6.1 Exploring Quadratic Relations

Quadratic Relation

A relation that can be written in the standard form $y = ax^2 + bx + c$, where $a \neq 0$.

The degree of all quadratic functions is 2. (highest exponent is 2)

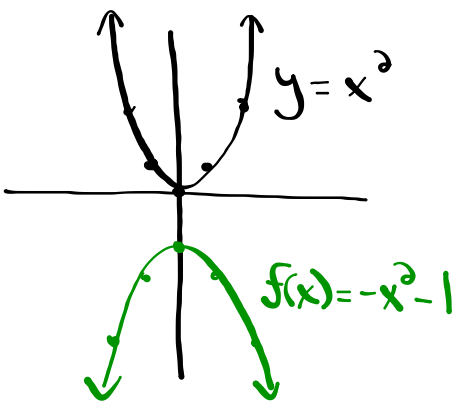
The constant term, "c", gives the value of the y-intercept for the corresponding graph.

Example of a Quadratic Relation: $y = 4x^2 + 2x + 1$

$a = 4$ $b = 2$ $c = 1$ (y-int) $(0,1)$

All quadratic functions create a parabola

degree 2



6.1 The Effect of Parameter "a"...

Consider the graphs of the following functions:

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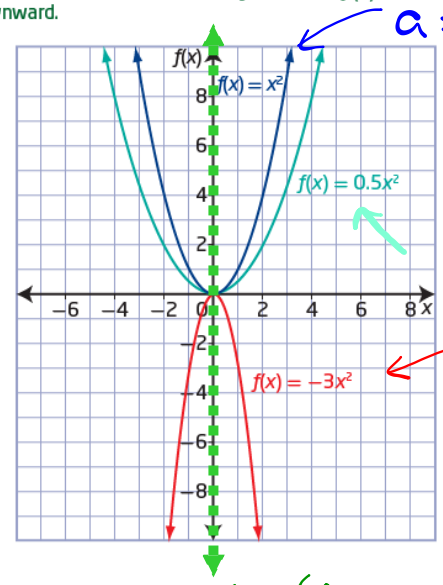
$f(x) = x^2$

$f(x) = 0.5x^2$ The parabola is wider in relation to the y-axis than $f(x) = x^2$ and opens upward.

$f(x) = -3x^2$ The parabola is narrower in relation to the y-axis than $f(x) = x^2$ and opens downward.

- Parameter a determines the orientation and shape of the parabola.



a = 1 SF = 1

parabola
The shape of the graph of any quadratic relation.

a = 0.5 SF = 0.5

a = -3
SF = |-3|
SF = 3

- Parameter "*a*" is also know as the "stretch factor" and is **ALWAYS POSITIVE.** SF = |a|
- When the sign in front of "*a*" is positive, the parabola will open upward and have a minimum point.
- When the sign in front of "*a*" is negative, the parabola will open downward and have a maximum value.
- When $a > 1$, the parabola will be stretched and become narrower.
- When $0 < a < 1$, the parabola will be compressed and become wider.

x = 0 (Axis of symmetry

- The maximum or minimum point on a parabola is otherwise know as the VERTEX.
- The equation of the imaginary line that divides a parabola in half vertically is otherwise know as the AXIS OF SYMMETRY.

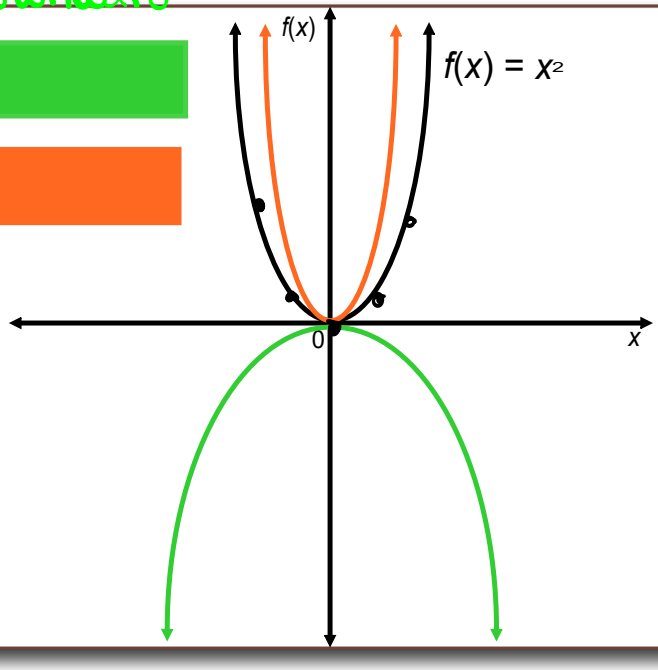
6.1 The Effect of Parameter *a* in $f(x) = ax^2$ on the Graph of $f(x) = x^2$

Match the functions and the parabolas. Highlight the functions using the coloured boxes to indicate your matching.

opens downward

$f(x) = -0.1x^2$

$f(x) = 2x^2$



Base:

$f(x) = x^2$

x	y
-2	4
-1	1
0	0
1	1
2	4

Check answer

In Summary

Key Ideas

- The degree of all quadratic functions is 2.
- The standard form of a quadratic function is

$$y = ax^2 + bx + c$$

where $a \neq 0$.

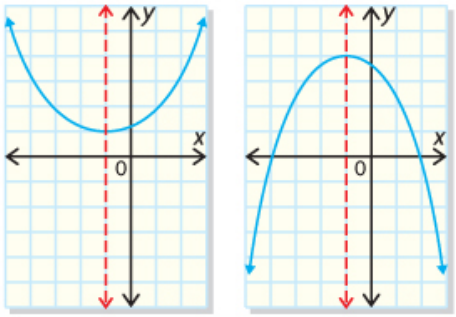
- The graph of any quadratic function is a parabola with a single vertical line of symmetry.

Need to Know

- A quadratic function that is written in standard form, $y = ax^2 + bx + c$, has the following characteristics:
  - The highest or lowest point on the graph of the quadratic function lies on its vertical line of symmetry.
  - If a is positive, the parabola opens up. If a is negative, the parabola opens down.

positive negative

$a > 0$ $a < 0$



  - Changing the value of b changes the location of the parabola's line of symmetry.
  - The constant term, c, is the value of the parabola's y-intercept.

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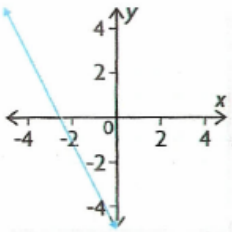
Questions 1, 2, 3, 4, 5, 6

SOLUTIONS => 6.1 Exploring Quadratic Relations

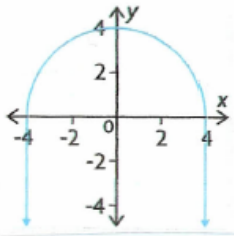
1. Which graphs appear to represent quadratic relations? Explain.

a)



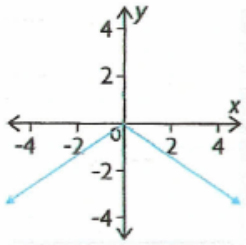
Not a quadratic relation.

b)



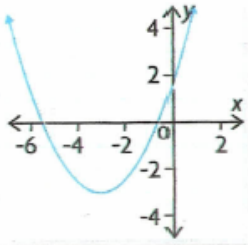
Not a quadratic relation.

c)



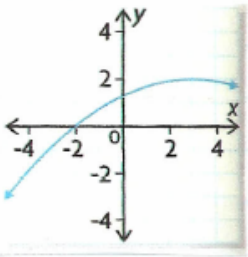
Not a quadratic relation.

d)



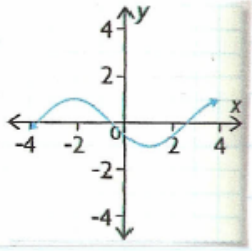
Quadratic relation

e)



Quadratic relation.

f)



Not a quadratic relation.

2. Which of the following relations are quadratic? Explain

a) $y = 2x - 7$

↳ Not a quadratic relation (degree 1)

b) $y = 2x(x+3)$

$y = 2x^2 + 6x$

↳ Quadratic relation (degree 2)

c) $y = (x+4)^2 + 1$

$y = (x+4)(x+4) + 1$

$y = x^2 + 4x + 4x + 16 + 1$

$y = x^2 + 8x + 17$

↳ Quadratic relation (degree 2)

d) $y = x^2 - 5x - 6$

↳ Quadratic relation (degree 2)

e) $y = 4x^3 + x^2 - x$

↳ Not a quadratic relation (degree 3)

f) $y = x(x+1)^2 - 7$

$y = x(x+1)(x+1) - 7$

$y = (x^2 + 1x)(x+1) - 7$

$y = x^3 + x^2 + x^2 + x - 7$

$y = x^3 + 2x^2 + x - 7$

↳ Not a quadratic relation (degree 3)

3. State the y-intercept for each quadratic relation in question 2.

b) $y = 2x^2 + 6x + \underset{\uparrow c}{0}$ y-intercept $\Rightarrow 0$

c) $y = x^2 + 8x + \underset{\uparrow c}{17}$ y-intercept $\Rightarrow 17$

d) $y = x^2 - 5x - \underset{\uparrow c}{6}$ y-intercept $\Rightarrow -6$

4. Explain why the condition $a \neq 0$ must be stated when defining the standard form, $y = ax^2 + bx + c$.

If $a = 0$, $y = ax^2 + bx + c$ would become $y = bx + c$ which is a linear relation (degree 1) and not a quadratic relation (degree 2).

5. Each of the following quadratic functions can be represented by a parabola. Does the parabola open up or down? Explain how you know.

a) $y = x^2 - 4$

Since the sign in from of the stretch factor ("a"), is positive, the parabola will open upward.

b) $y = -2x^2 + 6x.$

Since the sign in from of the stretch factor ("a"), is negative, the parabola will open downward.

c) $y = 9 - x + 3x^2$
$y = 3x^2 - x + 9$

Since the sign in from of the stretch factor ("a"), is positive, the parabola will open upward.

d) $y = -\frac{2}{3}x^2 - 6x + 1$

Since the sign in from of the stretch factor ("a"), is negative, the parabola will open downward.

6. Each table of values lists points in a quadratic relation. Decide without graphing, the direction in which the parabola opens.

a)

x	−4	−3	−2	−1	0	1
y	12	5	0	−3	−4	−3

Parabola opens upward.

b)

x	0	1	2	3	4	5
y	−13	−3	3	5	3	−3

Parabola opens downward.

c)

x	−5	−4	−3	−2	−1	0
y	3.0	−0.5	−3.0	−4.5	−5.0	−4.5

Parabola opens upward.

d)

x	0	1	2	3	4	5
y	−4	19	40	59	76	91

Parabola opens downward.

Attachments

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