Questions from Homework

Apply

6. Consider the function $f(x) = \frac{1}{4}\sqrt{5x}$.
   a) Identify the transformations represented by $f(x)$ as compared to $y = \sqrt{x}$.
   b) Write two functions equivalent to $f(x)$: one of the form $\boxed{y = a\sqrt{x}}$ and the other of the form $\boxed{y = \sqrt{bx}}$
   c) Identify the transformation(s) represented by each function you wrote in part b).
   d) Use transformations to graph all three functions. How do the graphs compare?

a) $a = \frac{1}{4} \rightarrow$ vertical stretch about the x-axis by a factor of $\frac{1}{4}$

$b = 5 \rightarrow$ horizontal stretch about the y-axis by a factor of $\frac{1}{5}$

b)(i) $y = \frac{1}{4}\sqrt{5x} = \frac{1}{4}\cdot\sqrt{5}\sqrt{x} = \frac{\sqrt{5}}{4}\sqrt{x}$ $(y = a\sqrt{x})$

$a = \frac{\sqrt{5}}{4}$

$VSF = \frac{\sqrt{5}}{4}$

(ii) $y = \frac{1}{4}\sqrt{5x} = \sqrt{\frac{1}{4}\cdot\frac{1}{4}\cdot 5x} = \sqrt{\frac{5}{16}x}$ $(y = \sqrt{bx})$

$b = \frac{5}{16}$

$HSF = \frac{16}{5}$

d) All three curves are the same.

9. $4 - y = \sqrt{3x}$

$$\frac{-y}{-1} = \frac{\sqrt{3x}}{-1} \quad \frac{-4}{-1}$$

$k \Rightarrow \uparrow 9$

$y = -\sqrt{3x} + 4$

translate 9 units up and reflect in x-axis

$a \rightarrow x(-1)$

$$\boxed{y = 1\sqrt{3x} + 13}$$

$a = 1 \quad b = 3 \quad h = 0 \quad k = 13$

a) D: $\{x \mid x \geq 0, x \in R\}$

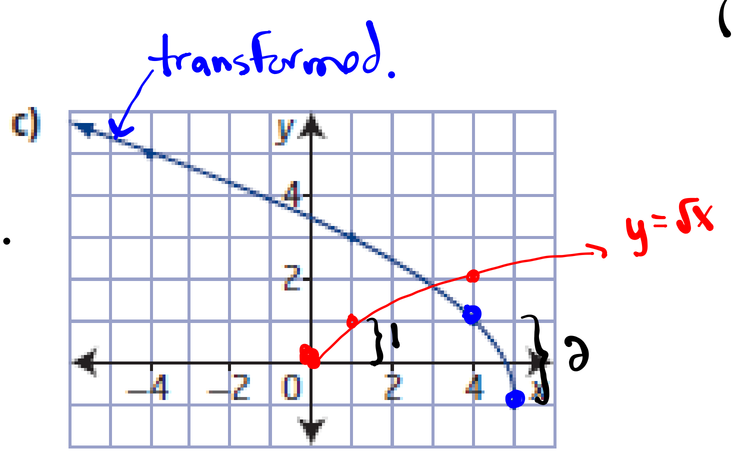
R: $\{y \mid y \geq 13, y \in R\}$

b) 0 units horizon

13 units up

⑩ Write equation in the form $y=a\sqrt{b(x-h)}+k$

c)



$(x,y) \rightarrow (-x+5,\ 2y-1)$

x	y
0	0
1	1
4	2
9	3

x	y
5	-1
4	1
1	3
-4	5

① Reflections: horizontal reflection in y-axis $(b<0)$

② $VSF = \frac{2}{1} = 2$ $(a=2)$

③ $HSF = 1$ $\left(b=\frac{1}{1}=-1\right)$

④ HT: $(0,0) \rightarrow (5,-1)$ 5 units right $(h=5)$

⑤ VT: $(0,0) \rightarrow (5,-1)$ 1 unit down $(k=-1)$

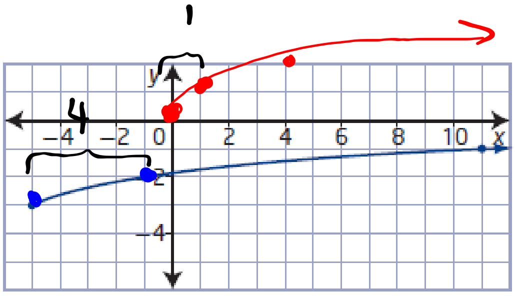
⑥ $y=a\sqrt{b(x-h)}+k$

$y=2\sqrt{-1(x-5)}+(-1)$

$\boxed{y=2\sqrt{-(x-5)}-1}$ or $y=\sqrt{-4(x-5)}-1$

(10) Write equation in the form $y = a\sqrt{b(x-h)} + k$

b)



① Reflections : none

② HSF $= \frac{4}{1} = 4$ $\quad b = \frac{1}{4}$

③ VSF → (skip) $\quad a = 1$

④ HT : $(0,0) \rightarrow (-5,-3)$ $\quad h = -5$

⑤ VT : $(0,0) \rightarrow (-5,-3)$ $\quad k = -3$

⑥ $y = 1\sqrt{\frac{1}{4}(x+5)} - 3$ or $y = \frac{1}{2}\sqrt{(x+5)} - 3$

Square Root of a Function

Focus on...

- sketching the graph of $y = \sqrt{f(x)}$ given the graph of $y = f(x)$
- explaining strategies for graphing $y = \sqrt{f(x)}$ given the graph of $y = f(x)$
- comparing the domains and ranges of the functions $y = f(x)$ and $y = \sqrt{f(x)}$, and explaining any differences

square root of a function

- the function $y = \sqrt{f(x)}$ is the square root of the function $y = f(x)$
- $y = \sqrt{f(x)}$ is only defined for $f(x) \geq 0$

The function $y = \sqrt{2x + 1}$ represents the **square root of the function** $y = 2x + 1$. linear

x	$y = 2x + 1$	$y = \sqrt{2x + 1}$
0	1	$\sqrt{1} = 1$
4	9	$\sqrt{9} = 3$
12	25	$\sqrt{25} = 5$
24	49	$\sqrt{49} = 7$
⋮	⋮	⋮

when $x = 0$

$y = 2x + 1$

$y = 2(0) + 1$

$y = 0 + 1$

$y = 1$

$y = 2x + 1$

x	y
0	1
4	9
12	25
24	49

$y = \sqrt{2x + 1}$

x	y
0	1
4	3
12	5
24	7

Example 1

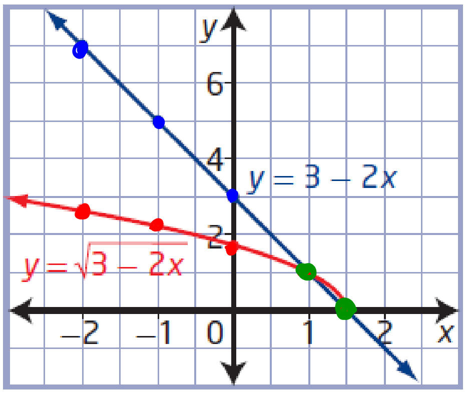
Compare Graphs of a Linear Function and the Square Root of the Function

linear

a) Given $f(x) = 3 - 2x$, graph the functions $y = f(x)$ and $y = \sqrt{f(x)}$.
b) Compare the two functions.

Use a table of values to graph $y = 3 - 2x$ and $y = \sqrt{3 - 2x}$.

x	$y = 3 - 2x$	$y = \sqrt{3 - 2x}$
−2	7	$\sqrt{7} = 2.7$
−1	5	$\sqrt{5} = 2.2$
0	3	$\sqrt{3} = 1.7$
1	1	$\sqrt{1} = 1$
1.5	0	$\sqrt{0} = 0$



Invariant points are (1, 1) and (1.5, 0)

$y = 3 - 2x$:

D: $\{x \mid x \in R\}$ or $(-\infty, \infty)$

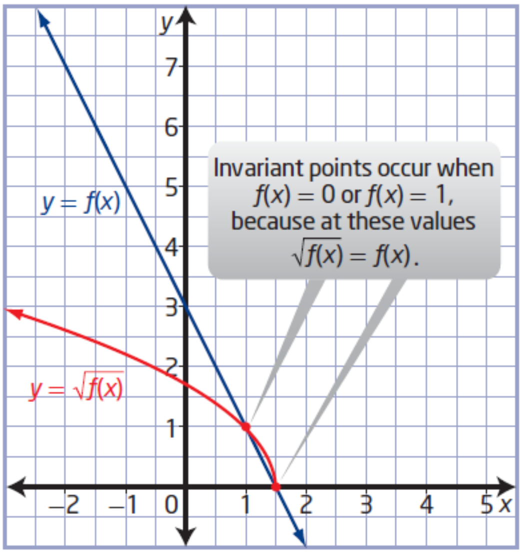
R: $\{y \mid y \in R\}$ or $(-\infty, \infty)$

$y = \sqrt{3 - 2x}$

D: $\{x \mid x \leq 1.5, x \in R\}$ or $(-\infty, 1.5]$

R: $\{y \mid y \geq 0, y \in R\}$ or $[0, \infty)$

b) Compare the graphs.



Why is the graph of $y = \sqrt{f(x)}$ above the graph of $y = f(x)$ for values of y between 0 and 1? Will this always be true?

For $y = f(x)$, the domain is $\{x \mid x \in \mathrm{R}\}$ and the range is $\{y \mid y \in \mathrm{R}\}$.
For $y = \sqrt{f(x)}$, the domain is $\{x \mid x \leq 1.5, x \in \mathrm{R}\}$ and the range is $\{y \mid y \geq 0, y \in \mathrm{R}\}$.

Invariant points occur at (1, 1) and (1.5, 0).

How does the domain of the graph of $y = \sqrt{f(x)}$ relate to the restrictions on the variable in the radicand? How could you determine the domain algebraically?

Relative Locations of $y = f(x)$ and $y = \sqrt{f(x)}$

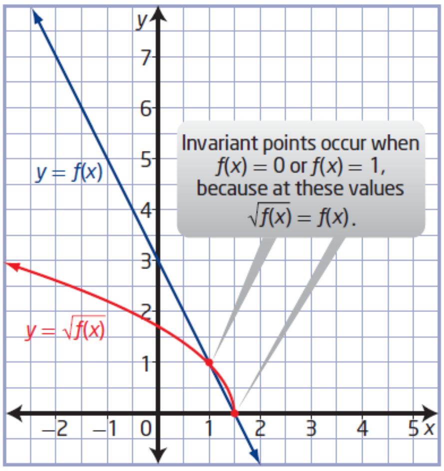
The domain of $y = \sqrt{f(x)}$ consists only of the values in the domain of $f(x)$ for which $f(x) \geq 0$.

The range of $y = \sqrt{f(x)}$ consists of the square roots of the values in the range of $y = f(x)$ for which $\sqrt{f(x)}$ is defined.

The graph of $y = \sqrt{f(x)}$ exists only where $f(x) \geq 0$. You can predict the location of $y = \sqrt{f(x)}$ relative to $y = f(x)$ using the values of $f(x)$.

	undefined	I.P.	Above f(x)	I.P.	Below f(x)
Value of $f(x)$	$f(x) < 0$	$f(x) = 0$	$0 < f(x) < 1$	$f(x) = 1$	$f(x) > 1$
Relative Location of Graph of $y = \sqrt{f(x)}$	The graph of $y = \sqrt{f(x)}$ is undefined.	The graphs of $y = \sqrt{f(x)}$ and $y = f(x)$ intersect on the x-axis.	The graph of $y = \sqrt{f(x)}$ is above the graph of $y = f(x)$.	The graph of $y = \sqrt{f(x)}$ intersects the graph of $y = f(x)$.	The graph of $y = \sqrt{f(x)}$ is below the graph of $y = f(x)$.

b) Compare the graphs.



Why is the graph of $y = \sqrt{f(x)}$ above the graph of $y = f(x)$ for values of y between 0 and 1? Will this always be true?

Your Turn

a) Given $g(x) = 3x + 6$, graph the functions $y = g(x)$ and $y = \sqrt{g(x)}$.

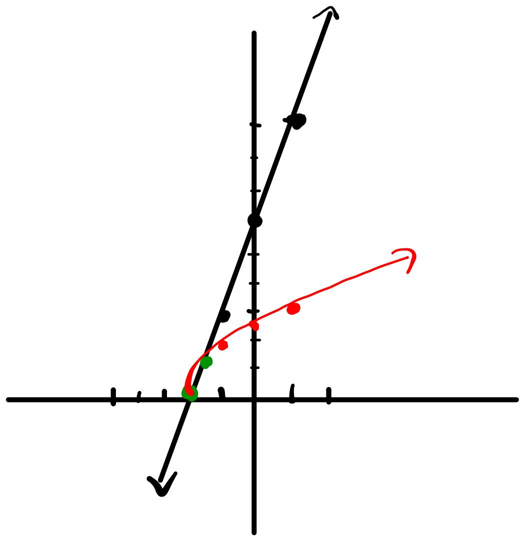
b) Identify the domain and range of each function and any invariant points.

$y = 3x + 6$

x	y
-2	0
-1	3
0	6
1	9
2	12

$y = \sqrt{3x+6}$

x	y
-2	0
-1	1.7
0	2.4
1	3
2	3.5



D: $\{x \mid x \in R\}$

R: $\{y \mid y \in R\}$

D: $\{x \mid x \geq -2, x \in R\}$

R: $\{y \mid y \geq 0, y \in R\}$

Invariant Points: $(-2, 0)$ + $(-\frac{5}{3}, 1)$

$3x + 6 = 1$

$3x = -5$

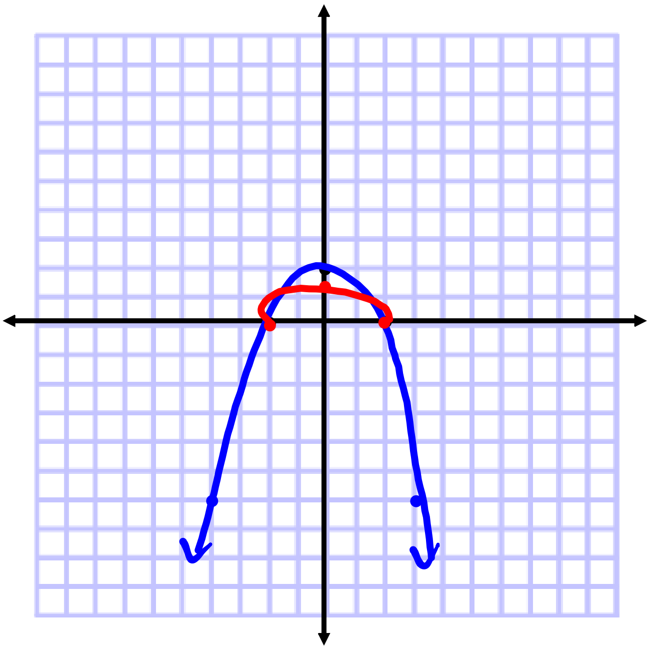
$x = \frac{-5}{3}$

Example 2

Compare the Domains and Ranges of $y = f(x)$ and $y = \sqrt{f(x)}$

Identify and compare the domains and ranges of the functions in each pair.

a) $y = 2 - 0.5x^2$ and $y = \sqrt{2 - 0.5x^2}$



$y = 2 - 0.5x^2$

x	y
-4	-6
-2	0
0	2
2	0
4	-6

$y = \sqrt{2 - 0.5x^2}$

x	y
-4	und.
-2	0
0	1.41
2	0
4	und.

D: $\{x \mid x \in R\}$

R: $\{y \mid y \leq 2, y \in R\}$

D: $\{x \mid -2 \leq x \leq 2, x \in R\}$

R: $\{y \mid 0 \leq y \leq 1.41, y \in R\}$

Example 3

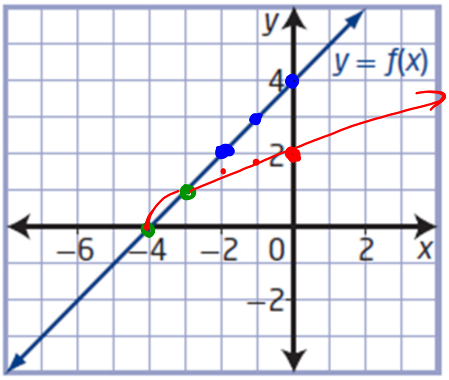
Graph the Square Root of a Function From the Graph of the Function

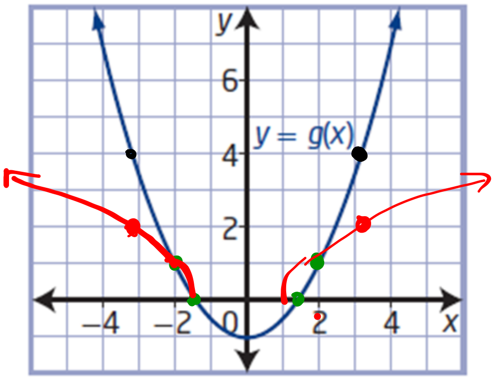
Step 1: Locate invariant points height of 0 or 1 $\left(\begin{matrix} y=0 \\ y=1 \end{matrix}\right)$

Step 2: Draw the portion of each graph between the invariant points $\left(\sqrt{f(x)} \text{ above } f(x)\right)$

Step 3: Locate other key points on $y = f(x)$ and $y = g(x)$ where the values are greater than 1. Transform these points to locate image points on the graphs of $y = \sqrt{f(x)}$ and $y = \sqrt{g(x)}$. $\left(\sqrt{f(x)} \text{ below } f(x)\right)$

Using the graphs of $y = f(x)$ and $y = g(x)$, sketch the graphs of $y = \sqrt{f(x)}$ and $y = \sqrt{g(x)}$.





$y = f(x)$

D: $\{x \mid x \in \mathbb{R}\}$ or $(-\infty, \infty)$

R: $\{y \mid y \in \mathbb{R}\}$ or $(-\infty, \infty)$

$y = \sqrt{f(x)}$

D: $\{x \mid x \geq -4, x \in \mathbb{R}\}$ or $[-4, \infty)$

R: $\{y \mid y \geq 0, y \in \mathbb{R}\}$ or $[0, \infty)$

$y = g(x)$

D: $\{x \mid x \in \mathbb{R}\}$ or $(-\infty, \infty)$

R: $\{y \mid y \geq -1, y \in \mathbb{R}\}$ or $[-1, \infty)$

$y = \sqrt{g(x)}$

D: $\{x \mid x \leq -1.5, x \geq 1.5, x \in \mathbb{R}\}$

or $(-\infty, -1.5] + [1.5, \infty)$

R: $\{y \mid y \geq 0, y \in \mathbb{R}\}$ or $[0, \infty)$

Key Ideas

- You can use values of $f(x)$ to predict values of $\sqrt{f(x)}$ and to sketch the graph of $y = \sqrt{f(x)}$.
- The key values to consider are $f(x) = 0$ and $f(x) = 1$.
- The domain of $y = \sqrt{f(x)}$ consists of all values in the domain of $f(x)$ for which $f(x) \geq 0$.
- The range of $y = \sqrt{f(x)}$ consists of the square roots of all values in the range of $f(x)$ for which $f(x)$ is defined.
- The y-coordinates of the points on the graph of $y = \sqrt{f(x)}$ are the square roots of the y-coordinates of the corresponding points on the original function $y = f(x)$.

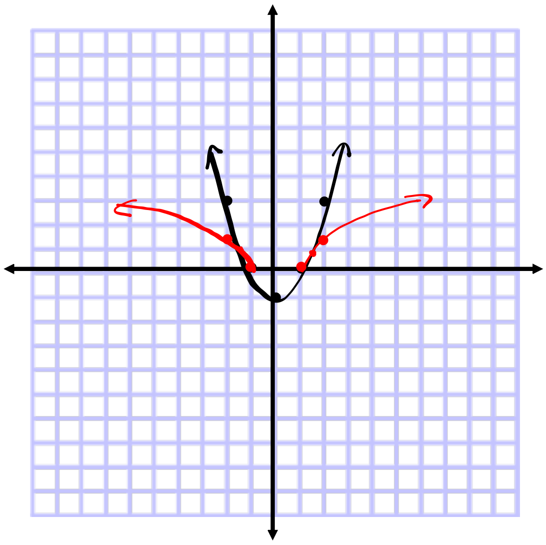
What do you know about the graph of $y = \sqrt{f(x)}$ at $f(x) = 0$ and $f(x) = 1$? How do the graphs of $y = f(x)$ and $y = \sqrt{f(x)}$ compare on either side of these locations?

Your Turn

1) Identify and compare the domains and ranges of the functions $y = x^2 - 1$ and $y = \sqrt{x^2 - 1}$. Verify your answers.

$y = x^2 - 1$

x	y
-2	3
-1	0
0	-1
1	0
2	3



$y = \sqrt{x^2 - 1}$

x	y
-2	$\sqrt{3} = 1.7$
-1	$\sqrt{0} = 0$
0	$\sqrt{-1}$ = undefined
1	$\sqrt{0} = 0$
2	$\sqrt{3} = 1.71$

D: $\{x \mid x \in R\}$ or $(-\infty, \infty)$

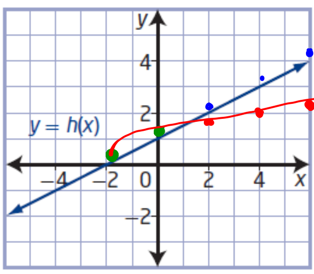
R: $\{y \mid y \geq -1, y \in R\}$ or $[-1, \infty)$

D: $\{x \mid x \leq -1$ and $x \geq 1, x \in R\}$

$(-\infty, -1]$ and $[1, \infty)$

R: $\{y \mid y \geq 0, y \in R\}$ or $[0, \infty)$

2) Using the graph of $y = h(x)$, sketch the graph of $y = \sqrt{h(x)}$.



(i) Locate invariant points $(-2, 0) + (0, 1)$

(ii) draw the portion of square root curve between invariant points. (above the original)

(iii) Locate key points on original and transform them (take square root of y-values)

$y = h(x)$

D: $\{x \mid x \in R\}$ or $(-\infty, \infty)$

R: $\{y \mid y \in R\}$ or $(-\infty, \infty)$

$y = \sqrt{h(x)}$

D: $\{x \mid x \geq -2, x \in R\}$ or $[-2, \infty)$

R: $\{y \mid y \geq 0, y \in R\}$ or $[0, \infty)$

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