$
$\Rightarrow 120°$.

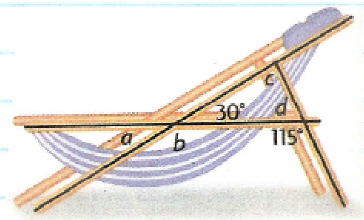
7. Prove SY || AD.



PROOF

Statement	Justification
<ASY = 53°	Sum of angles in a triangle is 180°
<SAD = 127°	Given
<ASY + <SAD = 180°	Property of equality.
SY || AD	Interior angles of the same side of the transversal are supplementary.

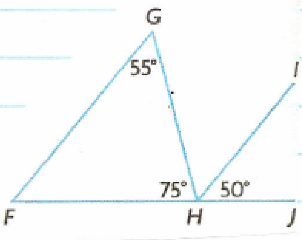
11. A manufacturer is designing a reclining lawn chair, as shown. Determine the measures of <a, <b, <c, and <d.



$\angle a = 30°$ (vertically opposite)
$\angle b = 180° - 30°$
$= 150°$ (supplementary angles)
$\angle d = 180° - 115°$
$= 65°$ (supplementary angles)
$\angle c = 180° - 65° - 30°$
$= 85°$ (angle sum of a Δ)

12.

a) Tim claims that FG is not parallel to HI because <FGH ≠ <IHJ. Do you agree or disagree? Justify your decision.

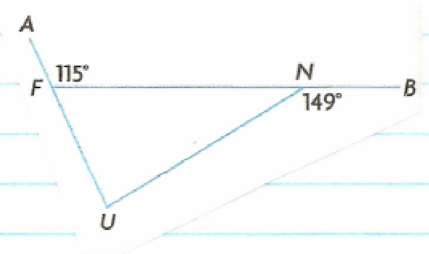


I disagree with Tim since <FGH and <IHJ are not corresponding, alternate interior, or alternate exterior angles.

b) How else could you justify your decision? Explain.

Since <GFH = 180°-55°-75° or 50°, this makes <GFH = <IHJ. Therefore, FG || HI since corresponding angles are equal.

14. Determine the measures of the interior angles of ΔFUN.



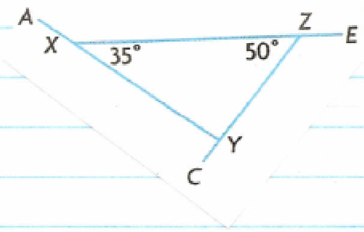
$\angle NFU = 180° - 115°$
$= 65°$
(Supplementary Angles)

$\angle FNU = 180° - 149°$
$= 31°$
(Supplementary Angles)

$\angle FUN = 180° - 65° - 31°$
$= 84°$
(Angle sum of a Δ)

15.

a) Determine the measures of <AXZ, <XYC, and <EZY.



$\angle AXZ = 180° - 35°$
$= 145°$

$\angle XYC = 180° - \angle XYZ$ *
$= 180° - 95°$
$= 85°$

* $\angle XYZ = 180° - 35° - 50°$
$= 95°$

$\angle EZY = 180° - 50°$
$= 130°$

Supplementary Angles

b) Determine the sum of the three exterior angles.

$\text{Sum} = 145° + 85° + 130°$
$= 360°$

Attachments

2s3e1 finalt.mp4

2s3e3 finalt2.mp4