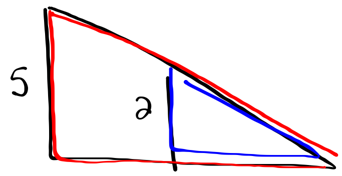
Questions From Homework

①



$\frac{dx}{dt} = 1.5 m/s$

$\frac{x+y}{5} = \frac{y}{2}$

$2x + 2y = 5y$

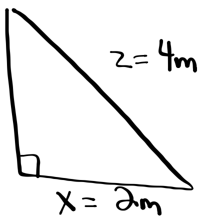
$2x = 3y$

$2\frac{dx}{dt} = 3\frac{dy}{dt}$

$2(1.5) = 3\frac{dy}{dt}$

$\boxed{1 m/s = \frac{dy}{dt}}$

②



$y = 3.46$

$\frac{dy}{dt} =$

$\frac{dx}{dt} = 0.3 m/s$

(1) Find y

$x^2 + y^2 = z^2$

$(2)^2 + y^2 = (4)^2$

$y^2 = 12$

$y = 3.46 m$

(2) Find $\frac{dy}{dt}$ constant

$x^2 + y^2 = z^2$

$2x\frac{dx}{dt} + 2y\frac{dy}{dt} = 0$

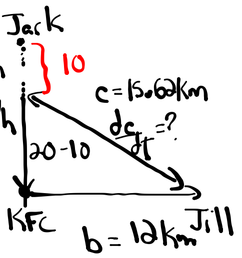
$2(2)(0.3) + 2(3.46)\frac{dy}{dt} = 0$

$1.2 + 6.92\frac{dy}{dt} = 0$

$6.92\frac{dy}{dt} = -1.2$

$\boxed{\frac{dy}{dt} = -0.1734 m/s}$

⑤



$a = 10km$

$\frac{da}{dt} = -5km/h$

$b = 12km$

$\frac{db}{dt} = +6km/h$

(i) Find c

$a^2 + b^2 = c^2$

$10^2 + 12^2 = c^2$

$100 + 144 = c^2$

$244 = c^2$

$15.62 = c$

(ii) Find $\frac{dc}{dt}$

$a^2 + b^2 = c^2$

$2a\frac{da}{dt} + 2b\frac{db}{dt} = 2c\frac{dc}{dt}$

$2(10)(-5) + 2(12)(6) = 2(15.62)\frac{dc}{dt}$

$-100 + 144 = 31.24\frac{dc}{dt}$

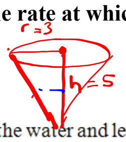
$44 = 31.24\frac{dc}{dt}$

$\boxed{1.41 km/h = \frac{dc}{dt}}$

Jack is headed south at 60 km/h towards JMH and Jill is headed west towards the school at 50 km/h. At what rate is the distance between them closing when Jack is 2 km and Jill is 3 km from the school?

(Hint: draw a diagram)

A water tank is built in the shape of a circular cone with height 5 m and diameter 6 m at the top. Water is being pumped into the tank at a rate of 1.6 m^3/min. Find the rate at which the water level is rising when the water is 2 m deep?



Let V be the volume of the water and let r and h be the radius of the surface and the height at time t, where t is measured in minutes. We are given the rate of increase of V, that is:

$$\frac{dV}{dt} = 1.6m^3/\text{min}$$

Given:
$\frac{dV}{dt} = 1.6m^3/\text{min}$ $h = 2m$
$\frac{dh}{dt} = ?$

We are asked to find $\frac{dh}{dt}$ when $h = 2m$.

The quantities V and h are related by the equation:

$$V = \frac{1}{3}\pi r^2 h$$

But we have to express V as a function of h alone. To eliminate r we look for a relationship between r and h. We use similar triangles in the figure to write.

$\frac{r}{h} = \frac{3}{5}$ Thus $r = \frac{3}{5}h$ and we have:

$$5r = 3h$$

$$\boxed{r = \frac{3h}{5}}$$

$$V = \frac{1}{3}\pi r^2 h \quad \text{(express V in terms of h only)}$$

$$V = \frac{1}{3}\pi \left(\frac{3h}{5}\right)^2 h$$

$$V = \frac{1}{3}\pi \left(\frac{9h^2}{25}\right) h$$

$$V = \frac{9\pi h^3}{75}$$

$$V = \frac{3}{25}\pi h^3$$

$$\frac{dV}{dt} = \frac{9}{25}\pi h^2 \frac{dh}{dt}$$

$$1.6 = \frac{9}{25}\pi (2)^2 \frac{dh}{dt}$$

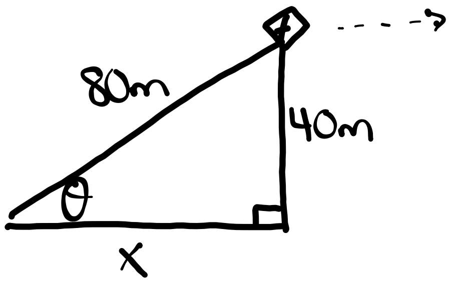
$$1.6 = \frac{36\pi}{25}\frac{dh}{dt}$$

$$1.6 \div \frac{36\pi}{25} = \frac{dh}{dt}$$

$$1.6 \times \frac{25}{36\pi} = \frac{dh}{dt}$$

$$0.3536\text{m/min} = \frac{dh}{dt}$$

A kite 40 m above the ground moves horizontally at a rate of 3 m/s. At what rate is the angle between the string and the horizontal decreasing when 80 m of string is let out?



Find $\frac{d\theta}{dt}$

$\frac{dx}{dt} = 3\text{ m/s}$

$\tan\theta = \frac{40}{x}$

$\tan\theta = 40x^{-1}$

$x^2+y^2=z^2$

$x^2+(40)^2=(80)^2$

$x^2 = 6400-1600$

$\boxed{x^2 = 4800}$

$\sec^2\theta \frac{d\theta}{dt} = -40x^{-2}\frac{dx}{dt}$

$\sec^2\theta \frac{d\theta}{dt} = -\frac{40}{x^2}\cdot\frac{dx}{dt}$

$\frac{d\theta}{dt} = \boxed{\frac{1}{\sec^2\theta}}\cdot -\frac{40}{x^2}\cdot\frac{dx}{dt}$

$\cos^2\theta = \frac{adj^2}{hyp^2}$

$\cos^2\theta = \frac{4800}{(80)^2}$

$\cos^2\theta = \frac{4800}{6400}$

$\cos^2\theta = \frac{3}{4}$

$\frac{d\theta}{dt} = \cos^2\theta\cdot -\frac{40}{x^2}\cdot\frac{dx}{dt}$

$\frac{d\theta}{dt} = \left(\frac{\cancel{3}^{\,1}}{\cancel{4}_{\,1}}\right)\left(-\frac{\cancel{40}^{\,\cancel{-10}\ -1}}{\cancel{4800}_{\,\cancel{1600}\ 160}}\right)(3)$

$\frac{d\theta}{dt} = -\frac{3}{160}$ rads/s

$\frac{d\theta}{dt} = -0.01875$ rads/s

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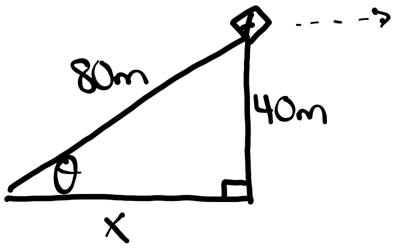


Find $\frac{d\theta}{dt}$

$$\tan\theta = \frac{40}{x}$$

$$\frac{dx}{dt} = 3\,m/s$$

$$\tan\theta = 40x^{-1}$$

$$\theta = \tan^{-1}(40x^{-1})$$

$$x^2+y^2=z^2$$

$$x^2+(40)^2=(80)^2$$

$$x^2=6400-1600$$

$$\boxed{x^2=4800}$$

$$\frac{d\theta}{dt} = \frac{1}{1+\left(\frac{40}{x}\right)^2} \cdot -40x^{-2} \cdot \frac{dx}{dt}$$

$$\frac{d\theta}{dt} = \frac{1}{1+\frac{1600}{x^2}} \cdot -\frac{40}{x^2} \cdot \frac{dx}{dt}$$

$$\frac{d\theta}{dt} = \frac{1}{1+\frac{\cancel{1600}^{1}}{\cancel{4800}_{3}}} \cdot \frac{\cancel{-40}^{-1}}{\cancel{4800}_{120}} \cdot 3$$

$$\frac{d\theta}{dt} = \frac{1}{4/3} \cdot \frac{-1}{120} \cdot 3$$

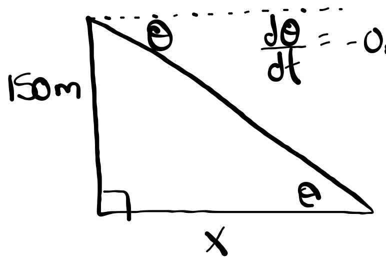
$$\frac{d\theta}{dt} = \frac{\cancel{3}^{1}}{4} \cdot -\frac{1}{\cancel{120}_{40}} \cdot 3$$

$$\frac{d\theta}{dt} = \frac{-3}{160} \text{ rads/sec}$$

$$\frac{d\theta}{dt} = -0.01875 \text{ rads/sec}$$

A car passes directly under a police helicopter 150 m above a straight and level highway. After the car has travelled another 20.0 m, the angle of depression of the car from the helicopter is decreasing at the rate of 0.215 rad/s. what is the speed of the car? Ans: 32.82 m/s

A car passes directly under a police helicopter 150 m above a straight and level highway. After the car has travelled another 20.0 m, the angle of depression of the car from the helicopter is decreasing at the rate of 0.215 rad/s. what is the speed of the car? Ans: 32.82 m/s



$$\frac{d\theta}{dt} = -0.215 \text{ m/s}$$

$$\frac{dx}{dt} = ?$$

$$\tan\theta = \frac{150}{x}$$

$$\tan\theta = 150x^{-1}$$

$$\theta = \tan^{-1}(150x^{-1})$$

$$\frac{d\theta}{dt} = \frac{1}{1+\left(\frac{150}{x}\right)^2} \cdot -150x^{-2} \cdot \frac{dx}{dt}$$

$$\frac{d\theta}{dt} = \frac{1}{1+\left(\frac{22500}{x^2}\right)} \cdot \frac{-150}{x^2} \cdot \frac{dx}{dt}$$

$$-0.215 = \frac{1}{1+\frac{22500}{400}} \cdot \frac{-150}{400} \cdot \frac{dx}{dt}$$

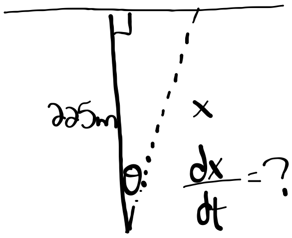
$$-0.215 = \frac{1}{\frac{4}{4}+\frac{225}{4}} \cdot -\frac{3}{8} \cdot \frac{dx}{dt}$$

$$-0.215 = \frac{\overset{1}{\cancel{4}}}{229} \cdot \frac{-3}{{}_2\cancel{8}} \cdot \frac{dx}{dt}$$

$$-0.215 = \frac{-3}{458} \frac{dx}{dt}$$

$$\boxed{32.82 \text{ m/s} = \frac{dx}{dt}}$$

A searchlight is 225 m from a straight wall. As the beam moves along the wall, the angle between the beam and the perpendicular to the wall is increasing at the rate of 1.5°/s. How fast the length of the beam increasing when is 315 m long?



$\cos\theta = \frac{225}{x}$

$\cos\theta = 225x^{-1}$

$\theta = \cos^{-1}(225x^{-1})$

$\frac{d\theta}{dt} = 1.5°/s \cdot \frac{\pi}{180}$

$\frac{d\theta}{dt} = 0.02618\, rads/s$

$$\frac{d\theta}{dt} = \frac{-1}{\sqrt{1-\left(\frac{225}{x}\right)^2}} \cdot \frac{-225}{x^2} \cdot \frac{dx}{dt}$$

$$0.02618 = \frac{-1}{\sqrt{1-\left(\frac{225}{315}\right)^2}} \cdot \frac{-225}{315^2} \cdot \frac{dx}{dt}$$

$$0.02618 = \frac{-1}{\sqrt{1-\frac{50625}{99225}}} \cdot \frac{-\cancel{225}^{\,1}}{\cancel{99225}_{\,441}} \cdot \frac{dx}{dt}$$

$$0.02618 = \frac{-1}{\sqrt{\frac{49}{49}-\frac{25}{49}}} \cdot \frac{-1}{441} \cdot \frac{dx}{dt}$$

$$0.02618 = \frac{-1}{\sqrt{\frac{24}{49}}} \cdot \frac{-1}{441} \cdot \frac{dx}{dt}$$

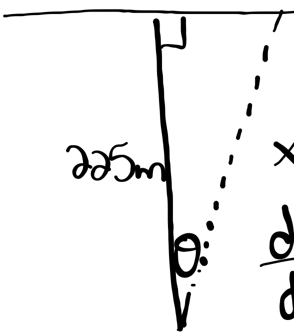
$$0.02618 = \frac{-1}{\frac{\sqrt{24}}{7}} \cdot \frac{-1}{441} \cdot \frac{dx}{dt}$$

$$0.02618 = \frac{-\cancel{7}^{\,1}}{\underbrace{\sqrt{24}}_{2\sqrt{6}}} \cdot \frac{-1}{\cancel{441}_{\,63}} \cdot \frac{dx}{dt}$$

$$0.02618 = \frac{1}{126\sqrt{6}}$$

$$\boxed{8.08\ m/s = \frac{dx}{dt}}$$

A searchlight is 225 m from a straight wall. As the beam moves along the wall, the angle between the beam and the perpendicular to the wall is increasing at the rate of 1.5°/s. How fast the length of the beam increasing when is 315 m long?



$\frac{dx}{dt} = ?$

$$\frac{d\theta}{dt} = 1.5^\circ/s$$

$$\frac{d\theta}{dt} = \underline{\underline{0.02618 \text{ rads/s}}}$$

when $x = 315$

$$\cos\theta = \frac{225}{315}$$

$$\theta = \cos^{-1}\left(\frac{225}{315}\right)$$

$$\theta = \underline{\underline{0.7752}} \text{ rads}$$

$$\cos\theta = \frac{225}{x}$$

$$\cos\theta = 225x^{-1}$$

$$-\sin\theta \frac{d\theta}{dt} = -225x^{-2}$$

$$-\sin\theta \frac{d\theta}{dt} = \frac{-225}{x^2}$$

$$\sin\theta \frac{d\theta}{dt} = \frac{225}{x^2}$$

$$\sin(0.7752) \cdot (0.02618) = \frac{225}{315^2} \frac{dx}{dt}$$

$$(0.6999)(0.02618) = \frac{225}{99225} \cdot \frac{dx}{dt}$$

$$0.018323 = 0.002268 \frac{dx}{dt}$$

$$\boxed{8.08 \text{ m/s} = \frac{dx}{dt}}$$