

## Review 1

$$\textcircled{1} \text{ a) } x^2 + 2xy + y^2 = 7$$

$$2x + 2y + 2x \frac{dy}{dx} + 2y \frac{dy}{dx} = 0$$

$$2x \frac{dy}{dx} + 2y \frac{dy}{dx} = -2x - 2y$$

$$\frac{dy}{dx} (2x + 2y) = -2x - 2y$$

$$\frac{dy}{dx} = \frac{-2x - 2y}{2x + 2y}$$

$$\frac{dy}{dx} = \frac{-2(x+y)}{2(x+y)} = \boxed{-1}$$

$$\text{b) } 6x^3 + 3x^2y = 4y^2$$

$$18x^2 + 6xy + 3x^2 \frac{dy}{dx} = 8y \frac{dy}{dx}$$

$$18x^2 + 6xy = 8y \frac{dy}{dx} - 3x^2 \frac{dy}{dx}$$

$$18x^2 + 6xy = \frac{dy}{dx} (8y - 3x^2)$$

$$\boxed{\frac{18x^2 + 6xy}{8y - 3x^2} = \frac{dy}{dx}}$$

## Review 1

$$\textcircled{a} \quad s = t^3 - 4.5t^2 + 6t$$

$$s' = v = 3t^2 - 9t + 6$$

$$s'' = a = 6t - 9$$

a)  $v$  @ 3 sec.

$$v = 3(3)^2 - 9(3) + 6$$

$$v = 6 \text{ m/s}$$

b)  $a$  @ 4 sec.

$$a = 6(4) - 9$$

$$a = 15 \text{ m/s}^2$$

c) at rest ( $v=0$ )

$$0 = 3t^2 - 9t + 6$$

$$0 = 3(t^2 - 3t + 2)$$

$$0 = 3(t-1)(t-2)$$

$$t-1=0 \quad | \quad t-2=0$$

$$t=1 \quad | \quad t=2$$

d)  $a > 0$

$$6t - 9 > 0$$

$$6t > 9$$

$$t > 1.5 \text{ s}$$

Review 1

③ Given

$$\frac{dv}{dt} = 2 \text{ m}^3/\text{min}$$

$$\underline{r = 2h}$$

$$\frac{dh}{dt} = ?$$

$$h = 4 \text{ m}$$



$$V = \frac{1}{3} \pi r^2 h$$

Express in terms of h only

$$V = \frac{1}{3} \pi (2h)^2 h$$

$$V = \frac{1}{3} \pi 4h^2 \cdot h$$

$$V = \frac{4\pi h^3}{3}$$

$$\frac{dv}{dt} = 4\pi h^2 \frac{dh}{dt}$$

$$2 = 4\pi (4)^2 \frac{dh}{dt}$$

$$2 = 64\pi \frac{dh}{dt}$$

$$\frac{2}{64\pi} \text{ m}^3/\text{min} = \frac{dh}{dt}$$

$$0.009947 \text{ m}^3/\text{min} = \frac{dh}{dt}$$

## Review 1

④ Given:

$$l = 5\text{cm}$$

$$\frac{dv}{dt} = 125\text{cm}^3/\text{s}$$

$$v = l^3$$

$$\frac{dv}{dt} = 3l^2 \frac{dl}{dt}$$

$$125 = 3(5)^2 \frac{dl}{dt}$$

$$125 = 75 \frac{dl}{dt}$$

$$\frac{5\text{cm/s}}{3} = \frac{dl}{dt}$$

$$1.\bar{6}\text{cm/s} = \frac{dl}{dt}$$

## Review 1

⑤ Given:

$$\frac{dr}{dt} = 10 \text{ cm/s}$$

$$t = 3 \text{ s}$$

(i) Find  $r$ :

$$r = 10 \text{ cm/s} \times 3 \text{ s}$$

$$r = 30 \text{ cm}$$

(ii) Find  $\frac{da}{dt}$ :

$$a = \pi r^2$$

$$\frac{da}{dt} = 2\pi r \frac{dr}{dt}$$

$$\frac{da}{dt} = 2\pi (30)(10)$$

$$\frac{da}{dt} = 600\pi \text{ cm}^2/\text{s}$$

$$\frac{da}{dt} = 1884.96 \text{ cm}^2/\text{s}$$

## Review 1

⑥ Given:

$$\frac{dv}{dt} = 6\text{m}^3/\text{min}$$

$$\frac{dr}{dt} = ?$$

$$r = 2\text{m}$$

$$V = \frac{4\pi r^3}{3}$$

$$\frac{dv}{dt} = 4\pi r^2 \frac{dr}{dt}$$

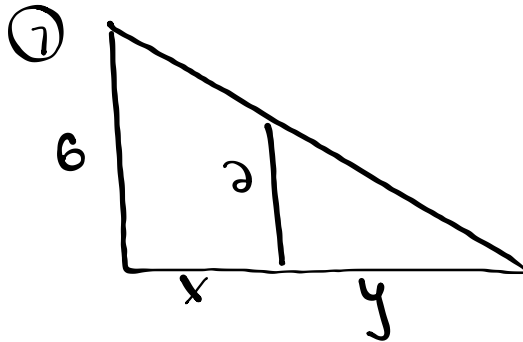
$$6 = 4\pi(2)^2 \frac{dr}{dt}$$

$$6 = 16\pi \frac{dr}{dt}$$

$$\frac{3}{8\pi} \text{ m/min} = \frac{dr}{dt}$$

$$0.119 \text{ m/min} = \frac{dr}{dt}$$

## Review 1



Given:

$$\frac{dx}{dt} = -1.5 \text{ m/s} \quad (\text{towards} = (-))$$

using similar triangles:

$$\frac{x+y}{6} = \frac{y}{2}$$

$$2x + 2y = 6y$$

$$2x = 4y$$

$$2 \frac{dx}{dt} = 4 \frac{dy}{dt}$$

$$2(-1.5) = 4 \frac{dy}{dt}$$

$$-3 = 4 \frac{dy}{dt}$$

$$-\frac{3}{4} \text{ m/s} = \frac{dy}{dt}$$

$$-0.75 \text{ m/s} = \frac{dy}{dt}$$

## Review 1

8 If  $x^2y + y^2 = 22$  Find  $\frac{dx}{dt}$  when  $y=2$  and  $\frac{dy}{dt} = 4$

(i) Find  $x$ :

$$x^2(2) + (2)^2 = 22$$

$$2x^2 + 4 = 22$$

$$2x^2 = 18$$

$$x^2 = 9$$

$$x = \pm 3$$

(ii) Find  $\frac{dx}{dt}$

$$x^2y + y^2 = 22$$

$$2x \frac{dx}{dt} y + x^2 \frac{dy}{dt} + 2y \frac{dy}{dt} = 0$$

$$(i) 2(3) \frac{dx}{dt} (2) + (3)^2(4) + 2(2)(4) = 0$$

$$12 \frac{dx}{dt} + 36 + 16 = 0$$

$$12 \frac{dx}{dt} = -52$$

$$\frac{dx}{dt} = -4.3$$

$$(ii) 2(3) \frac{dx}{dt} (2) + (3)^2(4) + 2(2)(4) = 0$$

$$-12 \frac{dx}{dt} + 36 + 16 = 0$$

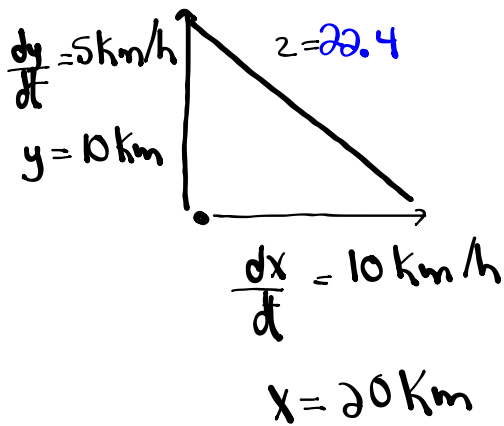
$$-12 \frac{dx}{dt} = -52$$

$$\frac{dx}{dt} = 4.3$$



## Review 1

⑨

(1) Find  $z$ :

$$x^2 + y^2 = z^2$$

$$(20)^2 + (10)^2 = z^2$$

$$400 + 100 = z^2$$

$$500 = z^2$$

$$\pm 22.4 = z$$

$$\boxed{22.4 = z}$$

(2) Find  $\frac{dz}{dt}$ :

$$x^2 + y^2 = z^2$$

$$2x \frac{dx}{dt} + 2y \frac{dy}{dt} = 2z \frac{dz}{dt}$$

$$2(20)(10) + 2(10)(5) = 2(22.4) \frac{dz}{dt}$$

$$400 + 100 = 44.8 \frac{dz}{dt}$$

$$500 = 44.8 \frac{dz}{dt}$$

$$\boxed{11.2 \text{ km/h} = \frac{dz}{dt}}$$

Review 2:

$$\textcircled{1} \text{ a) } x^3 + 2x^2y + y^3 = 12$$

$$3x^2 + 4xy + 2x^2 \frac{dy}{dx} + 3y^2 \frac{dy}{dx} = 0$$

$$2x^2 \frac{dy}{dx} + 3y^2 \frac{dy}{dx} = -3x^2 - 4xy$$

$$\frac{dy}{dx} (2x^2 + 3y^2) = -3x^2 - 4xy$$

$$\frac{dy}{dx} = \frac{-3x^2 - 4xy}{2x^2 + 3y^2} = -\frac{3x^2 + 4xy}{2x^2 + 3y^2}$$

$$\text{b) } 2xy^2 - y^3 = x^2$$

$$2y^2 + 2x \cdot 2y \frac{dy}{dx} - 3y^2 \frac{dy}{dx} = 2x$$

$$\textcircled{2y^2} + 4xy \frac{dy}{dx} - 3y^2 \frac{dy}{dx} = 2x$$

$$\frac{dy}{dx} (4xy - 3y^2) = 2x - 2y^2$$

$$\boxed{\frac{dy}{dx} = \frac{2x - 2y^2}{4xy - 3y^2}}$$

Review 2

$$\textcircled{a} \quad s = 2t^3 - 21t^2 + 60t$$

$$v = 6t^2 - 42t + 60$$

$$a = 12t - 42$$

$$\text{a) } v = 6(3)^2 - 42(3) + 60$$

$$v = 54 - 126 + 60$$

$$v = -12 \text{ m/s}$$

$$\text{b) } 0 = 6t^2 - 42t + 60$$

$$0 = 6(t^2 - 7t + 10)$$

$$0 = 6(t-2)(t-5)$$

$$\begin{array}{l|l} t-2=0 & t-5=0 \\ t=2 \text{ sec} & t=5 \text{ sec} \end{array}$$

$$\text{c) } a = 12(4) - 42$$

$$a = 48 - 42$$

$$a = 6 \text{ m/s}^2$$

$$\text{d) } 12t - 42 > 0$$

$$12t > 42$$

$$t > 3.5 \text{ sec}$$

Review 2

③ Given:

$$A = 4\pi r^2$$

$$\frac{dA}{dt} = -6 \text{ m}^2/\text{hour}$$

$$\frac{dA}{dt} = 8\pi r \frac{dr}{dt}$$

$$\frac{dr}{dt} = ?$$

$$-6 = 8\pi(3) \frac{dr}{dt}$$

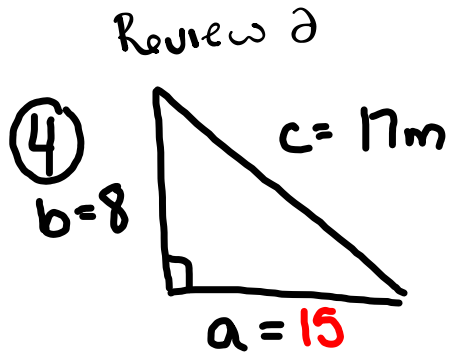
$$r = 3 \text{ m}$$

$$-6 = 24\pi \frac{dr}{dt}$$

$$\frac{-6}{24\pi} = \frac{dr}{dt}$$

$$-\frac{1}{4\pi} \text{ m/hour} = \frac{dr}{dt}$$

$$-0.0796 \text{ m/hour} = \frac{dr}{dt}$$



Given

$$\frac{db}{dt} = -4\text{m/s} \quad \frac{da}{dt} = ?$$

$$b = 8$$

$$a = \underline{15}$$

$$a^2 + b^2 = c^2$$

$$a^2 + b^2 = 17^2$$

$$2a \frac{da}{dt} + 2b \frac{db}{dt} = 0$$

$$2(15) \frac{da}{dt} + 2(8)(-4) = 0$$

$$30 \frac{da}{dt} = 64$$

$$\frac{da}{dt} = 2.13\text{m/s}$$

Review 2

⑤ Given:

$$h \frac{r}{h} = \frac{3}{6} h$$

$$r = \frac{3}{6} h = \frac{1}{2} h$$

$$\frac{dV}{dt} = \pi m^3 / \text{min}$$

$$h = 3$$

$$\frac{dh}{dt} = ?$$

$$V = \frac{1}{3} \pi r^2 h$$

$$V = \frac{1}{3} \pi \left(\frac{1}{2} h\right)^2 h$$

$$V = \frac{1}{3} \pi \frac{h^2}{4} \cdot h$$

$$V = \frac{1}{12} \pi h^3$$

$$\frac{dV}{dt} = \frac{1}{4} \pi h^2 \frac{dh}{dt}$$

$$\pi = \frac{1}{4} \pi (3) \frac{dh}{dt}$$

$$\pi = \frac{9\pi}{4} \frac{dh}{dt}$$

$$\frac{4}{9} m / \text{min} = \frac{dh}{dt}$$

$$0.4 \bar{4} m / \text{min} = \frac{dh}{dt}$$

## Review 2

⑥ Given:

$$\frac{dV}{dt} = 12 \text{ m}^3/\text{min}$$

$$r = 4 \text{ m}$$

$$\frac{dr}{dt} = ?$$

$$V = \frac{4}{3} \pi r^3$$

$$\frac{dV}{dt} = 4\pi r^2 \frac{dr}{dt}$$

$$12 = 4\pi (4)^2 \frac{dr}{dt}$$

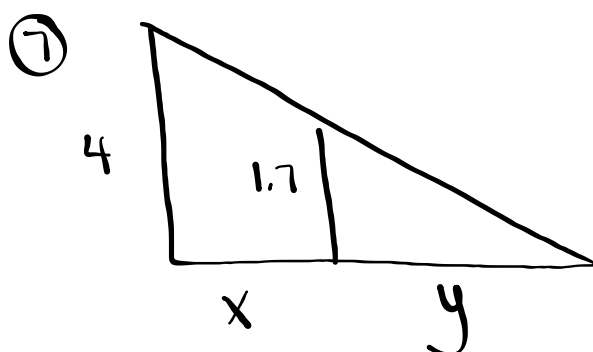
$$12 = 64\pi \frac{dr}{dt}$$

$$\frac{12}{64\pi} = \frac{dr}{dt}$$

$$\frac{3}{16\pi} \text{ m/min} = \frac{dr}{dt}$$

$$0.0597 \text{ m/min} = \frac{dr}{dt}$$

Review 2



$$\frac{dx}{dt} = -0.8 \text{ m/s} \quad \frac{dy}{dt} = ?$$

$$\frac{x+y}{4} = \frac{y}{1.7}$$

$$1.7x + 1.7y = 4y$$

$$1.7x = 2.3y$$

$$1.7 \frac{dx}{dt} = 2.3 \frac{dy}{dt}$$

$$1.7(-0.8) = 2.3 \frac{dy}{dt}$$

$$-1.36 = 2.3 \frac{dy}{dt}$$

$$-0.59 \text{ m/s} = \frac{dy}{dt}$$



Review 2

⑧  $x = y^3 + y$  show that  $y'' = \frac{-6y}{(3y^2+1)^3}$

$$1 = 3y^2 \frac{dy}{dx} + \frac{dy}{dx}$$

$$1 = \frac{dy}{dx} [3y^2 + 1]$$

$$\frac{1}{3y^2+1} = \frac{dy}{dx}$$

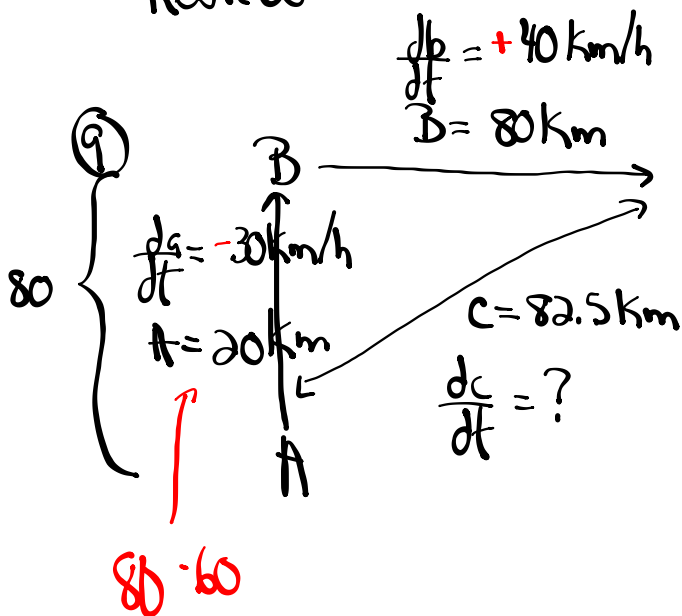
$$\frac{dy}{dx} = y' = \frac{1}{3y^2+1} = (3y^2+1)^{-1}$$

$$y'' = -1(3y^2+1)^{-2} \left(6y \frac{dy}{dx}\right)$$

$$y'' = \frac{-6y \frac{dy}{dx}}{(3y^2+1)^2} = \frac{-6y \left(\frac{1}{3y^2+1}\right)}{(3y^2+1)^2} = \frac{-6y}{3y^2+1} \times \frac{1}{(3y^2+1)^2}$$

$$= \frac{-6y}{(3y^2+1)^3}$$

Review 2

(1) Find  $C$ 

$$a^2 + b^2 = c^2$$

$$20^2 + 80^2 = c^2$$

$$400 + 6400 = c^2$$

$$\sqrt{6800} = c$$

$$82.5 = c$$

(ii) Find  $\frac{dc}{dt}$ 

$$a^2 + b^2 = c^2$$

$$2a \frac{da}{dt} + 2b \frac{db}{dt} = 2c \frac{dc}{dt}$$

$$2(20)(-30) + 2(80)(40) = 2(82.5) \frac{dc}{dt}$$

$$-1200 + 6400 = 165 \frac{dc}{dt}$$

$$5200 = 165 \frac{dc}{dt}$$

$$31.5 \text{ km/h} = \frac{dc}{dt}$$

### Steps in Logarithmic Differentiation

1. Take logarithms of both sides of an equation. (ln)
2. Differentiate implicitly with respect to  $x$ .
3. Solve the resulting equation for  $y'$

Use Logarithmic Differentiation to Differentiate the following:

$$y = \frac{e^x \sqrt{x^2 + 1}}{(x^2 + 2)^3}$$

$$\ln y = \ln \left[ \frac{e^x (x^2 + 1)^{1/2}}{(x^2 + 2)^3} \right]$$

$$\ln y = \ln e^x + \ln (x^2 + 1)^{1/2} - \ln (x^2 + 2)^3$$

$$\ln y = x \ln e + \frac{1}{2} \ln (x^2 + 1) - 3 \ln (x^2 + 2)$$

$$\ln y = x + \frac{1}{2} \ln (x^2 + 1) - 3 \ln (x^2 + 2)$$

$$\frac{y'}{y} = 1 + \frac{1}{2} \left( \frac{2x}{x^2 + 1} \right) - 3 \left( \frac{2x}{x^2 + 2} \right)$$

$$\frac{y'}{y} = \left( 1 + \frac{x}{x^2 + 1} - \frac{6x}{x^2 + 2} \right) \cdot y$$

$$y' = \left[ 1 + \frac{x}{x^2 + 1} - \frac{6x}{x^2 + 2} \right] \left[ \frac{e^x \sqrt{x^2 + 1}}{(x^2 + 2)^3} \right]$$

$$y = x^{x^5}$$
$$\ln y = \ln x^{x^5}$$
$$\ln y = x^5 \ln x \quad \leftarrow \text{product rule}$$

$$\frac{y'}{y} = 5x^4 \ln x + x^5 \left( \frac{1}{x} \right)$$

$$\cancel{y} \cdot \frac{y'}{\cancel{y}} = 5x^4 \ln x + x^4 \cdot y$$
$$y' = (5x^4 \ln x + x^4)(x^{x^5})$$

$$A = l^2 \text{ (Square)}$$

$$V = l^3 \text{ (Cube)}$$

$$A = \pi r^2 \text{ (Circle)}$$

$$A = 4\pi r^2 \text{ (Sphere)}$$

$$V = \frac{4}{3}\pi r^3 \text{ (Sphere)}$$

$$A = \frac{1}{2}bh \text{ (Triangle)}$$

$$V = \frac{1}{3}\pi r^2 h \text{ (Cone)}$$

$$x^2 + y^2 = z^2 \leftarrow z \text{ is constant (Ladder)}$$

$$x^2 + y^2 = z^2 \text{ (Intersection)}$$

$$\frac{h^{\text{of post}}}{x+y} = \frac{h^{\text{of man}}}{y} \text{ (Lamppost)}$$