

Questions From Homework

$$\textcircled{1} \text{ b) } x^2 - 9x + 14 \quad \begin{array}{l} -2 \times -7 = 14 \\ -2 + -7 = -9 \end{array}$$

$$(x-7)(x-2)$$

$$\textcircled{1} \text{ f) } 6y^2 - 11y + 3 \quad \begin{array}{l} -2 \times -9 = 18 \\ -2 + 9 = -11 \end{array}$$

$$(y-\frac{2}{6})(y-\frac{9}{6})$$

$$(y-\frac{1}{3})(y-\frac{3}{2})$$

$$(3y-1)(2y-3)$$

1, 8, 27, 64, 125... perfect cubes

$$\textcircled{2} \text{ c) } t^3 + 64 \quad a^3 + b^3 = (a+b)(a^2 - ab + b^2)$$

$$(t+4)(t^2 - 4t + 16)$$

$$\text{b) } x^3 - 1 \quad a^3 - b^3 = (a-b)(a^2 + ab + b^2)$$

$$(x-1)(x^2 + x + 1)$$

$$\textcircled{3} \text{ a) } (x^3 - x^2) - 16x + 16$$

$$x^2(x-1) - 16(x-1)$$

$$(x-1)(x^2 - 16) \quad \leftarrow \text{Diff of square } a^2 - b^2 = (a-b)(a+b)$$

$$(x-1)(x+4)(x-4)$$

$$\textcircled{3} \text{ b) } x^3 + 2x^2 - 11x - 12 \quad x = -1$$

$$(-1)^3 + 2(-1)^2 - 11(-1) - 12 \quad (x+1) \text{ is a factor}$$

$$-1 + 2 + 11 - 12$$

$$0$$

$$\begin{array}{r} x^3 + 2x^2 - 11x - 12 \\ \underline{-(x^3 + x^2)} \\ x^2 - 11x - 12 \\ \underline{-(x^2 + x)} \\ -12x - 12 \\ \underline{-(-12x - 12)} \\ 0 \end{array} \quad \begin{array}{l} \frac{4}{4} x^2 = -b \\ \frac{4}{4} + -3 = 1 \end{array}$$

$$(x+1)(x^2 + x - 12)$$

$$(x+1)(x+4)(x-3)$$

Questions From Homework

$$\textcircled{1} \text{ e) } 5x^2 + 13x + 6 \quad \begin{array}{l} 10 \times 3 = 30 \\ 10 + 3 = 13 \end{array}$$

$$(x + \frac{10}{5})(x + \frac{3}{5})$$

$$(x + 2)(5x + 3)$$

$$\textcircled{1} \text{ g) } t^3 + 2t^2 - 3t$$

$$t(t^2 + 2t - 3) \quad \begin{array}{l} -1 \times 3 = -3 \\ -1 + 3 = 2 \end{array}$$

$$t(t-1)(t+3)$$

$$\textcircled{3} \text{ f) } x^6 + 8 \quad a^3 + b^3 = (a+b)(a^2 - ab + b^2)$$

$$(x^2 + 2)(x^4 - 2x^2 + 4)$$

$$\textcircled{2} \text{ g) } x^4 - 16$$

$$(x^2 + 4)(x^2 - 4)$$

$$(x^2 + 4)(x + 2)(x - 2)$$

$$\textcircled{3} \text{ b) } x^3 + 0x^2 - 7x + 6 \quad x=1$$

$$(1)^3 + 0(1)^2 - 7(1) + 6 \quad (x-1) \text{ is a factor}$$

$$1 + 0 - 7 + 6$$

$$0$$

$$\begin{array}{r|l} x-1 \sqrt{x^3 + 0x^2 - 7x + 6} & (x-1)(x^2 + x - 6) \\ \underline{-(x^3 - x^2)} & (x-1)(x+3)(x-2) \\ \hline x^2 - 7x + 6 & \\ \underline{-(x^2 - x)} & \\ \hline -6x + 6 & \\ \underline{-(-6x + 6)} & \\ \hline 0 & \end{array}$$

$$x^3 + 0x^2 - 7x + 6 \quad x=2$$

$$(2)^3 + 0(2)^2 - 7(2) + 6 \quad (x-2) \text{ is a factor}$$

$$8 + 0 - 14 + 6$$

$$0$$

$$\begin{array}{r|l} 2 \downarrow & 1 \quad 0 \quad -7 \quad 6 \\ & \underline{2 \quad 4 \quad -6} \\ & 1 \quad 2 \quad -3 \quad 0 \end{array}$$

$$(x-2)(x^2 + 2x - 3)$$

$$(x-2)(x+3)(x-1)$$

Questions From Homework

$$\textcircled{3} \text{ e) } 4x^3 + 12x^2 + 5x - 6$$

$$x = -2$$

$$4(-2)^3 + 12(-2)^2 + 5(-2) - 6$$

$$x + 2 = 0$$

$$-32 + 48 - 10 - 6 = 0$$

$$\begin{array}{r} \underline{4x^2 + 4x - 3} \\ x+2 \overline{) 4x^3 + 12x^2 + 5x - 6} \\ \underline{-(4x^3 + 8x^2)} \\ 4x^2 + 5x - 6 \\ \underline{-(4x^2 + 8x)} \\ -3x - 6 \\ \underline{-(-3x - 6)} \\ 0 \end{array}$$

$$\begin{array}{l} -2 + 6 = 4 \\ -2 \times 6 = -12 \end{array}$$

$$(x+2)(4x^2 + 4x - 3)$$

$$(x+2)(x - \frac{2}{4})(x + \frac{6}{4})$$

$$(x+2)(x - \frac{1}{2})(x + \frac{3}{2})$$

$$(x+2)(2x-1)(2x+3)$$

$$\textcircled{2} \text{ h) } r^8 - 1$$

$$x^2 - 4$$

$$(x-2)(x+2)$$

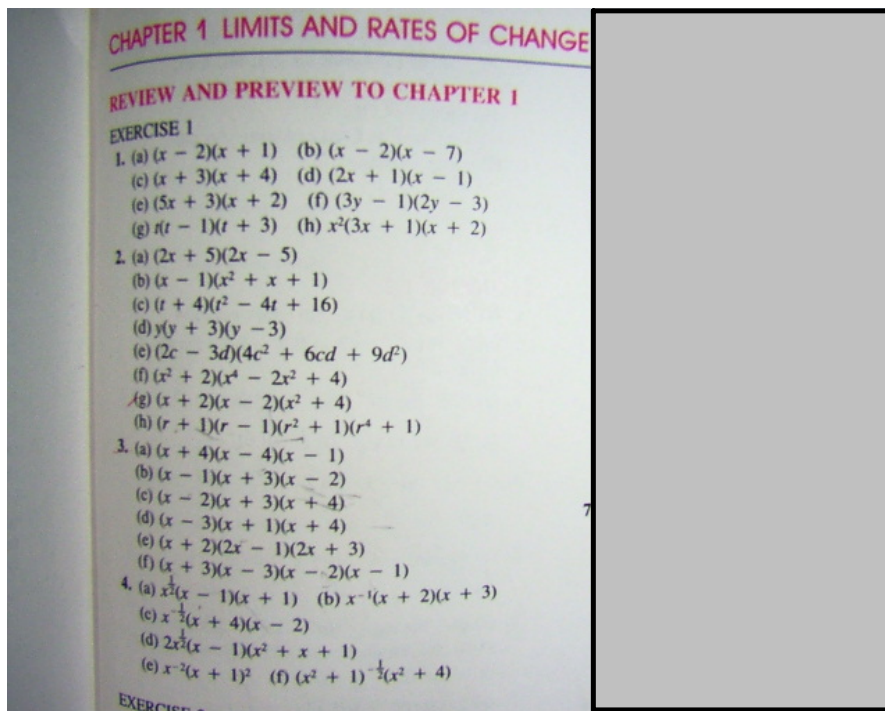
$$(r^4 - 1)(r^4 + 1)$$

$$(r^2 - 1)(r^2 + 1)(r^4 + 1)$$

$$(r-1)(r+1)(r^2+1)(r^4+1)$$

Answers to short sheet

Short Sheet 1-3



Synthetic Substitution

Factor using synthetic substitution $x^3 - 7x^2 - 4x + 28$

Find a value of x that makes it equal 0

$$\begin{aligned} (2)^3 - 7(2)^2 - 4(2) + 28 \\ 8 - 28 - 8 + 28 = 0 \end{aligned}$$

(x-value)

2

(Coefficients of the polynomial)

1	-7	-4	28
	2	-10	-28
1	-5	-14	

Bring down the first coefficient

Multiply the first coefficient by the x-value and place under the second coefficient. ADD.

Repeat the steps. The coefficients of the other factor are in the bottom row.

↓

$$(x-2)(x^2 - 5x - 14)$$

$$\begin{aligned} -7 \times 2 &= -14 \\ -7 + 2 &= -5 \end{aligned}$$

$$(x-2)(x-7)(x+2)$$

$$\begin{aligned} & (x^3 - 7x^2)(4x + 28) \\ & x^2(x-7) \cdot 4(x+7) \\ & (x-7)(x^2-4) \\ & (x-7)(x+2)(x-2) \end{aligned}$$

$$x^3 + 5x^2 - 2x - 24 \quad \text{Find a value of } x \text{ that makes it equal } 0$$

$$\begin{aligned} (-3)^3 + 5(-3)^2 - 2(-3) - 24 \\ -27 + 45 + 6 - 24 = 0 \end{aligned}$$

(x-value)

$$\underline{-3} \Big|$$



$$(x+3)(x^2+2x-8)$$

$$(x+3)(x-2)(x+4)$$

(Coefficients of the polynomial)

1	5	-2	-24
	-3	-6	24

1	2	-8
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$$\underline{-2} \times \underline{4} = -8$$

$$\underline{-2} + \underline{4} = 2$$

Homework

②① a) $x^3 - 4x^2 + x + 6 = 0$ $x = -1$ is a root

$$(-1)^3 - 4(-1)^2 + (-1) + 6 = 0$$

$$-1 - 4 - 1 + 6 = 0$$

$$0 = 0$$

③① a) $(x^3 - x^2 - 14x + 24) \div (x - 2)$

$$\begin{array}{r|rrrr} 2 & 1 & -1 & -14 & 24 \\ & & 2 & 2 & -24 \\ \hline & 1 & 1 & -12 & \end{array}$$

$$(x-2)(x^2 + x - 12)$$

$$(x-2)(x+4)(x-3)$$

Types of polynomials:

- 1 term \rightarrow monomial
- 2 terms \rightarrow binomial
- 3 terms \rightarrow trinomial
- 4 or more \rightarrow polynomial

Degree is the highest exponent

Find the product:

$$(x+2)(2x^2+3x-4)$$

$$\underline{2x^3} + \underline{3x^2} - \underline{4x} + \underline{4x^2} + \underline{6x} - 8$$

$$\boxed{2x^3 + 7x^2 + 2x - 8}$$

Factor:

$$\underline{2x^2} + \underline{7x} + \underline{6}$$

$$\begin{array}{l} \underline{3} + \underline{4} = 7 \\ \underline{3} \times \underline{4} = 12 \end{array}$$

$$(x+3)(x+4)$$

$$(2x+3)(x+2)$$

Solve:

$$\underline{2x^2} + \underline{7x} + \underline{6} = 0 \quad \begin{array}{l} \underline{3} + \underline{4} = 7 \\ \underline{3} \times \underline{4} = 12 \end{array}$$

$$(x + \underline{3})(x + \underline{4}) = 0$$

$$(2x+3)(x+2) = 0$$

$$2x+3=0 \quad | \quad x+2=0$$

$$2x = -3$$

$$\boxed{x = -2}$$

$$\boxed{x = -\frac{3}{2}}$$

Solve for y

$$-5y + 8 \geq y - 7$$

$$-5y - y \geq -7 - 8$$

$$\frac{-6y}{-6} \geq \frac{-15}{-6}$$

$$y \leq \frac{5}{2}$$

$$-5y + 8 \geq y - 7$$

$$8 + 7 \geq y + 5y$$

$$\frac{15}{6} \geq \frac{6y}{6}$$

$$\frac{5}{2} \geq y$$

$$x^4 + \underline{6}x^2 + \underline{8} = 0$$

$$(x^2 + 2)(x^2 + 4) = 0$$

$$\begin{array}{l|l} \cancel{x^2 + 2 = 0} & \cancel{x^2 + 4 = 0} \\ \cancel{x^2 = -2} & \cancel{x^2 = -4} \end{array}$$

$$\begin{array}{r} \underline{2} + \underline{4} = \underline{6} \\ \underline{2} \times \underline{4} = \underline{8} \end{array}$$

$$\begin{array}{r} 8 \\ 1 \times 8 \\ \textcircled{2 \times 4} \end{array}$$

Factoring Checklist

- ① • Try to take out a common factor
- 2 terms \rightarrow (Diff of Squares, Diff of Cubes, Sum of Cubes)
- 3 terms \rightarrow (Simple Trinomial or Hard Trinomial)
- 4 or more terms \rightarrow (Grouping)
- If nothing is working try Factor Theorem or Synthetic Substitution

ANSWERS

Factoring All Polynomials

- | | | |
|------------------------------|--------------------------|--|
| 1. $(3x+2)(3x-2)$ | 2. $(x+4)(x^2-4x+16)$ | 3. $50(2x+1)(2x-1)$ |
| 4. $7x(x+1)(x+1)$ | 5. Prime | 6. $3(x^2+27)$ |
| 7. $(2x-3)(x+1)$ | 8. $(x+5)(x-2)$ | 9. $(x+4)(x+4)$ |
| 10. $(2x-5)(2x-5)$ | 11. $(x-2)(2x-1)$ | 12. $(3x+4)(x-5)$ |
| 13. $(x^2-5)(x-3)$ | 14. $(2x+7)(2x-7)$ | 15. $(3x^2+4)(x^2-5)$ |
| 16. $(x-9)(x-9)$ | 17. $(4x+9)(4x-9)$ | 18. $(x^2+2)(2x-3)$ |
| 19. $(2x-3y)(4x^2+6xy+9y^2)$ | 20. $(x^2-3)(x+1)(x-1)$ | 21. $6ab(2x^2+ax^3-5b^2)$ |
| 22. $5a(a-5)$ | 23. $3a^3bm(ab-25m^3)$ | 24. $3(x+4)(x-4)$ |
| 25. $8(x-2)(x^2+2x+4)$ | 26. $(x+6)(x-3)$ | 27. $50(2x+1)(2x-1)$ |
| 28. $(6x^2+1)(3x+5)$ | 29. $3(x+2)(x^2-2x+4)$ | 30. $2(5x^2-1)(x-2)$ |
| 31. $(x-7)(5x+3)$ | 32. $(2x+9)(2x+1)$ | 33. $5(x^2+5)(3x-5)$ |
| 34. $(x+7)(x+8)$ | 35. $7x(2x+1)(2x-1)$ | 36. $(6x+1)(36x^2-6x+1)$ |
| 37. $(2x-7)(6x-1)$ | 38. $3(2x+7)$ | 39. $(2x^2y+3z)(2x^2y-3z)(4x^4y^2+9z^2)$ |
| 40. $(9x-1)(2x^2+3)$ | 41. $4(2x-1)(4x^2+2x+1)$ | 42. $(4x-5)(2x+5)$ |
| 43. $(x+9)(x-4)$ | 44. $2(x+6)(x+4)$ | |

Ex: $x^2 - 4$
 $(x+2)(x-2)$

$$\begin{array}{r} x^2 - 4 \\ (2)^2 - 4 \\ 4 - 4 \\ 0 \end{array}$$

$$\begin{array}{r} 2 \overline{) \quad 1 \quad 0 \quad -4} \\ \underline{\quad 2 \quad 4} \\ \quad 1 \quad 2 \quad \cancel{0} \end{array}$$

$(x-2)(x+2)$